Dimensioning Field-Grading Materials in Cable Joints by Adjoint Transient Finite-Element Sensitivity Analysis

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An adjoint sensitivity analysis of a nonlinear transient finite-element model allows to determine the sensitivities of the key performance indicators of a cable joint with respect to the parameters of an embedded field-grading material. The example considers a high-voltage DC cable joint submitted to an overvoltage event.

Keywords:
sensitivity analysis,
adjoint method,
finite element method,
electrothermal simulation,
HVDC cable joint

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I Introduction

In high-voltage equipment, field grading materials (FGMs) are applied to mitigate spots with high electric field strengths. The field and temperature dependencies of FGMs can be represented by the expression

$$\sigma_{\mathrm{FGM}}(E,\vartheta) = p_1 \frac{1+p_4^{(E-p_2)/p_2}}{1+p_4^{(E-p_3)/p_2}} e^{-p_5\left(\frac{1}{\vartheta}-\frac{1}{\vartheta_0}\right)} , \tag{1}$$

for the FGM's conductivity σ_{FGM}, with E the magnitude of the electric field strength $\boldsymbol{E}(\boldsymbol{r},t)$, $\vartheta(\boldsymbol{r},t)$ the temperature, ϑ_0 a reference temperature and $\{p_1,\dots,p_5\}$ a set of design parameters shaping the material characteristic [1]. Since recently, FGMs can be tailored to the specific design requirements of the HV device [2]. This, however, requires (a) calculating the behaviour of the HV device under transient stresses and (b) calculating the sensitivities of the quantities of interest (QoIs) with respect to a larger number of design parameters characterising the FGM.

II Transient Electrothermal Simulation

The transient excitation and the field and temperature dependence of FGMs necessitates a transient nonlinear multiphysically-coupled finite-element (FE) simulation for determining the nominal electrothermal fields. Adapting the electroquasistatic approximation of the Maxwell equations, the electric field strength is represented by $\boldsymbol{E} = -\nabla\Phi$ with $\Phi(\boldsymbol{r},t)$ the electric scalar potential. The formulation reads

$$-\nabla\cdot(\sigma\nabla\Phi) - \nabla\cdot\frac{\partial}{\partial t}(\varepsilon\nabla\Phi) = 0\,; \tag{2a}$$

$$-\nabla\cdot(\lambda\nabla\vartheta) + \frac{\partial}{\partial t}(c_{\mathrm{V}}\vartheta) = \dot{q}_{\mathrm{Joule}}\,, \tag{2b}$$

with $\varepsilon(\boldsymbol{r},\boldsymbol{E},\vartheta)$ the permittivity, $\lambda(\boldsymbol{r},\vartheta)$ the thermal conductivity, $c_{\mathrm{V}}(\boldsymbol{r},\vartheta)$ the volumetric heat capacity and $\dot{q}_{\mathrm{Joule}} = \sigma E^2$ the Joule loss density. Eq. (2) is discretised in space by lowest-order nodal FE shape functions and in time by the backward Euler method. To cope with the substantially different time scales of both subproblems, a weakly coupled multirate timestepping scheme is employed [3] (Fig. 1).

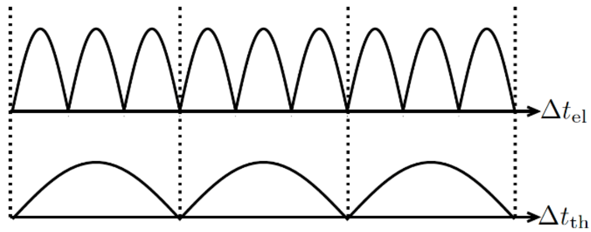

Figure 1: Multirate weakly coupled time-integration scheme: the electroquasistatic subproblem is time-stepped by small time steps Δt_{el}, whereas the thermal subproblem is time-stepped by larger time steps Δt_{th}.

The outcome of the nominal simulation is a set of N_{qoi} QoIs q_i, which are post-processed from (Φ, ϑ) on the computational domain Ω and for the time span $[t_0, t_{\mathrm{f}}]$ by

$$q_i = \int_\Omega \int_{t_0}^{t_{\mathrm{f}}} g_i\,(r, t, \Phi, \vartheta)\mathrm{d}\Omega\mathrm{d}t. \tag{3}$$

Here, the kernels $g_i(r, t, \Phi, \vartheta)$ allow to represent space- or time-integrated QoIs as well as localised or instantaneous QoIs. As examples, the time variation of E at position $\boldsymbol{r}_1$ is extracted by $g_1 = |-\nabla\Phi|\delta(\boldsymbol{r} - \boldsymbol{r}_1)$, whereas the *z*-component of the electric field strength on a line (r_2, φ_2, z), $z \in [z_a, z_b]$ at time instant t_2 is extracted by $g_2 = -\nabla\Phi \cdot \boldsymbol{e}_z \delta(r - r_2)\delta(\varphi - \varphi_2)H(z - z_a)H(z_b - z)\delta(t - t_2)$, with $\delta(s)$ the Dirac-delta function and $H(s)$ the Heaviside step function.

III Adjoint Sensitivity Analysis

In addition to the nominal QoIs q_i, engineers need to know the sensitivities $\frac{\mathrm{d}q_i}{\mathrm{d}p_j}$ of the QoIs with respect to N_{par} design parameters p_j. In a direct sensitivity method, such sensitivities would be post-processed from the derived solutions $\left(\frac{\mathrm{d}\Phi}{\mathrm{d}p_j}, \frac{\mathrm{d}\vartheta}{\mathrm{d}p_j}\right)$, which on their turn are computed from FE solutions of (2) differentiated with respect to each of the design parameters p_j. This becomes prohibitely expensive for large N_{par}, which is the case here, with already 5 parameters for each FGM in addition to the device's geometric parameters.

For a moderate N_{qoi}, the adjoint sensitivity method is preferred [4], [5]. For each kernel g_i, an adjoint solution (η_i, ξ_i) is solved from the adjoint transient linear multiphysically-coupled FE problem

$$-\nabla \cdot (\bar{\bar{\sigma}}_{\mathrm{d}} \nabla \eta_i) + \nabla \cdot \left(\bar{\bar{\varepsilon}}_{\mathrm{d}} \frac{\partial}{\partial t} \nabla \eta_i\right) + \nabla \cdot \left((\bar{\bar{\sigma}}_{\mathrm{d}} \boldsymbol{E} + \boldsymbol{J})\xi_i\right) = \frac{\partial g_i}{\partial \Phi} \; ; \tag{4}$$

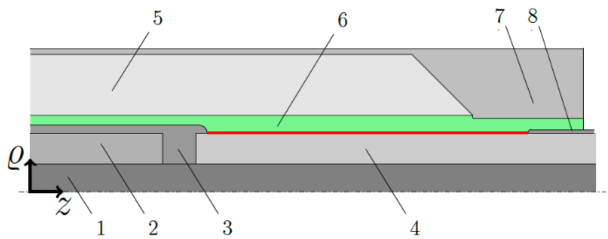

Figure 2: Cross section of a HVDC cable joint with copper conductor (1), aluminum connector (2), conductive silicone rubber (3), cross-linked polyethylene (4), insulating silicone rubber (5), nonlinear field-grading material (6), outer aluminum body (7) and outer cable semiconductor (8).

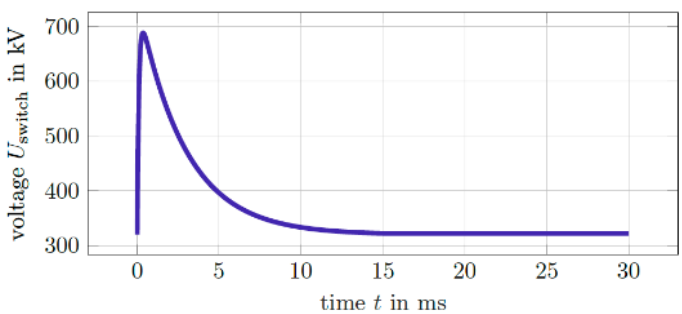

Figure 3: Switching impulse applied to the HVDC cable joint.

$$\begin{aligned} -\nabla \cdot (\lambda \nabla \xi_i) - \left(c_{\mathrm{V}} + \frac{\partial c_{\mathrm{V}}}{\partial \vartheta} \vartheta\right) \frac{\partial \xi_i}{\partial t} + \frac{\partial \lambda}{\partial \vartheta} \nabla \vartheta \cdot \nabla \xi_i \\ - \frac{\partial \sigma}{\partial \vartheta} E^2 \xi_i - \frac{\partial \sigma}{\partial \vartheta} \boldsymbol{E} \cdot \nabla \eta_i + \frac{\partial \varepsilon}{\partial \vartheta} \boldsymbol{E} \cdot \frac{\partial}{\partial t} \nabla \eta_i = \frac{\partial g_i}{\partial \vartheta} \, , \end{aligned} \tag{5}$$

where $\bar{\bar{\sigma}}_{\mathrm{d}}$ and $\bar{\bar{\varepsilon}}_{\mathrm{d}}$ the differential conductivity and permittivity [6], evaluated for the nominal solution (Φ_0, ϑ_0) [7]. The adjoint formulation steps backward in time and considers the nonlinear operation points that were already determined by the nominal solver.

IV Example: HVDC Cable Joint

The nominal and adjoint transient electrothermal analysis tools are applied to a HVDC cable joint (Fig. 2) submitted to a transient overvoltage event (Fig. 3). The nominal solution illustrates the field-grading effect (Fig. 4), i.e., the magnitude of the field stays within an acceptable range, despite the heating of the FGM during the overvoltage event. The adjoint solutions allow to determine the sensitivities of the Joule heat $\dot{q}_{\text{Joule}}$ ($g_{\text{Joule}} = \boldsymbol{J} \cdot \boldsymbol{E}$) generated during the overvoltage event with respect to the design parameters of the FGM (Fig. 5, dashed lines). For validation, $\dot{q}_{\text{Joule}}(p_1, p_2)$ has also been calculated by a standard parameter study (Fig. 5, solid lines). The multirate scheme allows to choose a coarse thermal time step (Fig. 6).

V Conclusions

Tuning the properties of a field-grading material, employed within the insulation system of a high-voltage devices, is possible with adjoint sensitivity analysis exerted on the nonlinear transient electrothermal FE model of the device.

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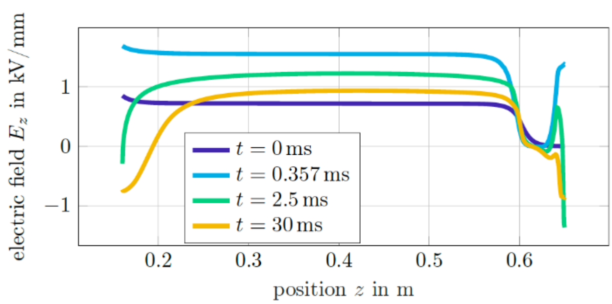

Figure 4: Electric field strength in the FGM, tangentially to the interface between FGM and XLPE (along the red line in Fig. 2).

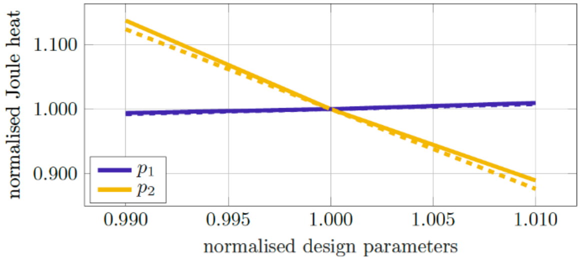

Figure 5: Sensitivities $\frac{\partial \dot{q}_{\mathrm{Joule}}}{\partial p_1}$ and $\frac{\partial \dot{q}_{\mathrm{Joule}}}{\partial p_2}$ calculated by the adjoint sensitivity method, shown as dashed lines at the nominal point $(p_{\mathrm{nom},1}, p_{\mathrm{nom},2})$, which is normalised to (1,1); Dependencies $\dot{q}_{\mathrm{Joule}}(p_1, p_{\mathrm{nom},2})$, and $\dot{q}_{\mathrm{Joule}}(p_{\mathrm{nom},1}, p_2)$ calculated by parameter variations, shown as solid lines, used for validation.

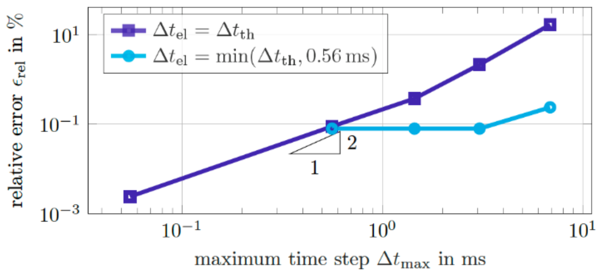

Figure 6: Convergence of the solution with respect to the time-step size: identical time step for both subproblems (dark blue line); time step for the electroquasistatic subproblem limited to 0.56 ms (light blue line).

References

[1] R. Hussain and V. Hinrichsen, "Simulation of thermal behavior of a 320 kV HVDC cable joint with nonlinear resistive field grading under impulse voltage stress," in CIGR´E Winnipeg 2017 Colloquium, 2017.

[2] M. Secklehner, R. Hussain, and V. Hinrichsen, "Tailoring of new field grading materials for HVDC systems," in 13th International Electrical Insulation Conference (INSUCON 2017), 05 2017.

[3] Y. Späck-Leigsnering, E. Gjonaj, H. De Gersem, T. Weiland, M. Gießel, and V. Hinrichsen, "Electroquasistatic-thermal modeling and simulation of station class surge arresters," IEEE Trans. Magn., vol. 52, no. 3, p. 9100104, 03 2016.

[4] S. Li and L. R. Petzold, "Adjoint sensitivity analysis for time-dependent partial differential equations with adaptive mesh refinement," J. Comput. Phys., vol. 198, pp. 310–325, 2004.

[5] J. Sarpe, A. Klaedtke, and H. De Gersem, "A parallel-in-time adjoint sensitivity analysis for a B6 bridge-motor suppy circuit," IEEE Trans. Magn., vol. 0, no. 0, pp. 0–0, 11 2023.

[6] H. De Gersem, I. Munteanu, and T. Weiland, "Construction of differential material matrices for the orthogonal finite-integration technique with nonlinear materials," IEEE Trans. Magn., vol. 44, no. 6, pp. 710–713, 06 2008.

[7] M. G. Ruppert, Y. Späck-Leigsnering, J. Buschbaum, and H. De Gersem, "Adjoint variable method for transient nonlinear electroquasistatic problems," Electr. Eng., vol. 105, no. 4, pp. 2319–2325, 08 2023.