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NETWORK-BASED TRANSFORMER MODELS – A TRANSIENT ANALYSIS

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This contribution is about network-based transformer models. Two models (single- and three-phase) are derived, whereas Hopkinson's analogy is used to depict the magnetic domain. Therein, an energy-based hysteresis model is incorporated to represent the magnetic core. The solution strategy is a second-order variable step size backward differentiation formula (BDF2) in time domain, yielding the transient response of the two transformers. Finally, the simulation results are compared to measurements.

Keywords:
network-based transformer models,
Hopkinson's analogy,
energy-based hysteresis model,
backward differentiation formula (BDF2),
transient response simulation

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I Introduction

Network-based transformer models are an excellent opportunity to obtain reasonable simulation results computationally cheaply. All quantities are assumed to be homogeneous, and the number of unknowns is smaller than in an approach with finite elements. One significant aspect in a transformer model is the representation of the magnetic core. This contribution derives a single- and a three-phase transformer network based on a mutual and leakage flux approach and on a topological approach, respectively. The magnetic domain in both transformer models is described using Hopkinson's analogy [1]. An energybased hysteresis model [2] depicts the transformer's core. The resulting nonlinear and hysteretic differential algebraic equation (DAE) system is solved using a second-order variable step size backward differentiation formula (BDF2). The simulated idle currents are compared to measurements, and inrush simulation results for nominal excitation are presented.

II The transformer models

The primary voltage v_{p} of a transformer can be described by copper and stray losses, and an induced voltage due to the mutual magnetic flux ϕ_{M} in the magnetic core [1], reading

$$v_{\mathrm{p}} = i_{\mathrm{p}} R_{\mathrm{p}} + L_{\sigma,\mathrm{p}} \frac{\mathrm{d}i_{\mathrm{p}}}{\mathrm{d}t} + N_{\mathrm{p}} \frac{\mathrm{d}\phi_{\mathrm{M}}}{\mathrm{d}t} \tag{1}$$

where i_{p} equals the primary current. Analogous to (1),

$$v_s = -i_s R_s - L_{\sigma,s} \frac{\mathrm{d}i_s}{\mathrm{d}t} + N_s \frac{\mathrm{d}\phi_M}{\mathrm{d}t} \tag{2}$$

describes the secondary voltage equation. Next, *Hopkinson*'s analogy, which equals

$$\Theta = \frac{l}{\mu(\Theta/\mathrm{l})\mathrm{A}} \phi_M = R_{mag}(\Theta)\phi_M \tag{3}$$

with Θ, l, μ, and A the magnetomotive force, the length of the magnetic flux path, the permeability, and the core's cross section, respectively, can be used to model the magnetic domain. The nonlinear and hysteretic resistance R_{mag} represents the energy-based hysteresis model, which takes Θ as an input, and outputs the magnetic flux ϕ_M.

Combining (1), (2), and (3) in one network states the singlephase transformer, as shown in Figure 1. Note that the index *i* counts only for the three-phase transformer.

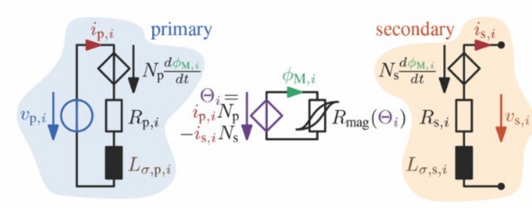

Figure 1: single-phase transformer network

A. Three-Phase Transformer

The electric domain of the three-phase transformer model is analogous to Figure 1 (three primary/secondary subnetworks with $i = \{1,2,3\}$), whereas the magnetic domain can be modeled according to the core topology, as Figure 2 depicts.

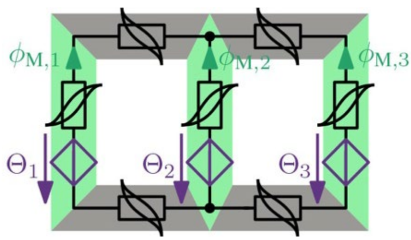

Figure 2: magnetic network of the three-phase transformer

Note that all magnetic resistances ( )are parametrized according to the length and cross section of the steel sheet regions.

III The differential algebraic equation system

The DAE system obtained from the networks can be stated with modified nodal analysis [3] as

$$\Lambda \frac{dx}{dt} + \gamma(x) = r, \tag{4}$$

where Λ, $\gamma(x)$, and r are the dynamic matrix containing derivative sources and inductances, the vector containing the linear and nonlinear and hysteretic current/voltage relations, and the exciting right hand side containing the independent sources, respectively. The vector $\boldsymbol{x}$ consists of the nodal voltages and currents over inductances and voltage sources. The system in (4) is discretized using BDF2, whereas the equation error in each time step is minimized using the *NewtonRaphson* method, enhanced with a line search algorithm.

IV Simulation results

The single- and the three-phase transformer are simulated for nominal primary voltage in the idle case, i.e. with no load connected to the secondary side(s).

A. Single-Phase Transformer

Figure 3 depicts a comparison between the simulated and measured primary current of the single-phase transformer in steady-state, which is instantaneously the case, if the sinusoidal primary voltage is switched on at ±90°.

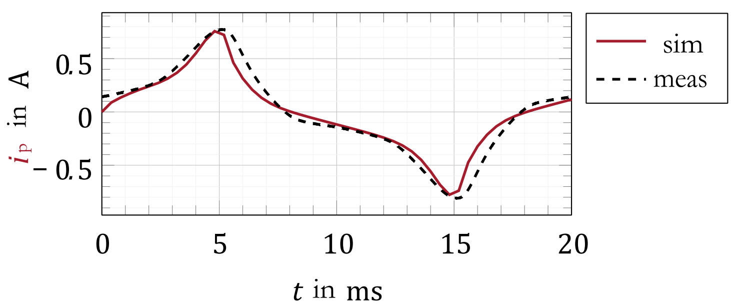

Figure 3: comparison between the simulated and measured idle current of the single-phase transformer in steady-state

Figure 4 depicts the simulation result of the inrush current of the single-phase transformer, if the sinusoidal primary voltage is switched on during the zero crossing.

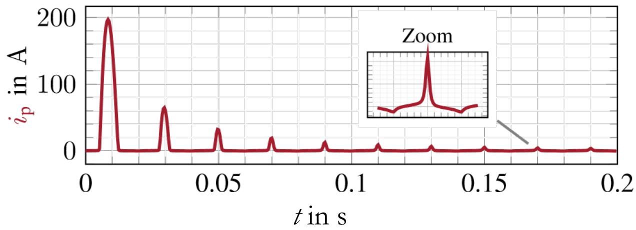

Figure 4: simulated inrush current of the single-phase transformer

Figure 5 depicts the simulated hysteresis of the single-phase transformer for a sinusoidal voltage excitation switched on at ±90°.

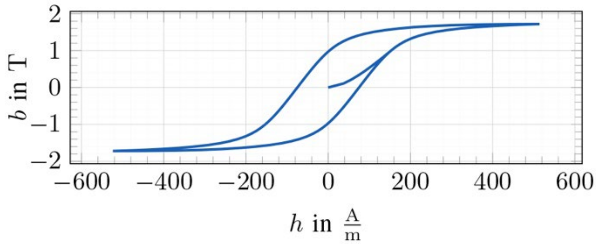

Figure 5: simulated hysteresis of the single-phase transformer

B. Three-Phase Transformer

The sinusoidal three-phase excitation for $v_{p,1,1}$, $v_{p,2}$, and $v_{p,3}$is 0°, 120°, and −120°, respectively.

Figure 6 depicts a comparison between the simulated and measured primary current of the three-phase transformer in steady-state.

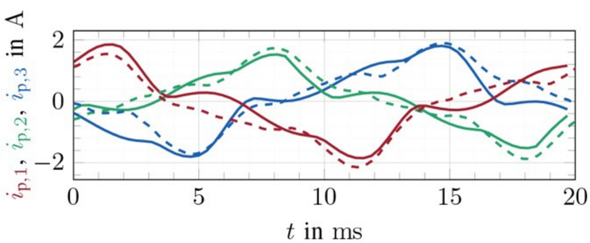

Figure 6: comparison between the simulated and measured idle current of the three-phase transformer in steady-state

Figure 7 depicts the simulation result of the inrush current of the three-phase transformer.

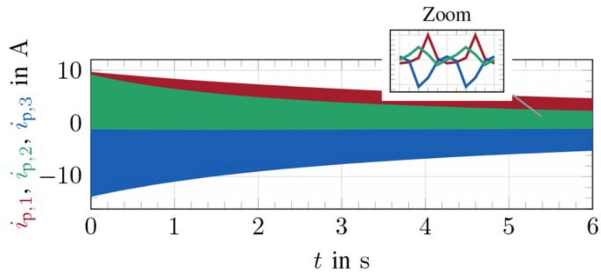

Figure 7: simulated inrush current of the three-phase transformer

V Conclusion and outlook

The algorithm is implemented in MATLAB. The *NewtonRaphson* iterations per time step are about 1 to 20, depending on the excitation level and point on the hysteresis trajectory; return points need on average more iterations. The parameters of the models are obtained from a short-circuit test; the hysteresis model is fitted to core measurements. The simulated idle currents coincide satisfactorily with the measured ones. In the full contribution, inrush measurements, the hysteresis model itself, modeling of stray paths, DC-biased excitation signals, the BDF2 algorithm, the DAE system, and different modeling topologies of the transformer's core are discussed.

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