Modelling of Nonlinear Magnetic Properties of Current Transformers with Piecewise Bézier Curves

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The magnetic properties of iron cores of contemporary electrical devices, such as current transformers, are highly nonlinear. This work proposes a simple modelling method for the nonlinear magnetic properties using piecewise Bézier curves. An anhysteretic curve is modelled with three measured points. Two second order Bézier curves are joined to form a composite curve based on those measured points. The modelled curve was then compared with a measured anhysteretic curve of a commercial electrical steel sheet. The presented results confirm that this approach with piecewise Bézier curves shows high potential to model magnetization curves.

Keywords:
anhysteretic curve,
Bézier curve,
current transformer,
differential evolution,
magnetic properties

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I Introduction

Current transformers (CT) represent an important element of contemporary electrical power systems. Their distorted secondary quantities have a high influence on the performance of protection relays. Therefore, the precise modelling of the highly nonlinear magnetic properties of CT's iron cores represents a very important topic that needs to be addressed. A universal approach for description of the nonlinear magnetic properties is yet to be found [1]. For that reason, many different methods have been proposed. Those methods are mostly based on simple analytic functions [2]. Recently, an approach based on curves instead of functions was presented [3].

The approaches based on curves offer more flexibility to describe the nonlinear magnetic properties. Classical Bézier curves were already used to approximate magnetization curves with high accuracy [3]. The classical Bézier curves are used for simple smooth approximations. For the description of complex shapes, classical Bézier curves can be joined into a composite curve. This approach is called piecewise (PW) Bézier curves. An approach with PW Bézier curves was, e.g., successfully used to approximate the characteristics of a PV cell [4].

In most cases, the magnetic properties of CTs are represented using limited measured data. This work proposes a simple method to model the nonlinear magnetic curves using PW Bézier curves when limited data is available. The ability to model magnetization curves was in this preliminary study presented on a measured anhysteretic curve of a non-oriented (NO) electrical steel sheet. This method is based on three measured points, i.e., the origin of the coordinate system, the saturation point, and an intermediate point on the curve. To model the anhysteretic curve two Bézier curves of second order were connected to create the modelled curve.

This work is divided into four sections. Section II presents the proposed modelling method for nonlinear magnetization curves based on PW Bézier curves. Section III contains the preliminary results of the research. A measured anhysteretic curve of a NO steel sheet was compared with the modelled curve. Finally, the concluding remarks are given in Section IV.

II Modelling of magnetic properties with PieceWise Bézier Curves

A. Bézier curves

Classical Bézier curves are parametric curves defined by their order n, a set of $n + 1$ control points $\mathrm{P_i}(x_\mathrm{i}, y_\mathrm{i})$ $(i = 0, \ldots, n)$ and parameter t ranging between 0 and 1 which are the start- and end-point of the curve respectively. Bézier curves are expressed by (1) [3]

$$B_*(t) = \sum_{i=0}^{n} b_{i,\mathrm{n}}(t) *_i\,, \;\; 0 \le t \le 1, \tag{1}$$

where $b_{i,\mathrm{n}}(t)$ are the Bernstein basis polynomials. Two polynomials must be calculated to define a Bézier curve, i.e., $B(t) = \left(B_\mathrm{x}(t), B_\mathrm{y}(t)\right)$. Therefore, $*$ is a placeholder for x and y depending on the control points' coordinates that are used. Bézier curves have two important properties that are exploited in engineering applications:

- Property 1: The first and last control points of a Bézier curve are interpolated by the curve.
- Property 2: The tangents formed by the first two and the last two control points define the initial and ending directions respectively [3].

B. Definition of variables

The anhysteretic curve was modelled using a PW Bézier curve in this paper. Two second order ($n = 2$) Bézier curves were joined to create one final curve. Fig. 1 presents an arbitrary curve formed by connecting two second order Bézier curves, i.e., curve $B_1(t) = \left(B_{\mathrm{x},1}(t), B_{\mathrm{y},1}(t)\right)$ and curve $B_2(t) = \left(B_{\mathrm{x},2}(t), B_{\mathrm{y},2}(t)\right)$. The first curve $B_1(t)$ was defined by control points $\mathrm{P_0}, \mathrm{P_1}$, and $\mathrm{P_2}$ and the second curve $B_2(t)$ by control points $\mathrm{P_2}, \mathrm{P_3}$, and $\mathrm{P_4}$ as shown in Fig. 1.

We assumed that three measured points of the anhysteretic curve are known, i.e., the origin $(0{,}0)$, the saturation point, and an intermediate point on the curve. The values of control points $\mathrm{P_0}$, $\mathrm{P_4}$ and $\mathrm{P_2}$ were set to be equal to those three measured points respectively.

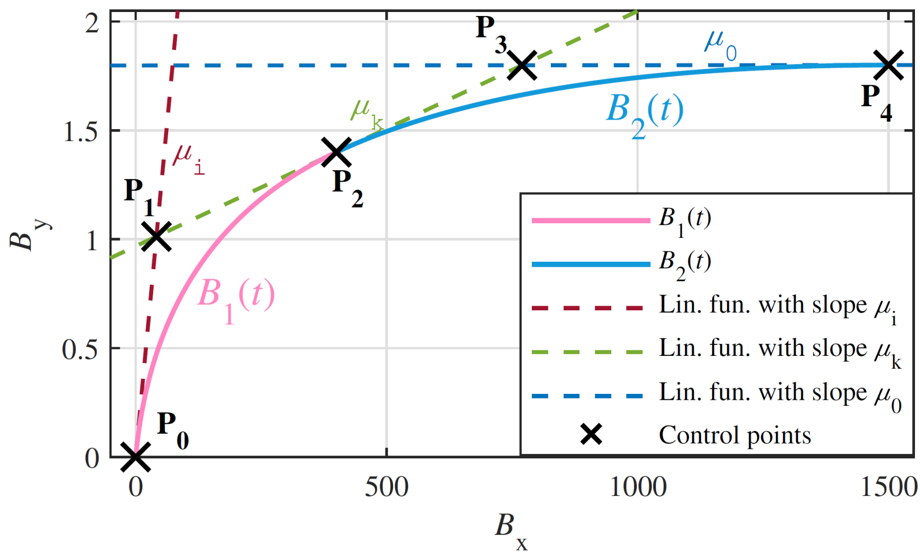

Figure 1: Definition of variables for the modelling approach with a PW Bézier curve.

The positions of the remaining control points P_1 and P_3 were uniquely defined by three permeabilities, i.e., μ_i, μ_k, and μ_0. The slope of the magnetization curve in the saturation region is known and equal to the relative permeability in vacuum, i.e., $\mu_0 = 4\pi \cdot 10^{-7}$ Vs/Am. The anhysteretic curve is the steepest in the low-field region. Consequently, the slope of the curve is here the largest and is defined with the initial permeability μ_i. Permeability μ_k defines the slope of the tangent line in control point P_2. Linear functions with arbitrary chosen μ_i and μ_k are presented in Fig. 1.

C. Parametric continuity rules

To ensure a curve with physical properties, the so-called parametric continuity rules were incorporated in the modelling process. They are defined as:

- C^0 continuity: curves intersect at one end point,
- C^1 continuity: equality of first derivatives at the intersecting point, and
- C^2 continuity: equality of second derivatives at the intersecting point.

Continuity C^0 was included by exploiting Property 1 where control point P_2 represented the last point of $B_1(t)$ and the first point of $B_2(t)$ (2)

$$B_1(t = 1) = B_2(t = 0) = \mathrm{P}_2\,. \tag{2}$$

Furthermore, continuity C^1 was considered by forcing control points $\mathrm{P}_1, \mathrm{P}_2$ and P_3 to lie on the same linear function with slope μ_k (3) as presented in Fig. 1.

$$\frac{\mathrm{d}B_{\mathrm{y},1}}{\mathrm{d}B_{\mathrm{x},1}}(t = 1) = \frac{\mathrm{d}B_{\mathrm{y},2}}{\mathrm{d}B_{\mathrm{x},2}}(t = 0) = \mu_\mathrm{k} \tag{3}$$

Continuity C^2 was used to express the objective function for the optimization process.

D. Methodology

Property 2 was exploited to prescribe the slope at the origin P_0 to μ_i and the slope at which the saturation point P_4 is reached to μ_0. This was achieved by placing P_1 to form a linear function with P_0 with slope μ_i. Similarly, P_3 was placed in such a way to form a linear function with P_4 with slope μ_0. Additionally, control points P_1, P_2 and P_3 must lie on the same linear function (here defined with μ_k) to obey the C^1 continuity rule. Finally, the intersection of linear functions with slopes μ_i and μ_k defined the position of P_1, and the intersection of linear functions with slopes μ_k and μ_0 defined the position of P_3. The determination of the positions of P_1 and P_3 is graphically depicted in Fig. 1.

However, permeabilities μ_i and μ_k are in most cases not known a priori. For this reason, we used the differential evolution (DE) algorithm to vary the values of slopes μ_i and μ_k and minimize the objective function (4).

$$F = \left(\frac{\mathrm{d}^2B_{\mathrm{y},1}}{\mathrm{d}B_{\mathrm{x},1}^2}(t = 1, \mu_\mathrm{i}, \mu_\mathrm{k}) - \frac{\mathrm{d}^2B_{\mathrm{y},2}}{\mathrm{d}B_{\mathrm{x},2}^2}(t = 0, \mu_\mathrm{i}, \mu_\mathrm{k})\right)^2 \tag{4}$$

The objective function (4) is equal to the square of the difference between the second derivatives of the second order Bézier curves in the joint point P_2. The constraints during the calculation were:

- $\mu_\mathrm{i} > 0$ and $\mu_\mathrm{k} > 0$;

- $x_0 < x_1 < x_2$ and $x_2 < x_3 < x_4$;
- $y_0 < y_1 < y_2$ and $y_2 < y_3 < y_4$.

III Results

To validate the proposed modelling approach, we measured the major loop of a NO27 steel sheet. The anhysteretic curve was then calculated as the mean value between the ascending and descending branches of the major loop [3]. Fig. 2 presents the measured and modelled anhysteretic curve of the NO27 steel sheet.

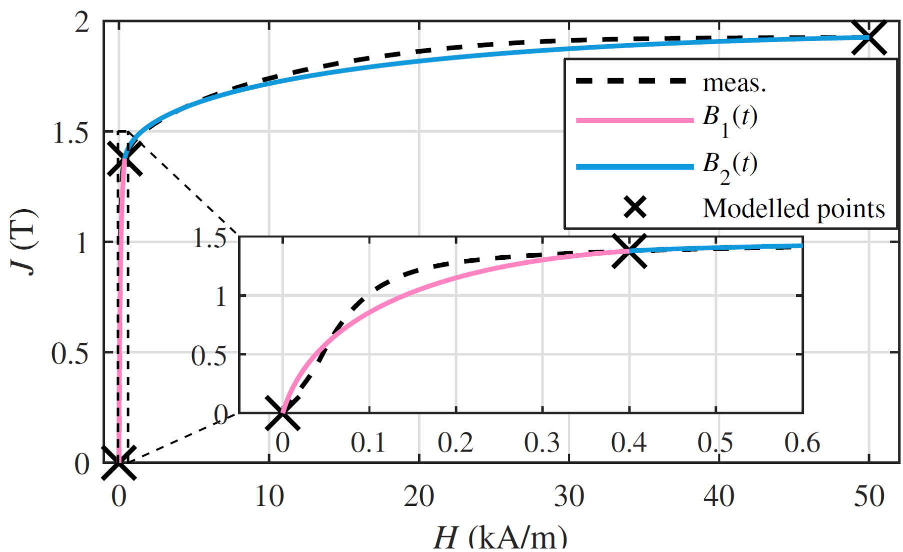

Figure 2: Modelled anhysteretic curve of a NO27 steel sheet with two joint second order Bézier curves.

The choice of the origin and saturation point is straightforward whereas the intermediate point was determined by varying its position along the measured curve. For each of those sets the PW Bézier curve was constructed and the NRMS error ε (Eq. (16) in [3]) was calculated. The set where the NRMS error was the lowest is plotted in Fig. 2, i.e., $\mathrm{P}_0(0{,}0)$, $\mathrm{P}_2(400{,}1.3765)$, $\mathrm{P}_4(50000{,}1.9252)$, $\mu_\mathrm{i} = 19753$, and $\mu_\mathrm{k} = 387.8757$.

The calculated NRMS deviation ε for the specific case in Fig. 2 was $\varepsilon = 0.015768$. Overall, the NRMS error ranged from 0.015768 to 0.0171494. This result is better than the results achieved by approximating measured data with most analytic functions presented in [3]. Compared with approximations with classical Bézier curves, the PW approach had slightly worse results than the sixth order Bézier curve [3]. This implies that the PW approach has high potential in the field of modelling magnetization curves when limited data is available.

IV Conclusions

Based on the presented results for a measured anhysteretic curve we concluded that the PW Bézier curves offer a simple way to model magnetization curves. This method needs less computational power than classical Bézier curves with comparable accuracy. However, the main challenge with Bézier curves remains to find a connection between the underlying physics and the parameter t.

The full paper will be extended with the application of the PW Bézier curves within a dynamic model of a CT with different iron cores. The PW Bézier curves will be validated on measurements performed on a CT.

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