Comparison of Simple Modeling Approaches of the Nonlinear Magnetic Properties of a Single-Phase Transformer

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This paper analyses the potential of several analytic functions to describe the nonlinear magnetic properties of a single phase transformer's iron core. A dynamic model of a single phase transformer with analytic functions describing magnetic properties was prepared in Simulink and used to calculate the inrush currents. The parameters of the chosen analytic functions were determined using differential evolution by minimizing the deviation between measured and calculated inrush currents in the time and frequency domains simultaneously. The ranking of the applied analytic functions based on the lowest deviation between measured and calculated currents is presented.

Keywords:
single-phase transformer,
nonlinear magnetic
properties,
analytic functions,
differential evolution,
inrush currents

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I Introduction

Transformers are commonly used devices in power systems and electronics applications. Because of their widespread use, it is essential to reliably model the transformer's behavior. The most complex task in the modeling approach is the description of the nonlinear magnetic properties of the iron core.

There are many different methods to represent the nonlinear magnetic properties of the iron core. A detailed description of the nonlinear magnetic properties requires the implementation of a hysteresis model, e.g., the well-known Preisach, Jiles-Atherton (JA) or Zirka-Moroz hysteresis models [1]. The implementation of such models is usually not straightforward, and non-standardized measurements are necessary for the parameter identification process in some cases. For this reason, researchers often use simplified descriptions of the iron cores of electrical machines. The simplest approach to model the nonlinear magnetic properties is using the single-valued history-independent anhysteretic curve. The anhysteretic curve is the mean value between the descending and ascending branches of the hysteresis major loop and represents the input for several finite-element tools.

In this research, we focused on a simple representation of the nonlinear magnetic properties of a transformer's iron core with analytic functions, e.g., the sigmoid functions. This approach is equal to the representation of the nonlinear magnetic properties with an anhysteretic curve. Recently, sigmoid functions were successfully used to model and approximate various magnetization curves [2], [3]. Their main feature is that they describe a curve in the shape of the letter »S«. In addition to sigmoid functions, we included a sum of exponential functions proposed in [4] in the analysis.

This paper contains five sections. In Section II a dynamic model of a single-phase transformer is described. Section III contains information about the methodology used in this research, i.e., the description of chosen analytic functions, determination of their parameters and a method for the evaluation of the obtained results. Results and concluding remarks are presented in Section IV and Section V, respectively.

II Single-phase transformer

Voltage balances in the primary and secondary windings are given by Eqs. (1) and (2), which represents the dynamic model of a single-phase transformer [5].

$$u_1 = i_1 R_1 + L_{\sigma 1}\frac{\mathrm{d}i_1}{\mathrm{d}t} + N_1 A \frac{\mathrm{d}B}{\mathrm{d}H}\left(\frac{N_1}{l}\frac{\mathrm{d}i_1}{\mathrm{d}t} + \frac{N_2}{l}\frac{\mathrm{d}i_2}{\mathrm{d}t}\right) \quad (1)$$

$$u_2 = i_2 R_2 + L_{\sigma 2}\frac{\mathrm{d}i_2}{\mathrm{d}t} + N_2 A \frac{\mathrm{d}B}{\mathrm{d}H}\left(\frac{N_1}{l}\frac{\mathrm{d}i_1}{\mathrm{d}t} + \frac{N_2}{l}\frac{\mathrm{d}i_2}{\mathrm{d}t}\right) \quad (2)$$

In Eqs. (1) and (2) u_1 and u_2 are the primary and secondary voltages, i_1 and i_2 are the primary and secondary currents. The magnetic nonlinear properties of the transformer's iron core are described by the term $\mathrm{d}B/\mathrm{d}H$, where B is the magnetic field density and H is the magnetic field strength. The definitions of the remaining parameters and their values for the transformer used in the analysis are given in Table 1.

Table 1: Single-phase transformer

Parameter	Value
Resistance of the primary winding R_1 (Ω)	11
Resistance of the secondary winding R_2 (Ω)	141.8
Leakage inductance of the primary winding $L_{\sigma 1}$ (mH)	32.97
Leakage inductance of the secondary winding $L_{\sigma 2}$ (mH)	32.97
Number of turns of the primary winding N_1	452
Number of turns of the secondary winding N_2	1722
Cross section area of the iron core A (m^2)	0.0012
Mean path length of the magnetic flux l (m)	0.308

III Methodology

A. Analytic functions

The magnetic properties $\mathrm{d}B/\mathrm{d}H$ are modeled using analytic functions. The analytic functions describe the nonlinear magnetic properties as a $B(H)$ relation. Such a choice is not coincidental, since these functions enable a straightforward calculation of their derivative, i.e., the derivative $\mathrm{d}B/\mathrm{d}H$. We decided to analyze the Langevin, Gompertz, Hyperbolic tangent, Algebraic, Logistic, Sigmoid, Elliot functions [2] and a sum of exponential functions (labeled exponential function) [4]. The derivatives of chosen analytic functions $\mathrm{d}B/\mathrm{d}H$ can be implemented directly in the dynamic model of a single-phase transformer.

B. Determination of function parameters

We used the Differential Evolution (DE) algorithm for the calculation of the parameters P_1–P_5 of the derivatives of sigmoid functions [2] and the parameters $\mathrm{C}_1, \mathrm{D}_1, \mathrm{C}_2, \mathrm{D}_2$ of the derivative of the exponential function [4]. The number of population members in the DE algorithm was set to 20, the mutation factor was 0.7, the crossover factor 0.5 and number of iterations 200. In each iteration, we solved Eqs. (1) and (2) using Matlab/Simulink and obtained the primary and secondary voltages and currents of the transformer.

The goal of the analysis was to achieve the smallest difference between the measured and the calculated current by the transformer model. We minimized the deviations from measurements and calculations in both, the time and frequency domain. Therefore, the objective function q (3) is the sum of two distinct parts, i.e., the mean square difference between calculated i_1 and measured $i_{1,\mathrm{m}}$ currents q_t and the mean square differences between the individual harmonic components q_f [4].

$$q = q_\mathrm{t} + q_\mathrm{f} \tag{3}$$

Equations for q_t and q_f are omitted due to the lack of space. They are thoroughly described in [4].

C. Evaluation of results

To evaluate the goodness of fit of the calculated currents, we applied the measure ε [5], defined by Eq. (4).

$$\varepsilon = \sqrt{\sum_{k=1}^{N} \frac{\left(i_{1,\mathrm{k}} - i_{1,\mathrm{m,k}}\right)^2}{N}} \tag{4}$$

In Eq. (4), $i_{1,\mathrm{k}}$ is the calculated primary current, $i_{1,\mathrm{m,k}}$ is the measured primary current of the transformer, N is the number of measured samples of $i_{1,\mathrm{m}}$ and $k = 1,2,\dots,N$.

IV Results

Measurements were conducted on a single-phase transformer with parameters given in Table 1. The inrush current $i_{1,\mathrm{m}}$ obtained in the no-load test was crucial for the analysis of the potential of analytic functions. The frequency of the primary and secondary quantities was 50 Hz. The measured primary voltage $u_{1,\mathrm{m}}$ and measured primary current $i_{1,\mathrm{m}}$ are shown in Fig. 1a) and b), respectively. The measured primary voltage $u_{1,\mathrm{m}}$ in Fig. 1a) was used as the input for the transformer's dynamic model. We calculated the parameters of the derivatives of the analytic functions and evaluated their ability to describe the inrush current of a single-phase transformer based on the deviation from $i_{1,\mathrm{m}}$.

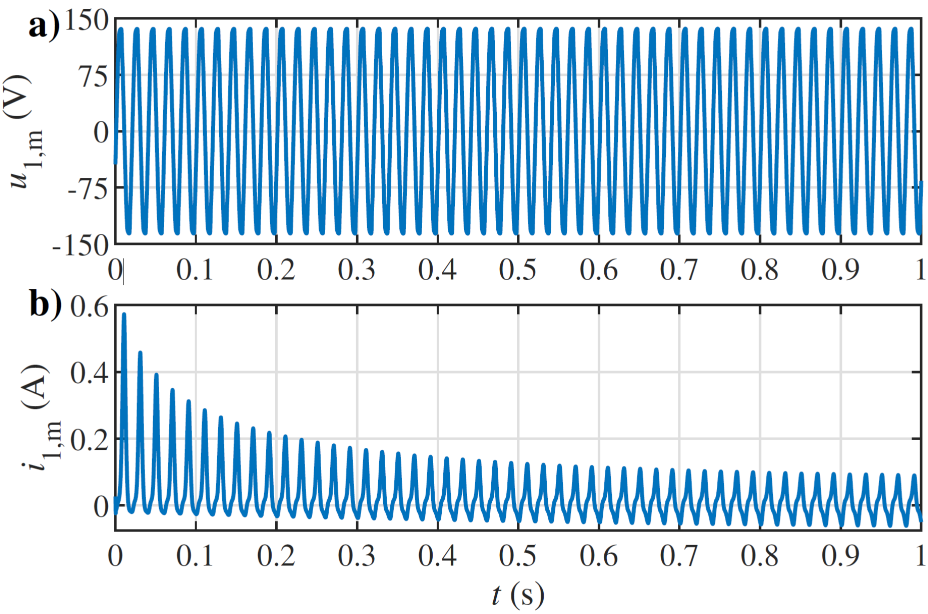

Figure 1: a) Measured primary voltage $u_{1,\mathrm{m}}$ and b) measured primary current $i_{1,\mathrm{m}}$.

The current calculated by applying the Logistic function has the lowest deviation from the measured current $i_{1,\mathrm{m}}$, as it is shown in Fig. 2b) and c). Therefore, we plotted the whole calculated current by applying the Logistic function separately in Fig. 2a). Further, this result was supported by calculating the measure ε given in Table 1. The current calculated by using the Logistic function achieved the lowest value of ε. Second best fit of the current was achieved w ith the Hyperbolic tangent function. Other analyzed functions have their shortcomings in the description of the inrush current of the transformer. Currents calculated with the Langevin and Gompertz functions have the highest deviation from the measured current. The Algebraic and Sigmoid functions are suitable for the description of the steady state of the current. The Elliot and exponential function have the appropriate shape of the current, but the amplitude is not accurate.

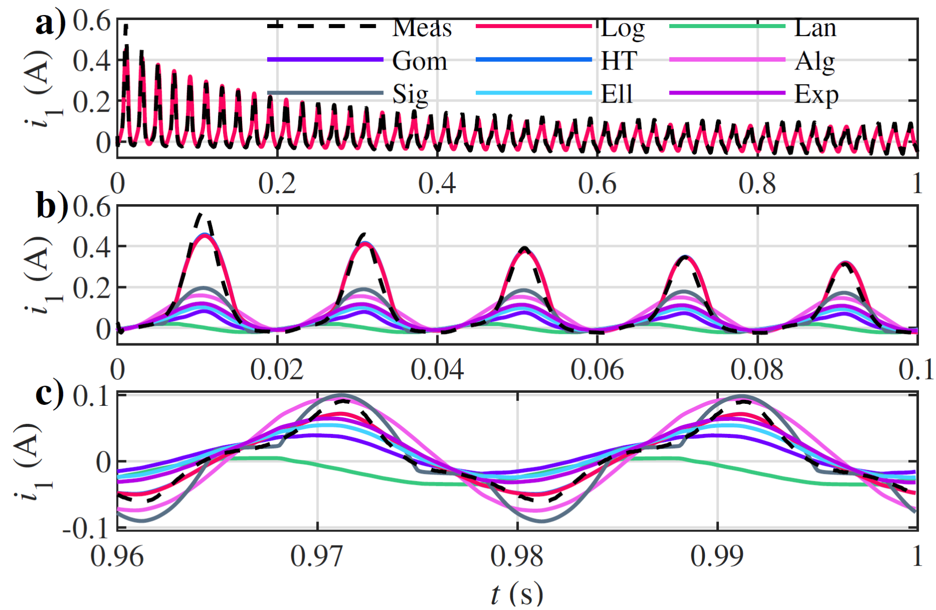

Figure 2: a) Measured current $i_{1,\mathrm{m}}$ and current i_1 calculated with the Logistic function, b) $i_{1,\mathrm{m}}$ and i_1 of all analytic functions in the first five periods and c) $i_{1,\mathrm{m}}$ and i_1 of all analytic functions in the steady state.

Table 2: Values of ε

Analytic function	Measure ε
Logistic	0.0134
Hyperbolic tangent	0.0135
Sigmoid	0.0324
Algebraic	0.0418
Exponential	0.0458
Elliot	0.0503
Gompertz	0.0589
Langevin	0.0810

V Conclusions

Based on the performed analysis, we concluded that simple analytic functions can adequately describe the nonlinear magnetic properties in the case of an inrush current of a single-phase transformer. We identified the Logistic and Hyperbolic tangent functions as the most suitable analytic functions. The final presentation will be extended with the implementation of the JA hysteresis model in the transformer's dynamic model. The results of the JA model will then be compared with results obtained with the analytic functions.

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