

# COMPARISON OF DIFFERENT OFFLINE MTPA TRAJECTORY ESTIMATION METHODS

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The main goal of this paper was to analyse and compare different offline Maximum-Torque Per Ampere (MTPA) trajectory estimation methods for Interior Permanent Magnet Synchronous Machines (IPMSMs). The analysis was performed based on Finite Element Analysis (FEA) data of IPMSMs. The obtained results show that despite neglecting all the non-linearities, the analytical MTPA trajectory calculation with constant IPMSM parameters can model the MTPA trajectory with adequately small difference when compared to optimization-based calculation. Consequently, the MTPA trajectory calculation was further simplified with a piece-wise linear approximation of the trajectory, resulting in simpler MTPA reference calculation within the controller and adequately small deviations from optimal MTPA operating points.

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## I Introduction

Nowadays, energy efficiency is the main driving force of development of contemporary electric machines and drives. Not only because of ever increasing energy demand and energy cost, but also due to material usage and because it improves the product usability. High efficiency electric drives can for example extend operating time of battery powered devices. The energy efficiency of electric drives can be improved not only by optimizing the drive's design but also by control techniques which minimize the power losses [1].

There are many approaches to power loss minimization which differ by the loss considered. The most significant losses of electric drives are the Joule losses, which are proportional to square of the magnitude of current. Consequently, the most used technique is Maximum Torque Per Ampere (MTPA) control, which minimizes the current consumption for the given torque, thus reducing the Joule losses and providing practical, but suboptimal solution, since total power losses might not be minimized [1]. However, there are many different methods of implementing MTPA control, and different methods for estimation of MTPA trajectories [1], [2].

## II MTPA Trajectory Estimation Methods

The MTPA trajectory estimation methods are mainly being used for pole salient synchronous motors and can be according to [1] divided into offline and online methods. All offline methods require motor parameters to be determined either by measurements or Finite-Element Analysis (FEA), while some online methods can track the MTPA trajectory without knowing any motor parameter [1]. This research is limited only to comparison of most used offline MTPA calculation methods in applied engineering.

The basis of MTPA trajectory estimation is the current-dependent Interior Permanent Magnet Synchronous Machine (IPMSM) model since the cross-coupling and rotor position dependency have very little effect on dynamic performance of such machines [3]. Consequently, the simplified, well-known, IPMSM model in d-q reference frame is defined by (1) and (2),

$$u_d = R_s i_d + L_d \frac{di_d}{dt} - L_q i_q \omega p \quad (1)$$

$$u_q = R_s i_q + L_q \frac{di_q}{dt} + (\Psi_m + L_d i_d) \omega p \quad (2)$$

where the  $R_s$  is stator resistance,  $L_d$  is direct axis inductance,  $L_q$  quadrature axis inductance,  $\Psi_m$  is flux linkage due to permanent magnets, and  $p$  the number of pole pairs and  $\omega$  is mechanical angular speed, while  $u_d$ ,  $u_q$  are direct and quadrature axis voltages, and  $i_d$ ,  $i_q$  corresponding currents. Based on (1) and (2), the torque equation (3) can be derived.

$$T = \frac{3}{2} p (\Psi_m i_q + (L_d - L_q) i_d i_q). \quad (3)$$

The nonlinear behaviour of flux-current relationship can be considered by expressing the apparent inductances and the permanent magnet flux linkage as functions of both current components, i.e.,  $L_d(i_d, i_q)$ ,  $L_q(i_d, i_q)$  and  $\Psi_m(0, i_q)$ . Based on (3), an infinite number of  $(i_d, i_q)$  current combinations can generate the desired torque. The nonlinear MTPA optimization problem stated by (4) returns the desired torque  $T^*$  at minimum current.

$$\min_{i_{dq}} \|i_{dq}\| \text{ s. t. } T(i_d, i_q) = T^* \quad (4)$$

The first analysed MTPA trajectory estimation method was based on an analytical approach. By assuming constant motor parameters (i.e., neglecting nonlinearities), the MTPA trajectory can be calculated based on (3) and obtaining (5) [4].

$$i_d = \frac{\Psi_m - \sqrt{\Psi_m^2 + 8(L_d - L_q)^2 I_s^2}}{4(L_q - L_d)} \quad (5)$$

Another very popular method exploits Look-Up tables (LUTs) because they can contain MTPA trajectories that include nonlinear effects. Such trajectories are in general calculated from measured or FEA data by applying offline methods [5]. The measured data is usually obtained by measuring the shaft torque and stator currents at steady-state conditions. The MTPA trajectory is then determined by searching for maximum torque at different current magnitudes, as defined by (4). Meanwhile, the

FEA data-based approach is often possible only for machine developers. The main drawback of such methods is the time-consuming process of obtaining the measurement or FEA data. The advantage of these methods is that they can consider the non-linear flux-current and torque-current relationships, thus providing more accurate MTPA trajectories. However, when FEA data is used, the difference between FEA and measured data may occur which will be further investigated.

The obtained MTPA trajectories can be, however, considered by fitting an adequate mathematical function [6]. Within this work the simplest approximation function is considered, namely the piece-wise linear approximation. Moreover, the linear approximation consists of only one function which was obtained from marked points of optimization based MTPA trajectory by using minimum root-mean square error criterion for fitting. However, the number of approximation functions can also be higher.

### III Results

The described analytical- and optimization-based methods for MTPA trajectory estimation were analysed based on FEA data of a V-shape IPMSM. Despite the estimation methods are very different, the results show only a small deviation between the obtained trajectories (Figure 1). In the analytically based estimation, the inductance values from no-load operation were considered. The trajectory fit could be even improved if these values would be optimized [3]. Figure 1 presents the FEA calculated constant torque lines in dependence of current components ( $i_d, i_q$ ) and the maximum stator current limit  $I_{sMAX}$ .

The most significant difference in calculated MTPA trajectories can be found in the current angle  $\beta$ , as presented in Figure 2. The highest difference in angle  $\beta$  between analytical and optimization based MTPA calculation is only up to  $-3^\circ$  and can be found around the maximum stator current value. The difference in current angle  $\beta$  between the linear and optimization-based calculation is up to  $-13^\circ$  at up to half of maximum stator current.

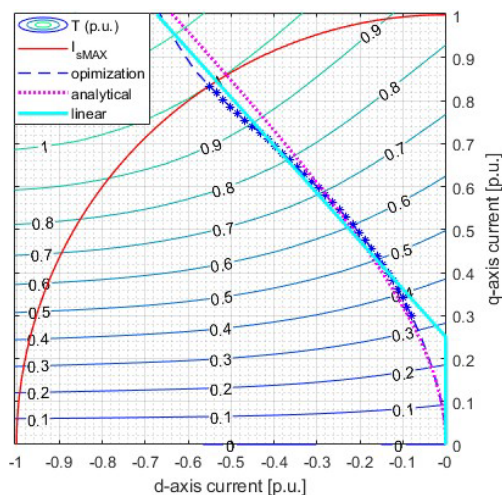


Fig.1. Constant torque lines with estimated MTPA curves

The difference in current angle  $\beta$  has very little effect on the generated torque. The highest difference in current needed to generate the desired torque can be found around nominal torque when comparing analytical calculation to optimization based MTPA trajectory calculation. Since the difference is less than 0,002 of maximum current, the impact on the Joule loss is negligible. Even in the case of piece-wise linear approximation of the MTPA trajectory, the difference in required current is less than 0,007 of the maximum current. Additionally, this difference occurs at low currents, where the Joule losses are also low. Because the drives are usually not operating at low torques, the proposed piece-wise linear approximation of the MTPA trajectory could be used. Consequently, the computation complexity and time within the implemented control system can be reduced which is enabling MTPA operation on simplest (and cheapest) microcontrollers.

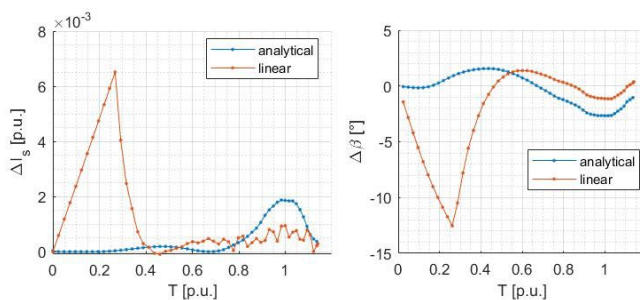


Fig.2. Comparison of analytical and optimization based MTPA trajectories

### III Conclusion

This paper analysed the difference between optimization-based calculation, analytical calculation, and piece-wise linear approximation of the MTPA trajectory. The analysis was systematically performed based on FEA data. The results show a very small difference in current amplitudes and angles required to generate the desired torque despite the difference due to errors in estimated MTPA trajectories. Consequently, the simpler MTPA calculation methods could be used to simplify the control algorithms for most basic microcontrollers at a very small efficiency trade-off.

The described MTPA trajectory estimation methods will be further analysed in a future research work, where the MTPA trajectories will be estimated also based on measurements within a laboratory experimental setup.

### References

- [1] Dianov, F. Tinazzi, S. Calligaro, S. Bolognani, "Review and Classification of MTPA Control Algorithms for Synchronous Motors, IEEE Trans. Power Electronics, Vol. 37, No. 4, pp. 3990-4007, 2022,
- [2] Yan M., Wen B., Cui Q., Peng X., "Parameter Identification for Maximum Torque per Ampere Control of Permanent Magnet Synchronous Machine under Magnetic Saturation", Control and Optimization of Power Converters and Drives, Electronics, Vol 13, No. 4, 2024
- [3] Garmut M., Steentjes S., Petrun M., "Parameter identification for MTPA control based on a nonlinear d-q dynamic IPMSM model", Compel, Vol 42., No. 4, pp. 846-860, 2023,
- [4] M. N. Uddin, T. S. Radwan, and M. A. Rahman, "Performance of interior permanent magnet motor drive over wide speed range," IEEE Trans. Energy Convers., vol. 17, no. 1, pp. 79–84, Mar. 2002,
- [5] H. W. de Kock, A. J. Rix, M. J. Kamper, "Optimal torque control of synchronous machines based on finite-element analysis" IEEE Trans. On Industrial Electronics, Vol. 57, No. 1, 2010.
- [6] Huang S., Chen Z., Huang K., Gao J., "Maximum Torque Per Ampere and Flux-weakening Control for PMSM Based on Curve Fitting", IEEE Vehicle Power and Propulsion Conference, pp. 1-5, 2010