University of Maribor Press

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ELECTRICAL AND ELECTROMECHANICAL CONVERTERS

LECTURE NOTATIONS

Maribor 2025

University of Maribor

Faculty of Electrical Engineering
and Computer Science

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Lecture Notations

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January 2025

Title Electrical and Electromechanical Converters

Subtitle Lecture Notations

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Assistant editors Matej Gajzer, Bojan Slemnik

Graphic material Sources are own unless otherwise noted. Zagradišnik, Ritonja (authors), 2025

Cover designer Jan Perša
(University of Maribor, University Press)

Published by University of Maribor, University Press
Slomškov trg 15, 2000 Maribor, Slovenia
https://press.um.si, zalozba@um.si

Issued by University of Maribor, Faculty of Electrical Engineering and Computer Science
Koroška cesta 46, 2000 Maribor, Slovenia
https://www.feri.um.si, feri@um.si

Edition 1st, translation of the original work from the Slovenian language Zagradišnik, I., Ritonja, J. (2022). *Električni in elektromehanski pretvorniki: Zapiski predavanj.* Univerza v Mariboru, Univerzitetna založba, doi: 10.18690/um.feri.11.2022

Published at Maribor, Slovenia, January 2025

Publication type E-book

Available at https://press.um.si/index.php/ump/catalog/book/937

CIP - Kataložni zapis o publikaciji
Univerzitetna knjižnica Maribor

621.313/.314(075.8)(0.034.2)

ZAGRADIŠNIK, Ivan
Electrical and electromechanical converters [Elektronski vir] : lecture notations / authors Ivan Zagradišnik, Jožef Ritonja ; [language editing Shelagh Hedges]. - 1st ed. - E-knjiga. - Maribor : University of Maribor, University Press, 2025

Način dostopa (URL): https://press.um.si/index.php/ump/catalog/book/937. - Prevod dela: Električni in elektromehanski pretvorniki
ISBN 978-961-286-945-8 (PDF)
doi: 10.18690/um.feri.1.2025
COBISS.SI-ID 222237955

ISBN 978-961-286-945-8 (pdf)

DOI https://doi.org/10.18690/um.feri.1.2025

Price Free copy

For publisher Prof. Dr. Zdravko Kačič, Rector of University of Maribor

Attribution Zagradišnik, I., Ritonja, J. (2025). *Electrical and Electromechanical Converters: Lecture Notations.* University of Maribor, University Press. doi: 10.18690/um.feri.1.2025

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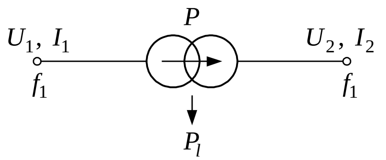
INTRODUCTION

Electrical and electromechanical converters (EEMCs) are devices that convert energy from one form to another. Types of conversion: electrical to electrical, electrical to mechanical, mechanical to electrical. The conversion is time-limited or unlimited.

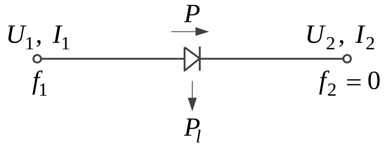
Electrical machines are also considered as EEMCs. In these, the energy conversion is always mediated by a magnetic field.

Examples of energy conversions

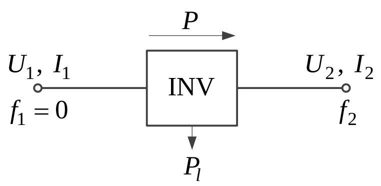
Electrical $\rightarrow$ electrical $\qquad$ P_l are losses.



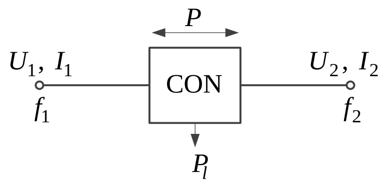
TRANSFORMER



RECTIFIER



INVERTER



(*DIRECT*)
CONVERTER

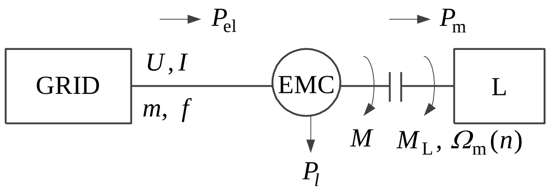
number of branches = number of phases

$m_1 \gtreqless m_2$

frequency

$f_1 \gtreqless f_2$

Electrical $\rightarrow$ mechanical



L is load.

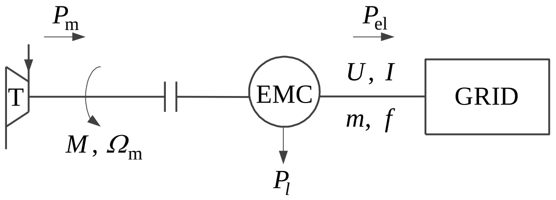
MOTOR

$$\eta = \frac{P_{el} - P_l}{P_{el}} = \frac{P_m}{P_{el}}$$

η: $0,6 \ldots 0,98$

EMC is an electromechanical converter.

Mechanical $\rightarrow$ electrical



T is turbine.

GENERATOR

$$\eta = \frac{P_{el}}{P_m}$$

η: $0,95 \ldots 0,98$

Magnetic field

The transfer of power or energy in electrical machines is carried out by a magnetic field or magnetic flux. The basic quantities (marked with an asterisk) and derived quantities are used for notation.

Basic quantities:

$$*B = \frac{\Phi}{A}$$

magnetic flux density

$$*\Phi = \iint_A \vec{B} \cdot \mathrm{d}\vec{A}$$ magnetic flux

$$*H = \frac{B}{\mu}$$ magnetic field intensity

$$*\mu = \frac{B}{H}$$ permeability

Derived quantities:

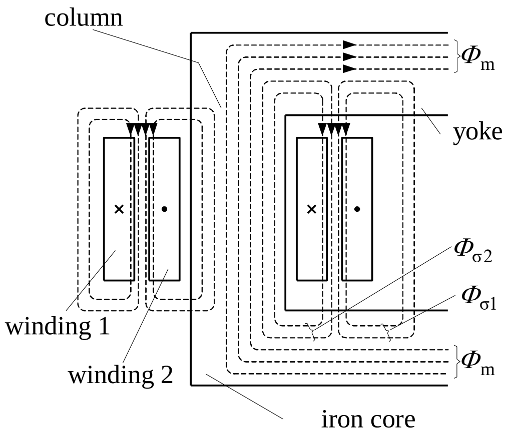
$$\Psi = \sum \Phi_\mathrm{i} N_\mathrm{i} = \iint_A \vec{B} \cdot \mathrm{d}\vec{A}$$ coil flux or magnetic linkage

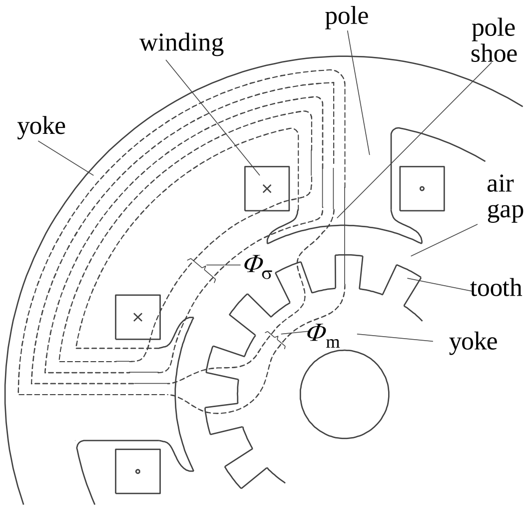
$$L = \frac{\Psi}{I}$$ inductance

$$F_\mathrm{m} = \oint_K \vec{H}_l \cdot \mathrm{d}\vec{l}$$ magnetomotive force (*MMF*) – excitation

$$\Lambda_\mathrm{m} = \frac{\Phi}{F_\mathrm{m}}$$ magnetic conductivity (permeance) or $R_\mathrm{m} = \frac{F_\mathrm{m}}{\Phi}$ resistance (reluctance)

Examples for the main and leakage mag. fields





Basic magnetic field laws

1) Law of current flow (Ampere's law)

$$\oint_K \vec{H}_l \cdot d\vec{l} = \iint_A \vec{J} \cdot d\vec{A} = IN = \Theta$$ (J is the current density.)

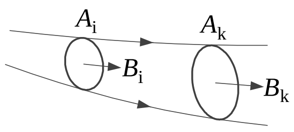
discretization $\vec{H}_l \rightarrow H_i$; $d\vec{l} = l_i$

$$\sum H_i l_i = IN$$

2) Flux preservation law

$$\Phi = \oiint_A \vec{B} \cdot d\vec{A} = 0$$

Flux tube



discretization $\vec{B} \rightarrow B_i$; $d\vec{A} \rightarrow A_i$

$$\sum B_i A_i = 0 \quad \Phi_i = B_i A_i$$

3) Material law

$$\vec{B} = f(\vec{B}_i) = \mu_0 \vec{H} + \vec{B}_i = \mu_0 \vec{H} + \mu_0 \kappa \vec{H} = \mu \vec{H} \qquad \mu = \mu_0 \mu_r = \mu_0 (1+\kappa)$$

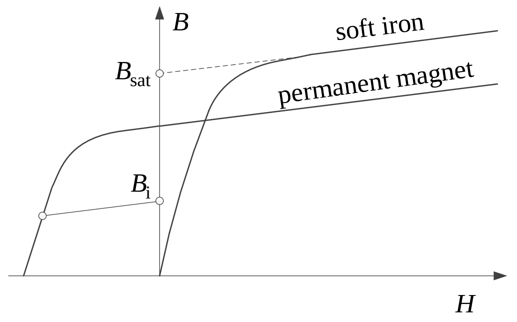
B_i is the magnetic polarization or intrinsic flux density, and κ the magnetic susceptibility.

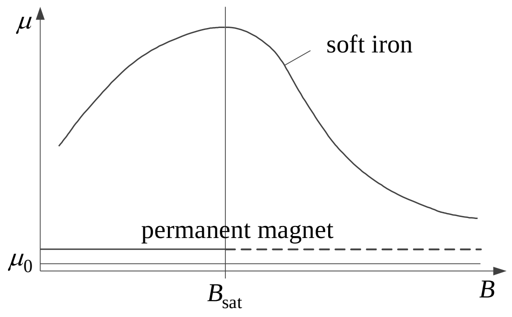
Magnetic curve (B-H curve)

The B-H curve is shown in the left Figure for soft iron and a permanent magnet and for the corresponding permeabilities in the right Figure. For a magnetic circuit, the following applies

without saturation: $\mu_{Fe} \rightarrow \infty$; $H_{Fe} \rightarrow 0$ in saturation: $\mu_{Fe} \rightarrow \mu_0$

$$B_{Fe} < B_{sat} \ (B_{sat} = B - \text{saturation}) \ B_\delta = \mu_0 \frac{IN}{\delta} \qquad B_{Fe} > B_{sat} \ B_\delta \approx \mu_0 \frac{IN}{\delta + l_{Fe}}$$



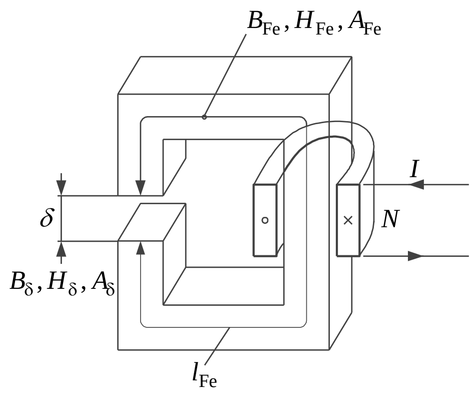


Flux preservation law − application $\rightarrow \Phi_\delta = \Phi_{Fe}$

Law of current flow – application:

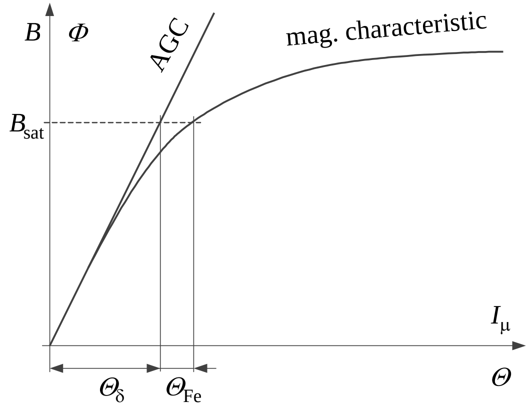
$$\Theta = \sum H_i l_i = I N$$

$$I_\mu = \frac{\sum H_i l_i}{\sqrt{2} N}$$, (if $H = \hat{H}$ is the peak value).



Magnetization characteristic

The magnetization characteristic gives the dependence for a magnetic circuit consisting of an air gap and an iron core. The air gap characteristic (AGC) is the tangent to the curve.



Change of measure: $B = f(\Theta)$

Ordinate: $\Phi_\delta = B_\delta A_\delta \rightarrow \Phi_\delta = f(\Theta_{sum})$

$$\Theta = \Theta_{sum} = \Theta_\delta + \Theta_{Fe}$$

Abscissa: $\Theta = I_\mu N \rightarrow B = f(I_\mu)$

To calculate the magnetic circuit: from $E \to \Phi \to B_\mathrm{i}$ on part i; from B_δ will be:

$$\Theta_\delta = \frac{B_\delta}{\mu_0}\delta,\quad \Theta_\mathrm{Fe} = H_\mathrm{Fe} l_\mathrm{Fe} \qquad (H_\mathrm{Fe} \to \text{ from B-H curve})$$

$$\hat{I}_\mu = \frac{\Theta_\mathrm{sum}}{N} \to I_\mu = \frac{\Theta_\mathrm{sum}}{\sqrt{2}N} \qquad \Phi = \frac{\Theta_\mathrm{sum}}{R_\mathrm{m}} = \frac{\sqrt{2} I_\mu N}{R_\mathrm{m}} = \sqrt{2} I_\mu N \Lambda_\mathrm{m}$$

inductance of the coil $L = \frac{\Psi}{\sqrt{2} I_\mu} = \frac{N\Phi}{\sqrt{2} I_\mu} = \frac{N^2}{R_\mathrm{m}} = N^2 \Lambda_\mathrm{m}$ and $\Lambda_\mathrm{m} = \frac{1}{R_\mathrm{m}} = \frac{\mu A}{l}$

Excitation of windings

Magnetic field of a concentric (cylindrical) winding

a) Example of a transformer

The individual windings are spatially close. The excitation is concentrated. The main flux connecting the windings is generated in the iron core. If the Fe core does not have an air gap, it will:

$$\delta \to 0 \text{ and } \mu_\mathrm{Fe} \to \infty \quad R_\mathrm{m} = \frac{l_\mathrm{Fe}}{\mu_\mathrm{Fe} A_\mathrm{Fe}} \to 0, \quad L_\mathrm{m} \to \infty, \quad L_\sigma \to 0.$$

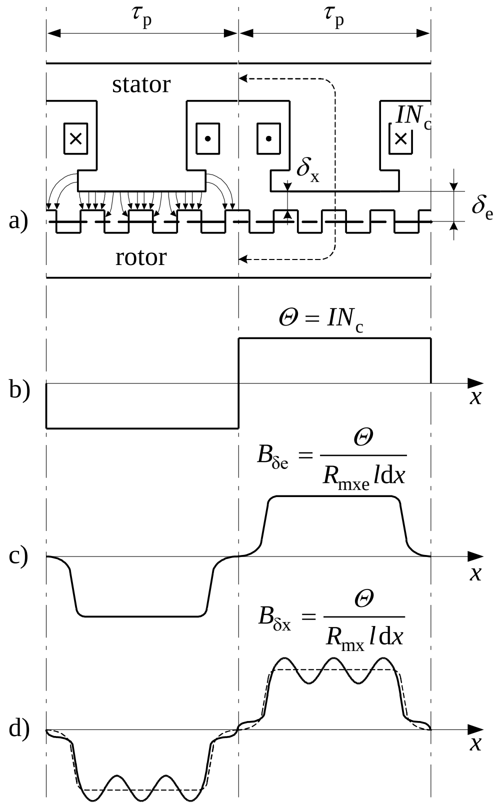
L_m is the magnetizing (main) inductance and L_σ the leakage inductance.

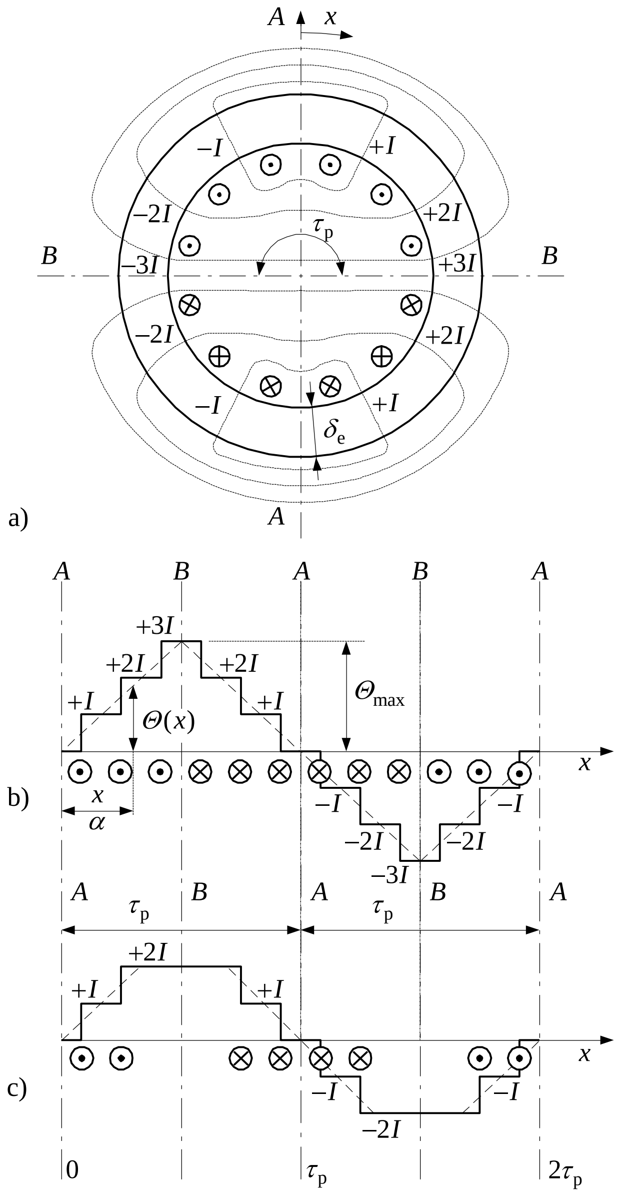
b) Example of salient poles for rotating machines

In rotating machines, the individual windings are separated by an air gap "δ" or equivalent (enlarged) air gap δ_e to take account of the effect of the machine's slotted openings.

$\Theta = I N_\mathrm{c}$ magnetic excitation by direct current of a single pole coil with N_c turns.

If the current oscillates, $i = \sqrt{2} I \cos(\omega t) = \hat{I} \cos(\omega t)$, Θ will also oscillate.





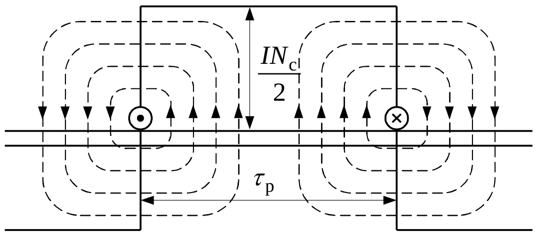
Magnetic field of a distributed winding

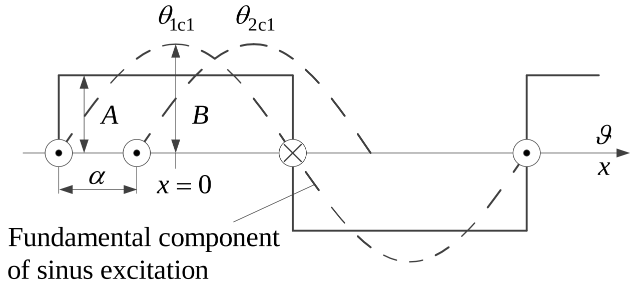
a) <u>DC power supply</u>

The left Figure (below) shows the coil excitation (*MMF*) curve for the developed rotor perimeter: $\Theta_c = \frac{IN_c}{2} \equiv A$

and the fundamental harmonic component of the coil excitation $\hat{\Theta}_{c1} = \frac{4}{\pi}\frac{IN_c}{2} \equiv B = \frac{4}{\pi}A$.

N_c are the number of turns of the coil. For p pole pairs are $N_c = \frac{N}{p}$ and $\hat{\Theta}_{c1} = \frac{4}{\pi}\frac{IN}{2p}$.

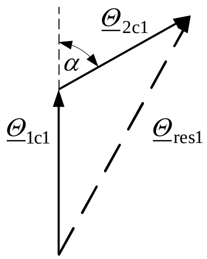




The space distribution of the *MMF* of the coil is: $\theta_x = \hat{\Theta}_{c1}\cos\left((x/\tau_p)\pi\right)$, if $\tau_p = \frac{D\pi}{2p} = \frac{r\pi}{p}$ and $x = 0$ is in the coil symmetry axis. Introducing the angle $\vartheta = (x/\tau_p)\pi$ is $\theta_x = \hat{\Theta}_{c1}\cos\vartheta$.

For two coils displaced by an angle in adjacent slots of a rotating electromechanical converter, the fundamental harmonic component of the *MMF* is: $\underline{\Theta}_{res1} = \underline{\Theta}_{1c1} + \underline{\Theta}_{2c1}$.

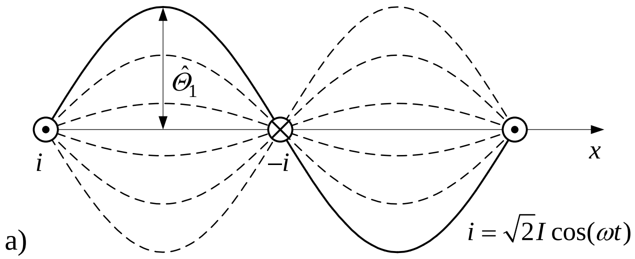
In general, for a distributed winding, $\hat{\Theta}_{res1} = \frac{4}{\pi}\frac{IN}{2p} f_w$, if f_w is the winding factor, i.e. the ratio of the geometric to the arithmetic sum of the *MMF* of two or more coils: $f_w = \frac{\sum_{\text{geom.}} \Theta}{\sum_{\text{arit.}} \Theta} = \frac{\Theta_{res1}}{\sum_{i=1}^{n} \Theta_{ic1}}$.

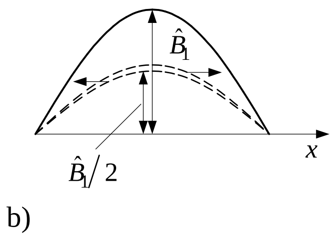


For a uniformly distributed winding of a large number of coils, the *MMF* equation is a straight line: $\theta_x = \Theta_{max}\frac{x}{\tau_p/2}$ for $0 \le x \le \tau_p/2$. (Valid to the right dotted Figure (b) on page 7.)

b) <u>AC power supply</u> $i = \sqrt{2} I\cos(\omega t)$

MMF $\theta(x,t) = \hat{\Theta}_1\cos\vartheta\cos(\omega t) \;\rightarrow\; b(x,t) = \frac{\mu_0\theta(x,t)}{\delta}$ (In the air gap, $b \propto \theta$.)





$$\theta(x,t)=\hat{\Theta}_1\cos\vartheta\cos(\omega t)=\frac{\hat{\Theta}_1}{2}\big(\cos(\vartheta-\omega t)+\cos(\vartheta+\omega t)\big) \text{ or for } b\propto\theta$$

$$b(x,t)=\frac{\hat{B}_1}{2}\big(\cos(\vartheta-\omega t)+\cos(\vartheta+\omega t)\big)$$

$$b(x,t)=b_{\mathrm{p}}(x,t)+b_{\mathrm{n}}(x,t)$$

direction of motion $\rightarrow \quad \leftarrow$

The amplitude of a positive (direct) or negative (inverse) wave travels at the speed obtained from the condition for the value of the argument:

$$\vartheta\mp\omega t=0 \text{ or } \frac{x}{\tau_{\mathrm{p}}}\pi\mp\omega t=0 \rightarrow x=\pm\omega t\frac{\tau_{\mathrm{p}}}{\pi},$$

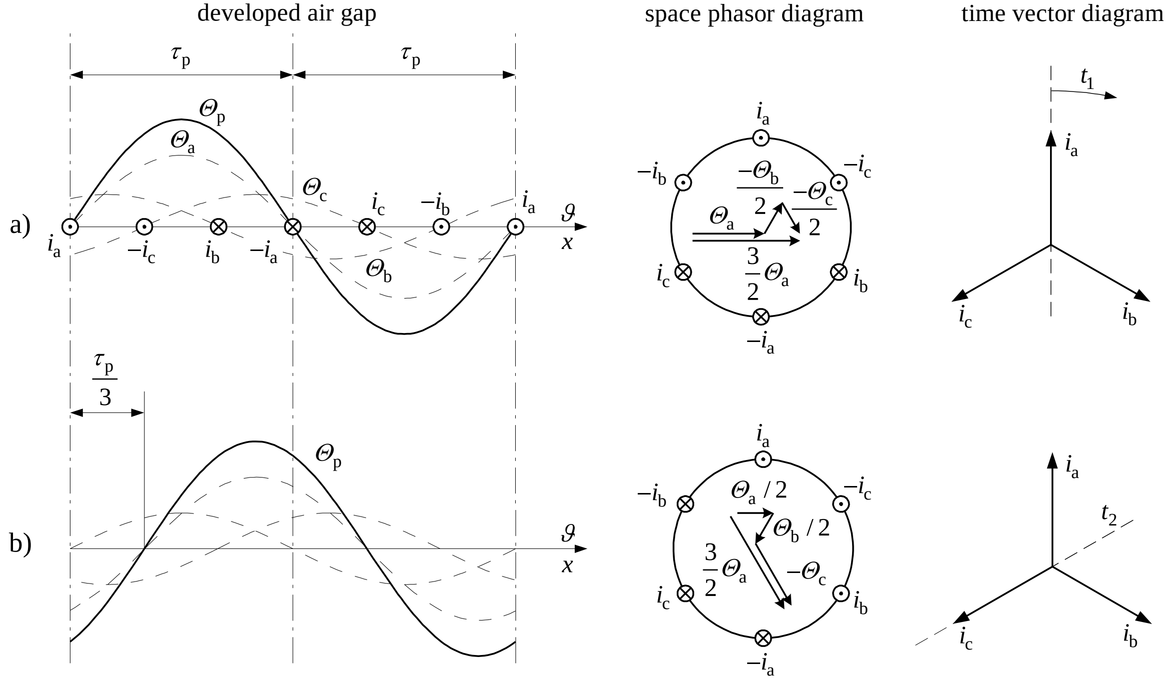
$$\upsilon=\frac{\mathrm{d}x}{\mathrm{d}t}=\pm\frac{\omega\tau_{\mathrm{p}}}{\pi}=\pm 2f\tau_{\mathrm{p}}=\pm\frac{2\tau_{\mathrm{p}}}{T}.$$

From the condition $\upsilon=D\pi n=\pm 2f\tau_{\mathrm{p}}=\pm 2f\dfrac{D\pi}{2p}$ the expression is given: $\boxed{n=\pm\dfrac{f}{p}}$.

This is the basic equation for the rotational speed of a rotating magnetic field in an electrical rotating machine.

A more practical equation for $f=50$ Hz is: $n=\pm\dfrac{60f}{p}=\pm\dfrac{3000}{p}\left(\mathrm{min}^{-1}\equiv\mathrm{rot./min}\right)$.

The magnetic field of a distributed three-phase winding or of three single-phase windings displaced in space by a mechanical angle $\alpha_{\mathrm{fm}} = \alpha_{\mathrm{f}} / p = (2\pi/3)/p \equiv 120^{\circ}/p$.



The Figure applies to the *MMF* of a two-pole machine which causes a magnetomotive field in the air gap with phase current phasors according to the time diagrams for $\omega t = 0^{\circ}$ and 60°.

Using the angle ϑ and the corresponding space and time phase shift, i.e., 120°, we get:

phase "a" $\frac{\hat{B}_1}{2}\left(\cos(\vartheta-\omega t)+\cos(\vartheta+\omega t)\right)=b_{\mathrm{ap}}+b_{\mathrm{an}}$,

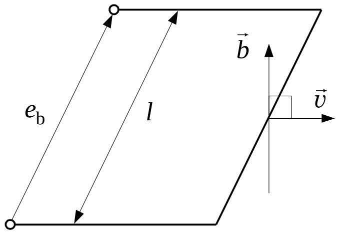
phase "b" $\frac{\hat{B}_1}{2}\left(\cos\left((\vartheta-120^{\circ})-(\omega t-120^{\circ})\right)+\cos\left((\vartheta-120^{\circ})+(\omega t-120^{\circ})\right)\right)=b_{\mathrm{bp}}+b_{\mathrm{bn}}$,

phase "c" $\frac{\hat{B}_1}{2}\left(\cos\left((\vartheta-240^{\circ})-(\omega t-240^{\circ})\right)+\cos\left((\vartheta-240^{\circ})+(\omega t-240^{\circ})\right)\right)=b_{\mathrm{cp}}+b_{\mathrm{cn}}$.

The sum of the positive field waves is: $b_\mathrm{p} = b_\mathrm{ap} + b_\mathrm{bp} + b_\mathrm{cp} = \frac{3}{2}\hat{B}_1 \cos(\vartheta - \omega t)$. The sum of the negative field waves is: $b_\mathrm{n} = b_\mathrm{an} + b_\mathrm{bn} + b_\mathrm{cn} = 0$, because the phasors of the negative wave of the magnetic field of the second and third phases are displaced by 240° and 120° respectively. In general, for a symmetrical *m*-phase system $b_\mathrm{p} = \frac{m}{2}\hat{B}_1 \cos(\vartheta - \omega t)$, if the field amplitude of each phase is: $\hat{B}_1 = \frac{\mu_0 \hat{\Theta}_1}{\delta}$ or *MMF* $\hat{\Theta}_1 = \frac{4}{\pi}\frac{N f_\mathrm{w}}{2p}\sqrt{2}I$ and for *m*-phases $\hat{\Theta}_{1\mathrm{m}} = \frac{m}{2}\frac{4}{\pi}\frac{N f_\mathrm{w}}{2p}\sqrt{2}I$.

Induced voltage

a) Induced voltage of a conductor (bar) due to motion in a magnetic field



$$e_\mathrm{b} = (\vec{v} \times \vec{B}) \cdot \vec{l} = -(\vec{B} \times \vec{v}) \cdot \vec{l}$$

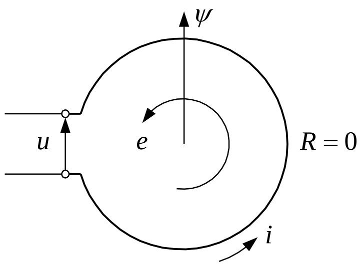
$$e_\mathrm{b} = v \cdot B \cdot l \quad \text{for } \vec{B} \perp \vec{v}$$

From $\mathrm{d}A = l\,\mathrm{d}x = -l\,\mathrm{d}s = -l\,v\,\mathrm{d}t$ (because $\mathrm{d}x = -\mathrm{d}s$)

we prove: $e_\mathrm{b} = \frac{vBl\mathrm{d}t}{\mathrm{d}t} = -\frac{B\mathrm{d}A}{\mathrm{d}t} = -\frac{\mathrm{d}\phi}{\mathrm{d}t}$.

b) Induced loop voltage due to a time-varying magnetic field

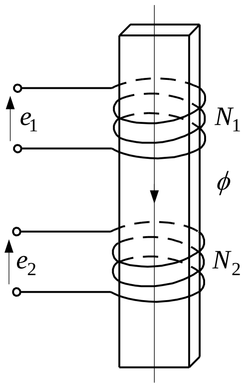
Induction law in integral form (Faraday's law):



$$e = \oint_K \vec{E}_\mathrm{l} \cdot \mathrm{d}\vec{l} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_A \vec{B} \cdot \mathrm{d}\vec{A} = -\frac{\mathrm{d}\psi}{\mathrm{d}t} .$$

E_l is the electromotive force (*EMF*).

Example of a transformer:



$$e_1 = -N_1 \frac{\mathrm{d}\phi}{\mathrm{d}t} \qquad e_2 = -N_2 \frac{\mathrm{d}\phi}{\mathrm{d}t}$$

$$\left|\frac{e_1}{e_2} = \frac{N_1}{N_2}\right| \qquad K_\mathrm{U} = \frac{N_1}{N_2} \text{ (voltage ratio)}$$

The general law of induction, if $\psi = \mathrm{f}(x,t)$:

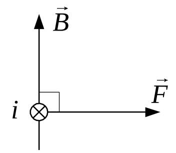
$$e = -\frac{\mathrm{d}\psi}{\mathrm{d}t} = -\left(\frac{\partial\psi}{\partial t} + \frac{\partial\psi}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t}\right) = -\left(\frac{\partial\psi}{\partial t} + \upsilon\frac{\partial\psi}{\partial x}\right) = e_\mathrm{t} + e_\mathrm{r},$$

transformer voltage + rotating or moving voltage.

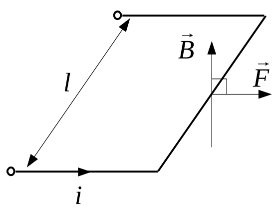
For two windings with mutual inductance L_{12}, there will be: $\psi = L_{12}\, i_\mu$ and

$$e = -\frac{\mathrm{d}\psi}{\mathrm{d}t} = -\left(L_{12}\frac{\mathrm{d}i_\mu}{\mathrm{d}t} + i_\mu \frac{\mathrm{d}L_{12}}{\mathrm{d}t}\right) = e_\mathrm{t} + e_\mathrm{r}.$$

Forces in a magnetic field



Force on the current conductor (Lorenz force)



$$\vec{F} = Q\cdot(\vec{\upsilon}\times\vec{B}) = q_\mathrm{l}\, l\cdot(\vec{\upsilon}\times\vec{B}) = l\cdot(\vec{i}\times\vec{B}) = (\vec{i}\times\vec{B})\cdot l,$$

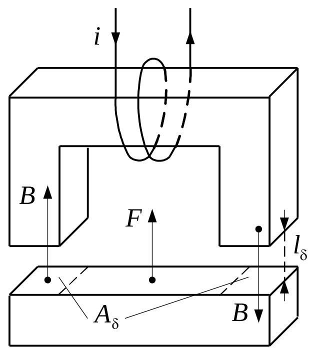
because $i = \upsilon q_\mathrm{l}$ will be a force $F = i\cdot B\cdot l$.

q_l is the line charge.

Motor: $\vec{F}$ acts in the direction of $\vec{\upsilon}$ or Ω_m.

Generator: $\vec{F}$ acts against the direction of $\vec{\upsilon}$ or Ω_m.

Force on the iron core



Energy in the air gap:

$$W_m = \frac{1}{2}\Phi_\delta \Theta_\delta \text{ and for } \Phi_\delta = \Theta_\delta \Lambda_\delta \text{ is } W_m = \frac{1}{2}\Theta_\delta^2 \Lambda_\delta .$$

$$F = \frac{dW_m}{dx} = \frac{1}{2}\Theta_\delta^2 \frac{d\Lambda_\delta}{dx} = -\frac{1}{2}\Theta_\delta^2 \frac{\mu_0 A_\delta}{l_\delta^2} \frac{dl_\delta}{dx}$$

For $\Lambda_\delta = \dfrac{\mu_0 A_\delta}{l_\delta}$ and $dl_\delta = -dx$ is force:

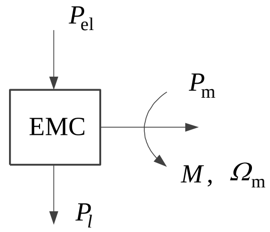
$$F = \frac{1}{2} \frac{\mu_0 \Theta_\delta^2 A_\delta}{l_\delta^2} .$$

For $B_\delta = \dfrac{\mu_0 \Theta_\delta}{l_\delta}$, we get the expression for the force on the iron core: $F = \dfrac{1}{2}\dfrac{B_\delta^2 A_\delta}{\mu_0} = \dfrac{1}{2}\dfrac{B_\delta^2}{\mu_0} A_\delta$.

Examples of applications: switches, relays, magnets, and step motors.

Energy and power transmission

Electrical $\rightarrow$ mechanical

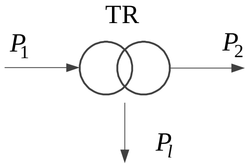


Stationary operation:

$$P_{el} - P_l = P_m$$

EMC – electromechanical converter (motor)

Electrical $\rightarrow$ electrical

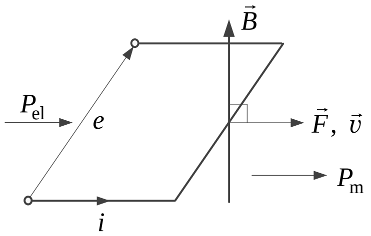


Stationary operation:

$$P_1 - P_l = P_2$$

TR – transformer

Example of a lossless linear electromechanical converter



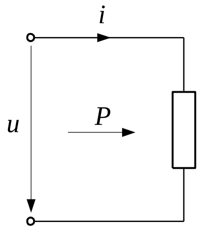
Accepted electrical power

$$P_{el} = e\,i = vBl\,i$$

Mechanical power output

$$P_m = Fv = iBlv$$

Arrows system



$P = u\,i$

$P > 0$ consumer (motor)

$P < 0$ producer (generator)

Torque

There are several ways to calculate the torque.

1) From electrical power $P_{el} = e\,i = P_m = M\,\Omega_m$; $\Omega_m = 2\pi n = v / r$ (mechanical angular or circular speed) we get shaft torque:

$$M = \frac{P_m}{\Omega_m}.$$

2) From induced voltage

a) From the induced voltage due to motion, we get: $e\,i = vBli = viBl = vF\frac{r}{r} = M_e\frac{v}{r} \rightarrow M_e$

$$M_e = iBlr = Fr \qquad \text{(e – electromagnetic torque)}$$

We need to know the magnetic field distribution (in the air gap).

b) The absolute value of the transformer induced voltage gives:

$$e = \left| -\frac{\mathrm{d}\psi}{\mathrm{d}t} \right| . \qquad e\,i = \frac{\mathrm{d}\psi}{\mathrm{d}t} i = M\Omega_\mathrm{m}$$

$$\Omega_\mathrm{m} = \frac{\mathrm{d}\alpha_\mathrm{m}}{\mathrm{d}t} = \frac{1}{p}\frac{\mathrm{d}\alpha}{\mathrm{d}t}$$ (Ω_m – mechanical angular velocity)

$$\alpha = p\alpha_\mathrm{m}$$ (α_m is the mechanical or "space" angel, α – electrical angel)

$$M_\mathrm{e} = \frac{ei}{\Omega_\mathrm{m}} = \frac{\mathrm{d}\psi}{\mathrm{d}t} i \frac{p\mathrm{d}t}{\mathrm{d}\alpha} = pi\frac{\mathrm{d}\psi}{\mathrm{d}\alpha} \qquad \left(M_\mathrm{e} = p\,i\,N\frac{\mathrm{d}\phi}{\mathrm{d}\alpha} \text{ is valid for } \psi = N\phi. \right)$$

We need to know the magnetic field distribution (in the air gap).

3.) From the magnetic field energy $\left(F = \frac{\mathrm{d}W_\mathrm{field}}{\mathrm{d}x} \right)$

$$M_\mathrm{e} = F\,r = \frac{\mathrm{d}W_\mathrm{field}}{\mathrm{d}x} r = \frac{\mathrm{d}W_\mathrm{field}}{\mathrm{d}\alpha} \qquad (\text{for } \mathrm{d}x = r\,\mathrm{d}\alpha)$$

The force acts in the direction of increasing the mutual magnetic linkage, or increasing the magnetic conductivity, or decreasing the magnetic resistance.

From the equation for energy, for a single excitation winding:

$$M_\mathrm{e} = \frac{1}{2}\,\Theta\frac{\mathrm{d}\Phi}{\mathrm{d}\alpha} = \frac{1}{2}\,\Theta^2\frac{\mathrm{d}\Lambda_\mathrm{m}}{\mathrm{d}\alpha} \text{ or } M_\mathrm{e} = -\frac{1}{2}\,\Phi\frac{\mathrm{d}\Theta}{\mathrm{d}\alpha} = -\frac{1}{2}\,\Phi^2\frac{\mathrm{d}R_\mathrm{m}}{\mathrm{d}\alpha}.$$

In the equations, we consider $\Phi = \Theta\Lambda_\mathrm{m}$ or $\Theta = \Phi R_\mathrm{m}$.

For two or more windings, the energy is expressed better in terms of inductances:

$$M_\mathrm{e} = \frac{1}{2}\,i_1^2\frac{\mathrm{d}L_1}{\mathrm{d}\alpha} + \frac{1}{2}\,i_2^2\frac{\mathrm{d}L_2}{\mathrm{d}\alpha} + i_1 i_2\frac{\mathrm{d}L_{12}}{\mathrm{d}\alpha}.$$

L_1 and L_2 are the self-inductances and L_{12} the mutual inductance of the two windings.

Losses and efficiency

1) Winding losses: a) joule losses, b) eddy current losses due to skin-effect

$P_{Cu} = I^2 R$ or additional losses

$R = R_{=} + R_{add}$

2) Losses in the Fe core: a) hysteresis, b) eddy-current

$P_{Fe} = P_{Feh} + P_{Fee} = k_h f B^x m_{Fe} + k_e f^2 B^2 m_{Fe}$,

where the exponent for hysteresis losses is $x = 1,6 \div 2,8$.

3) Mechanical losses in rotating machines

Efficiency

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{in} - P_l}{P_{in}} = 1 - \frac{P_l}{P_{in}} = 1 - \frac{P_l}{P_1}$$

Heating and cooling

Heat transfer

1. The propagation of heat − the heat flux − in a solid body is:

$\Phi_t = \Lambda_t (\vartheta_1 - \vartheta_2)$ λ – specific thermal conductivity $\left(\frac{W}{m \cdot K}\right)$

thermal conductivity $\Lambda_t = \frac{\lambda A}{d}$ A – surface (m^2), d – body thickness (m)

2. Thermal transmittance by convection

$\Phi_t = \Lambda_t (\vartheta_1 - \vartheta_2) = \alpha_c A_c (\vartheta_1 - \vartheta_2)$ α_c – convection coefficient $\left(\frac{W}{m^2 \cdot K}\right)$

For air, the approximate equation is: $\alpha_{\mathrm{c}} \approx 6,5+0,05(\vartheta_1-\vartheta_2)$. For natural convection, the convection coefficient for the temperature rise $40 \div 50\ \mathrm{K}$ is $\alpha_{\mathrm{c}} = (8,5 \div 9)\ \mathrm{W/(m^2 \cdot K)}$.

3) Radiative thermal transmittance

$$\Phi_{\mathrm{t}} = \Lambda_{\mathrm{t}}(\vartheta_1-\vartheta_2) = \alpha_{\mathrm{r}} A_{\mathrm{r}}(\vartheta_1-\vartheta_2)$$

α_{r} (radiation coefficient) is a function of temperature, temperature difference and material type.

$$\alpha_{\mathrm{r}} = C_1\left(\Theta_{\mathrm{w}}^4-\Theta_{\mathrm{a}}^4\right)\frac{1}{\Delta\vartheta}$$

Θ_{w} is the absolute wall temperature and Θ_{a} the absolute ambient temperature,

$C_1 = \varepsilon C_{\mathrm{r}}$ is the radiative constant of the surface of a body, ε is the absorption ratio $(\varepsilon < 1)$, i.e., the ratio of the received (absorbed) to the irradiated radiant energy.

$$C_{\mathrm{r}} = 5,67\cdot 10^{-8}\ \frac{\mathrm{W}}{\mathrm{m^2 \cdot K^4}}$$ black-body radiation constant

$$\Delta\vartheta = \vartheta_1-\vartheta_2 = \vartheta_{\mathrm{w}}-\vartheta_{\mathrm{a}}$$ the temperature difference between the wall and the ambient

For the temperature rise $(40 \div 50)\ \mathrm{K}$, calculate the radiation coefficient $\alpha_{\mathrm{r}} \approx 5+0,033(\vartheta_1-\vartheta_2)$, i.e., $\alpha_{\mathrm{r}} = (6,3 \div 6,65)\ \mathrm{W/(m^2 \cdot K)}$.

EEMC heating

Electrical and electromechanical converters are inhomogeneous bodies, but they are treated as homogeneous bodies. In the time differential $\mathrm{d}t$, thermal energy $P_l\,\mathrm{d}t$ is released, some of which is stored (temperature rise − first term on the right-hand side of the equation), some of which is dissipated to the surroundings by convection and radiation (second term on the right-hand side).

$$P_l\,\mathrm{d}t = m\,c\,\mathrm{d}(\Delta\vartheta) + \alpha A \Delta\vartheta\,\mathrm{d}t$$

The solution for the time differential is: $\mathrm{d}t = \dfrac{\dfrac{mc}{\Lambda_\mathrm{t}}}{\dfrac{P_l}{\Lambda_\mathrm{t}} - \Delta\vartheta}\,\mathrm{d}(\Delta\vartheta)$.

In the equation $\Lambda_\mathrm{t} = \alpha A$ is the thermal conductivity, the coefficient of conductivity $\alpha = \alpha_\mathrm{c} + \alpha_\mathrm{r}$ and the specific heat c $\left(\mathrm{W \cdot s/(kg \cdot K)}\right)$.

Time $t = -\dfrac{mc}{\Lambda_\mathrm{t}} \ln\left(\dfrac{P_l}{\Lambda_\mathrm{t}} - \Delta\vartheta\right) + C$

For $t = 0$ is $\Delta\vartheta = \Delta\vartheta_0$ or 0.

Time constant:

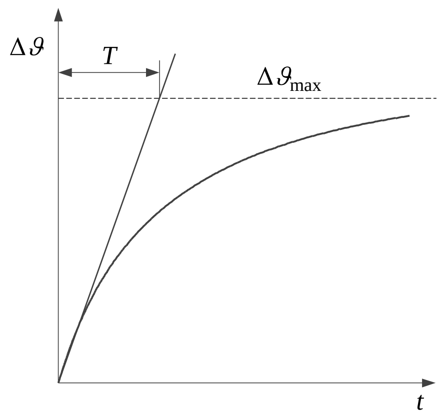
$T = \dfrac{mc}{\alpha A}$ (a few minutes to a few hours)

Equation for heating $\Delta\vartheta = \left(\dfrac{P_l}{\Lambda_\mathrm{t}} - \Delta\vartheta_0\right)\left(1 - \mathrm{e}^{-\frac{t}{T}}\right) + \Delta\vartheta_0$

The equation applies to heating at constant losses and constant cooling conditions.

$\Delta\vartheta_\mathrm{max} = \dfrac{P_l}{\Lambda_\mathrm{t}} = \dfrac{P_l}{\alpha A}$, if it is $\Delta\vartheta_0 = 0$, it will be:

$\Delta\vartheta = \Delta\vartheta_\mathrm{max}\left(1 - \mathrm{e}^{-\frac{t}{T}}\right)$.



EEMC cooling

$\Delta\vartheta_0 = \Delta\vartheta_\mathrm{max}$, $P_l = 0$

$\Delta\vartheta = \Delta\vartheta_\mathrm{max}\,\mathrm{e}^{-\frac{t}{T}}$

In general, the time constant T is larger for dimensionally larger EEMC (higher mass *m*).

For forced cooling it will be T smaller because it increases α (the heat transfer coefficient).

TRANSFORMER

Introduction

A transformer is a static power transmission device that converts an AC voltage and current system by electromagnetic induction into another voltage and current system, usually of different magnitudes and the same frequency.

Primary		Secondary
Single-phase system	$\rightarrow$	single-phase system
		or two-phase system
Three-phase system	$\rightarrow$	three-phase system
		or six-phase system
		or twelve-phase system

In general: m_1 phase system $\rightarrow$ m_2 phase system $(m_1 = m_2$ or $m_1 \neq m_2)$

The most elementary design of a single-phase transformer has two coils (windings):

1) a primary power supply coil,

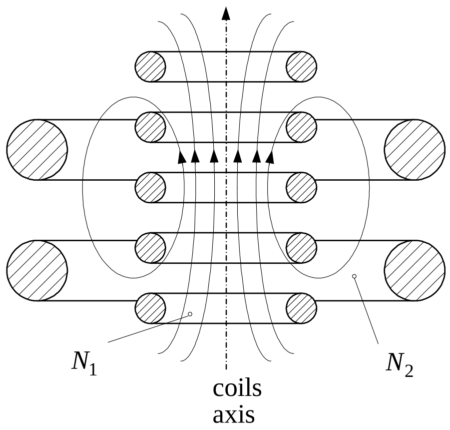
2) a secondary coil for the power output.

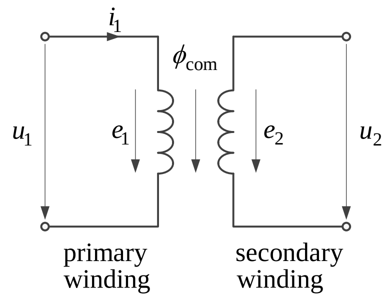
The two windings are usually separated galvanically.

Example: $N_1 = 5$ turns, $N_2 = 2$ turns

Current $i_1 \rightarrow \theta_1 = i_1 N_1 \rightarrow \phi_1$ (6 magnetic lines or density lines)

$$i_2 = 0 \rightarrow \theta_2 = i_2 N_2 = 0$$





Flux linkage of the primary winding:

$$\psi_1 = \left(3\times\frac{6}{6}+2\times\frac{4}{6}\right)\phi_1 = 4\frac{1}{3}\phi_1 \text{ (ideal } \psi_1 = 5\phi_1\text{)}$$

and of the secondary winding

$$\psi_2 = \left(2\times\frac{4}{6}\right)\phi_1 = 1\frac{1}{3}\phi_1 \text{ (ideal } \psi_2 = 2\phi_1\text{).}$$

Common flux: $\phi_{com} = \frac{4}{6}\phi_1 = \frac{2}{3}\phi_1$

Leakage flux: $\phi_{\sigma 1} = \frac{2}{6}\phi_1 = \frac{1}{3}\phi_1$

} Primary flux: $\phi_1 = \phi_{com} + \phi_{\sigma 1} = \frac{2}{3}\phi_1 + \frac{1}{3}\phi_1$

$$\psi_2 = \psi_{com2} = 1\frac{1}{3}\phi_1$$

$$\psi_1 = \psi_{com1} + \psi_{\sigma 1} = \left(5\times\frac{2}{3}\right)\phi_1 + \left(3\times\frac{1}{3}\right)\phi_1 = 4\frac{1}{3}\phi_1$$

Only the common flux ϕ_{com} is involved in the transformation process. The importance of the leakage flux, which is not involved in the transformation process, will be explained later.

The ideal transformer: $\psi_{\sigma 1} = \psi_{\sigma 2} = 0$.

The permeability of the medium (core) around which the winding is wound, $\mu \to \infty$,

$i_{1\mu}\ N_1 \to 0,\ i_{1\mu} \to 0$.

Operation mode

Assume $R_1 = R_2 = 0$

Primary winding $\to$ imposed voltage u_1. Secondary winding open.

The grid voltage u_1 drives a current through the primary winding:

$$i_1 \to \theta_1 \to \phi_1 \to \psi_1 .$$

The magnetic linkage ψ_1 must be such that equilibrium is created:

$$u_1 = -e_1 = -\left(-\frac{\mathrm{d}\psi_1}{\mathrm{d}t}\right) = \frac{\mathrm{d}\psi_1}{\mathrm{d}t} = \frac{\mathrm{d}\left(\psi_{\mathrm{com1}} + \psi_{\sigma 1}\right)}{\mathrm{d}t} = -\left(e_{\mathrm{com1}} + e_{\sigma 1}\right).$$

At the same time, an induced voltage appears in the secondary winding with N_2 turns:

$$-e_2 = -\frac{\left(-\mathrm{d}\psi_{\mathrm{sk2}}\right)}{\mathrm{d}t} = \frac{\mathrm{d}\psi_{\mathrm{sk2}}}{\mathrm{d}t} = u_2 .$$

The primary leakage flux $\phi_{\sigma 1}$, or magnetic linkage $\psi_{\sigma 1}$, creates an inductive voltage drop across the primary winding:

$-e_{\sigma 1} = \dfrac{\mathrm{d}\psi_{\sigma 1}}{\mathrm{d}t}$ and do not participate in the transformation process.

Transformer design

A two-coil transformer without an iron core (air transformer) has a large leakage flux $\phi_{\sigma 1}$ and $\phi_{\sigma 2}$ a small common flux $\phi_{\mathrm{com}} = \phi_1 - \phi_{\sigma 1}$.

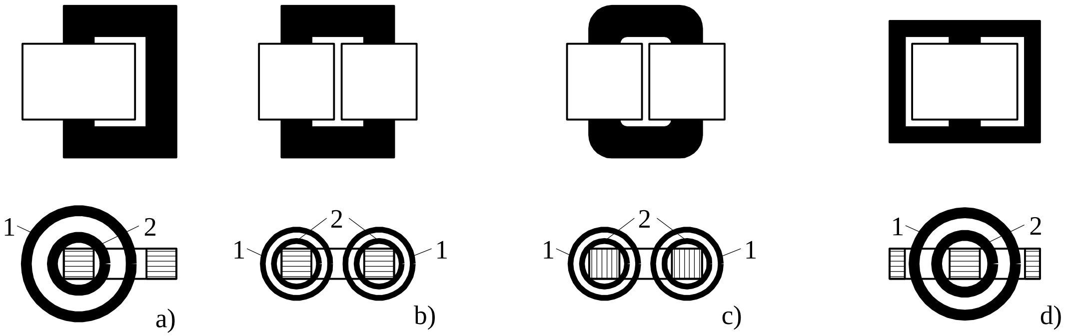
Therefore, a core of ferromagnetic material, i.e., an oriented transformer sheet, is used to guide the flux ($\mu_{\mathrm{rFe}} \approx 4000 \div 40000,$ maximum).

Design of single-phase transformer cores

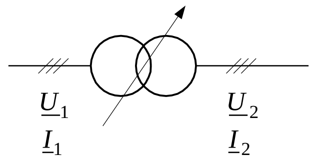
There is a core (column) type and a shell type.

In the core type, the winding is mounted on one or two columns connected by a yoke (Figures a and b). The same applies to the cutting ribbon core (Figure c).

In the shell type (Figure d), the flux in the yoke is half that in the column (half the cross section of the yoke $\rightarrow$ the lower the height of the transformer).



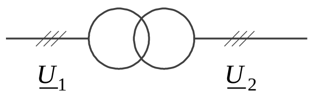
Transformer tasks

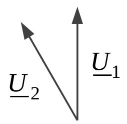


Energy transmission and

voltage adjustment

transmission $\rightarrow$ U_2 (high)

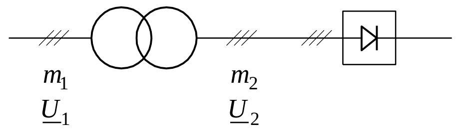


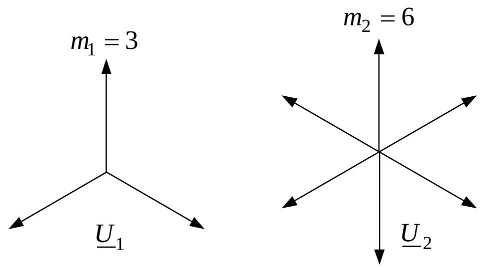


distribution $\rightarrow$ U_2 (low)

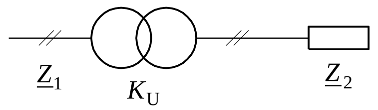
phase rotation

Industry – phase multiplier





Electrical circuits

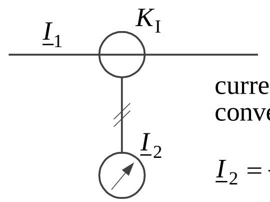


$\underline{Z}_1 = K_U^{\ 2}\,\underline{Z}_2$

impedance adjustment

K_U – voltage ratio

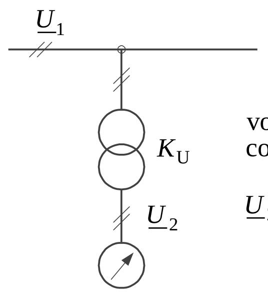
Measurement technique



current converter

$$\underline{I}_2 = \frac{\underline{I}_1}{K_I}$$

K_I – current ratio



voltage converter

$$\underline{U}_2 = \frac{\underline{U}_1}{K_U}$$

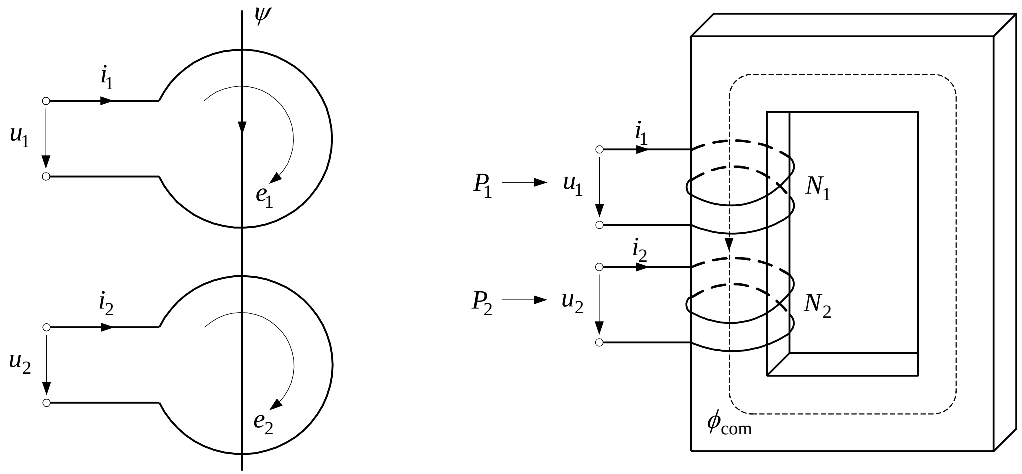
Single-phase transformer with iron core

Direction arrows

They are adapted as follows:

Both coils (windings) are wound clockwise.

The direction of the voltage drops across winding 1 is positive and so is the current. Since the magnetization of both windings are in the same direction, this determines the direction of the current in winding 2, and also the voltage drop across winding 2.



The positive direction also applies to the magnetic coupling through the coils (ψ), or the magnetic flux in the core (ϕ_{com}). The positive current in the coils generates positive ampere-turns for the integration path in the direction of the arrow through the coils.

Operation of an ideal transformer

The properties of the materials used are:

1) permeability of the magnetic circle $\mu_{\text{Fe}} = \infty$,

2) the electrical conductivity of the magnetic circuit $\gamma_{\text{Fe}} = 0$,

3) the permeability of air $\mu_{\text{air}} = 0$,

4) electrical conductivity of the conductors $\gamma_{\text{con}} = \infty$,

5) the magnetic circuit has no air gaps $\delta_{\text{gap}} = 0$.

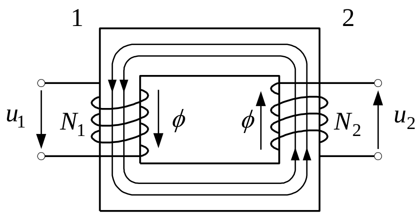
For a loop (winding) with ohmic resistance R and voltage drop Ri, the voltage equation is:

$$u = Ri - e = Ri + \frac{\mathrm{d}\psi}{\mathrm{d}t}.$$

Since $R = 0$, it applies to windings 1 and 2:

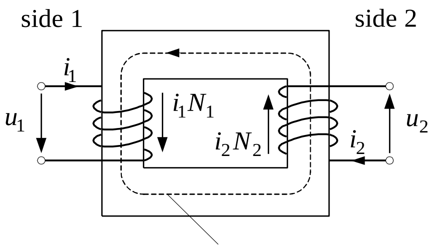
$$u_1 = -e_1 = \frac{\mathrm{d}\psi_1}{\mathrm{d}t},$$

$$u_2 = -e_2 = \frac{\mathrm{d}\psi_2}{\mathrm{d}t}.$$



Magnetic linkage of the two windings: $\psi_1 = N_1\phi$
$\psi_2 = N_2\phi$

This is true for an ideal transformer, because all the flux in the positive direction of the arrows passes through windings 1 and 2. It is therefore $\phi_1 = \phi_2 = \phi_{\mathrm{com}} = \phi$.



In the absence of eddy currents, ampere-turns operate along the integration path

i_1N_1 and i_2N_2.

$$\mu_{\mathrm{Fe}} = \infty \;\rightarrow\; \sum iN = \Theta = 0$$

$$i_1N_1 + i_2N_2 = 0$$

For the sinusoidal form of the primary applied voltage U_1 (rigid grid) and the magnetic linkage $(\underline{\Psi} = \hat{\Psi}\mathrm{e}^{\mathrm{j}\omega t})$ the complex notation of the ideal transformer equations is introduced:

$$\underline{U}_1 = -\underline{E}_1 = \mathrm{j}\omega\,\frac{\underline{\Psi}_1}{\sqrt{2}} = \mathrm{j}\omega\,N_1\frac{\underline{\Phi}}{\sqrt{2}},$$

$$\underline{U}_2 = -\underline{E}_2 = \mathrm{j}\omega\,\frac{\underline{\Psi}_2}{\sqrt{2}} = \mathrm{j}\omega\,N_2\frac{\underline{\Phi}}{\sqrt{2}},$$

$$\underline{I}_1N_1 + \underline{I}_2N_2 = 0.$$

The first two equations give the voltage ratio:

$$\frac{\underline{U}_1}{\underline{U}_2} = \frac{N_1}{N_2} \;\rightarrow\; \frac{U_1}{U_2}\,\mathrm{e}^{\mathrm{j}(\varphi_{\mathrm{u1}}-\varphi_{\mathrm{u2}})} = \frac{N_1}{N_2}.$$

$$K_U = \frac{U_1}{U_2} = \frac{N_1}{N_2} \text{ for } \varphi_{u1} = \varphi_{u2}$$

The voltage $\underline{U}_1$ or $\underline{U}_2$ dictates the magnitude (amplitude) and phase position of the flux.

$$\underline{\hat{\Phi}} = \frac{\sqrt{2}}{\omega}\frac{\underline{U}_1}{N_1} = \frac{\sqrt{2}}{\omega}\frac{\underline{U}_2}{N_2} \qquad \hat{\Phi} = \frac{\sqrt{2}}{\omega}\frac{U_1}{N_1} = \frac{\sqrt{2}}{\omega}\frac{U_2}{N_2}$$

$$\varphi_\Phi = \varphi_{u1} - \frac{\pi}{2} = \varphi_{u2} - \frac{\pi}{2} = \varphi_{e1} + \frac{\pi}{2} = \varphi_{e2} + \frac{\pi}{2} \qquad \left(4,44 = \frac{2\pi}{\sqrt{2}} = 4\frac{\pi/2}{\sqrt{2}} = 4 \cdot 1,11\right)$$

$E_1 = \omega N_1 \dfrac{\hat{\Phi}}{\sqrt{2}} = 4,44\, f\, N_1 \hat{\Phi}$ is the RMS value of the induced voltage in the primary and

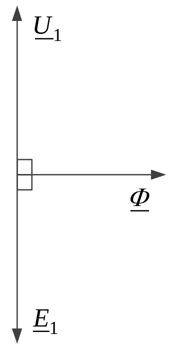
$E_2 = \omega N_2 \dfrac{\hat{\Phi}}{\sqrt{2}} = 4,44\, f\, N_2 \hat{\Phi}$ in the secondary winding.

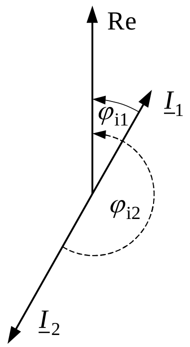
The phasor diagram shows the position $\underline{U}_1$, $\underline{E}_1$ and $\underline{\Phi}$. The third equation gives the current ratio K_I:

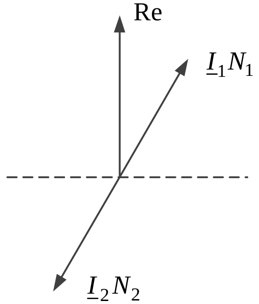
$$\frac{\underline{I}_1}{\underline{I}_2} = -\frac{N_2}{N_1} \rightarrow K_I = \frac{I_1}{I_2} = \frac{N_2}{N_1} \quad \text{for } \varphi_{i2} = \varphi_{i1} - \pi .$$

The current phasor diagram is shown in the middle Figure for an example $N_1 > N_2$.

The right Figure shows the ampere-turns for which: $\underline{I}_1 N_1 = -\underline{I}_2 N_2$.







No-load of an ideal transformer

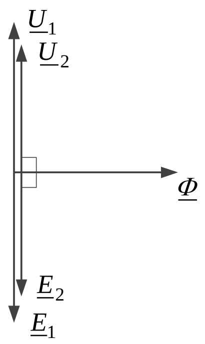
$\underline{U}_1 = \underline{U}_{\text{grid}}$ (grid voltage)

$\underline{I}_2 = 0$

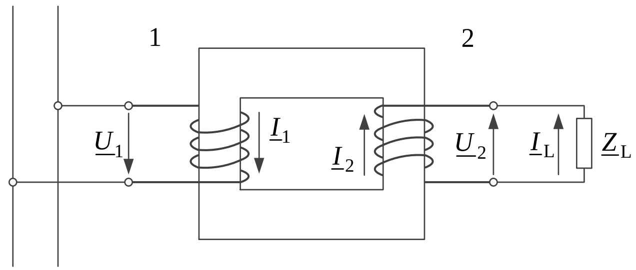
$$\underline{U}_2 = -\underline{E}_2 = \frac{N_2}{N_1}\underline{U}_1 = -\frac{N_2}{N_1}\underline{E}_1 = \mathrm{j}\omega N_2 \frac{\underline{\Phi}}{\sqrt{2}}$$

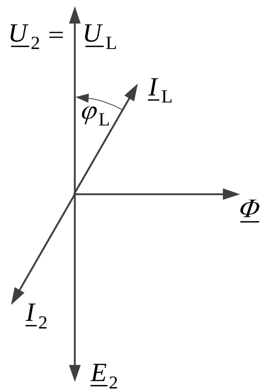
After $\underline{I}_2 = 0$ follows from $\underline{I}_1 N_1 + \underline{I}_2 N_2 = 0 \;\rightarrow\; \underline{I}_1 = 0$.

A phasor diagram is shown for the example $N_1 > N_2$.



Load of an ideal transformer





$\underline{U}_{\text{L}} = \underline{U}_2 \qquad \underline{Z}_{\text{L}} = \dfrac{\underline{U}_{\text{L}}}{\underline{I}_{\text{L}}}$

$\underline{I}_{\text{L}} = -\underline{I}_2$

Two current phasors appear on the secondary side of the diagram, whose phase position is shown in the example Figure: $\underline{Z}_{\text{L}} = R_{\text{L}} + \mathrm{j}X_{\text{L}}$.

According to the power flow theorem:

$$\mathrm{Re}(\underline{U}_{\text{L}}\underline{I}_{\text{L}}^*) > 0 \;\text{ and }\; \mathrm{Re}(\underline{U}_2\underline{I}_2^*) < 0\,.$$

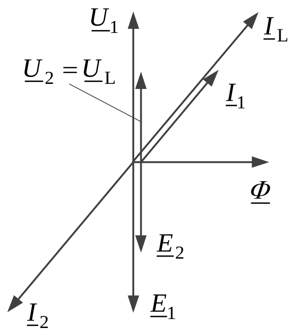
The magnitude and phase position of the voltage and flux are unaffected by the loading of the ideal transformer.

Valid: $\underline{U}_2 = -\underline{E}_2 = \frac{N_2}{N_1}\underline{U}_1 = -\frac{N_2}{N_1}\underline{E}_1 = \mathrm{j}\omega N_2 \frac{\Phi}{\sqrt{2}}$.

From the equilibrium condition $\underline{I}_1 N_1 + \underline{I}_2 N_2 = \frac{\Theta}{\sqrt{2}} = 0$ follow: $\underline{I}_1 = -\frac{N_2}{N_1}\underline{I}_2$.

At $\mu_{\mathrm{Fe}} = \infty$ or $R_\mathrm{m} = 0$ any deviation from the value $\Theta = 0$ would cause the flux and induced voltage to increase beyond all limits $(\Phi_\mathrm{m} = \Theta / R_\mathrm{m})$.

The following Figure shows a complete phasor diagram with the imposed voltage on the primary side $\underline{U}_1$ and the load $\underline{Z}_\mathrm{L} = R_\mathrm{L} + \mathrm{j}\,X_\mathrm{L}$ on the secondary side.



Energy balance:

$$P_1 = \mathrm{Re}(\underline{U}_1 \underline{I}_1^*) = -\mathrm{Re}\left(\frac{N_1}{N_2}\underline{U}_2 \frac{N_2}{N_1}\underline{I}_2^*\right) =$$

$$= -\mathrm{Re}(\underline{U}_2 \underline{I}_2^*) = -P_2 = \mathrm{Re}(\underline{U}_\mathrm{L}\underline{I}_\mathrm{L}^*) = P_\mathrm{L}$$

In an ideal transformer, power is transmitted without losses, only the voltage level changes according to the turn ratio:

$$U_2 = \frac{N_2}{N_1}U_1 = \frac{U_1}{K_\mathrm{U}}.$$

<u>Transformed</u> (reduced) quantities

It is not transparent for $\frac{N_1}{N_2} >> 1$ or $\frac{N_1}{N_2} << 1$ the phasor diagram.

Therefore, transformed quantities are introduced for voltage and current. For voltage it is:

$U_2' = \frac{N_1}{N_2}U_2 = K_\mathrm{U}U_2$. For the current, it is from the equal excitation condition:

$$\hat{\Theta}_2' = \sqrt{2}I_2' N_1 = \sqrt{2}I_2 N_2 = \hat{\Theta}_2 \;\rightarrow\; I_2' = \frac{N_2}{N_1}I_2 = K_\mathrm{I}I_2 = \frac{1}{K_\mathrm{U}}I_2.$$

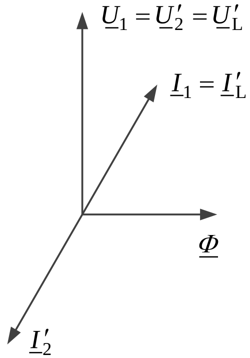
The power must also remain the same and valid:

$$U_2' I_2' = \frac{N_1}{N_2} U_2 \frac{N_2}{N_1} I_2 = U_2 I_2 \;\rightarrow\; \text{apparent power } S_2' = S_2 \text{ and therefore it is } P_2' = P_2 .$$

For an ideal transformer, the following equations now apply:

$$\underline{U}_1 = \mathrm{j}\omega \frac{\underline{\Psi}_1}{\sqrt{2}} = \mathrm{j}\omega N_1 \frac{\underline{\Phi}}{\sqrt{2}},$$

$$\underline{U}'_2 = \mathrm{j}\omega \frac{N_1}{N_2} \frac{\underline{\Psi}_2}{\sqrt{2}} = \mathrm{j}\omega \frac{N_1}{N_2} N_2 \frac{\underline{\Phi}}{\sqrt{2}} = \mathrm{j}\omega N_1 \frac{\underline{\Phi}}{\sqrt{2}}.$$



The difference of the two equations gives: $\underline{U}_1 - \underline{U}'_2 = 0$.

For magnetic flux linkage:

$$\Psi_2' = \frac{N_1}{N_2} \Psi_2 = K_U \Psi_2 \;\rightarrow\; \Psi_1 - \Psi_2' = 0 .$$

From the equation for the ampere-turns $\underline{I}_1 N_1 + \underline{I}_2 N_2 = 0$, the sum of $\underline{I}_1 + \underline{I}'_2 = 0$.

Under load, it is possible to write:

$$\underline{U}'_L = \frac{N_1}{N_2} \underline{U}_L \text{ and } \underline{I}'_L = \frac{N_2}{N_1} \underline{I}_L \rightarrow \underline{Z}'_L = \frac{\underline{U}'_L}{\underline{I}'_L} = \left(\frac{N_1}{N_2}\right)^2 \underline{Z}_L = K_U^2 \underline{Z}_L .$$

The above Figure shows the corresponding phasor diagram of an ideal transformer with transformed quantities of the secondary side to the primary side.

Real transformer operation

Because:

1. $\mu_{Fe} \neq \infty \rightarrow \oint_K \underline{\vec{H}}_{Fe} \cdot \mathrm{d}\vec{l} = \sqrt{2}\underline{I}_1 N_1 + \sqrt{2}\underline{I}_2 N_2 \neq 0$,

2. hysteresis loop $\rightarrow$ hysteresis losses in the iron P_{Feh},

3. $B = \mathrm{f}(H)$ (non-linear dependence between B and H) $\rightarrow$ higher harmonic components,

4. $\gamma_{\text{Fe}} \neq 0$ $\rightarrow$ eddy current $\rightarrow$ eddy current losses in the iron P_{Fee},

5. $\mu_{\text{air}} = \mu_0 \neq 0$ $\rightarrow$ leakage magnetic linkage $(\Psi_1 - \Psi_2' = \Psi_\sigma > 0)$,

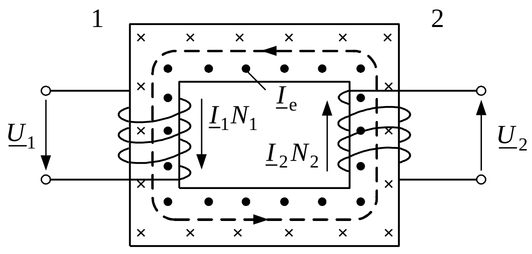
6. $\gamma_{\text{c}} \neq \infty$ $\rightarrow$ ohmic resistance of the conductors $\rightarrow$ voltage drops ($R_1 I_1$ and $R_2 I_2$) $\rightarrow$
   joule losses in the windings $P_{\text{Cu}} = R_1 I_1^2 + R_2 I_2^2$,

7. Ψ_σ $\rightarrow$ eddy current in the massive conductors $\rightarrow$
   additional eddy current losses P_{Cue},

8. ohmic and leakage reactance voltage drops $\rightarrow \underline{U}_1 - \underline{U}'_2 = 0$.

General equations

The crosses and dots indicate the direction of the eddy current $\underline{I}_{\text{e}}$.



The following equations apply:

$$\underline{U}_1 = R_1 \underline{I}_1 + \frac{1}{\sqrt{2}} \frac{\mathrm{d}\underline{\Psi}_1}{\mathrm{d}t},$$

$$\underline{U}_2 = R_2 \underline{I}_2 + \frac{1}{\sqrt{2}} \frac{\mathrm{d}\underline{\Psi}_2}{\mathrm{d}t},$$

$$\underline{I}_1 N_1 + \underline{I}_2 N_2 + \underline{I}_{\text{e}} = \frac{\underline{\Theta}}{\sqrt{2}}.$$

In order to facilitate the treatment of a real transformer and given the specific operating condition, the transformer is treated as:

a) a current-ideal voltage-real transformer,

b) a voltage-ideal current-real transformer.

Operation of a current-ideal voltage-real transformer

A real transformer is current-ideal for:

$\mu_{\mathrm{Fe}} = \infty$ and $\gamma_{\mathrm{Fe}} = 0$,
$\mu_{\mathrm{air}} = \mu_0 \neq 0$ and $\gamma_{\mathrm{c}} \neq \infty$.

No excitation is required to magnetize iron, and therefore

$$\underline{I}_1 N_1 + \underline{I}_2 N_2 = 0 \;\rightarrow\; \underline{I}_1 + \underline{I}'_2 = 0 .$$

As a result of the leakage, the $\underline{U}_1 - \underline{U}'_2 \neq 0$.

General equations describe the state of a current-ideal real transformer with transformed quantities (if the $\underline{\Psi} = \hat{\Psi} e^{j\omega t}$):

$$\underline{U}_1 = R_1 \underline{I}_1 + j\omega \frac{\underline{\Psi}_1}{\sqrt{2}} ,$$

$$\underline{U}'_2 = R'_2 \underline{I}'_2 + j\omega \frac{\underline{\Psi}'_2}{\sqrt{2}} ,$$

$$\underline{I}_1 + \underline{I}'_2 = 0 .$$

The equal loss condition gives: $I'^2_2 R'_2 = I^2_2 R_2 \rightarrow$

$$\rightarrow R'_2 = \left(\frac{I_2}{I'_2} \right)^2 R_2 = K^2_{\mathrm{U}} R_2 ,$$ because the third equation implies $\underline{I}'_2 = -\underline{I}_1$.

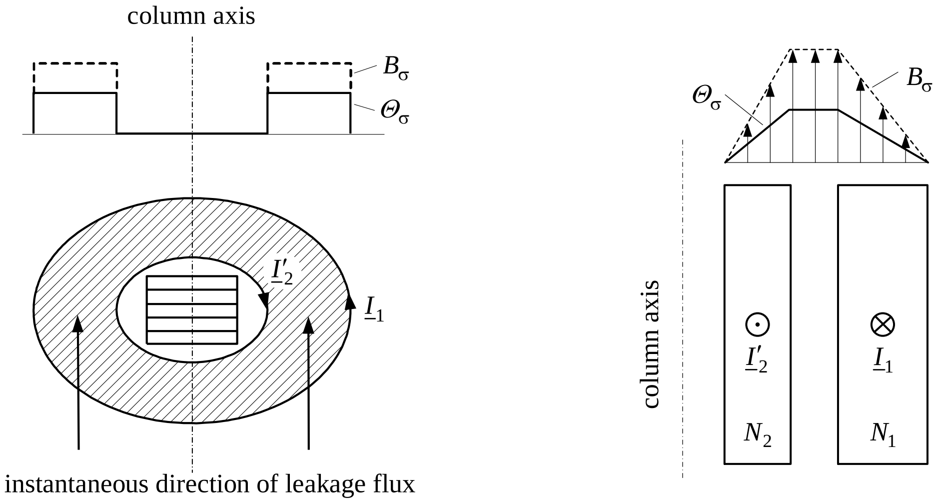
The voltage difference from the first two equations is:

$$\underline{U}_1 - \underline{U}'_2 = (R_1 + R'_2)\underline{I}_1 + j\omega(\underline{\Psi}_1 - \underline{\Psi}'_2)/\sqrt{2} = \underline{U}_{\mathrm{r}} + \underline{U}_{\sigma} .$$

The $\underline{U}_{\mathrm{r}} = (R_1 + R'_2)\underline{I}_1 = R\underline{I}_1$ part (ohmic voltage drop) is in phase with the current.

The $\underline{U}_{\sigma} = \mathrm{j}\omega(\underline{\Psi}_1 - \underline{\Psi}'_2)/\sqrt{2}$ part (inductive voltage drop) overtakes the leakage magnetic linkage by 90°.

Since $\underline{I}_1 = -\underline{I}'_2$, the ampere-turns in the core column are opposed, and the difference in magnetic linkage $\underline{\Psi}_1 - \underline{\Psi}'_2$ is present only in the space between the windings. In the Figure, each winding is shown with only one thin turn wound around the core in the same direction (usually the direction of the right-hand screw).



The winding always has a finite thickness. Therefore, the leakage in the cross-section has the shape of a trapezoid. The leakage linkage is written with the total inductance:

$\underline{\Psi}_1 - \underline{\Psi}'_2 = L_{\sigma}\underline{I}_1\sqrt{2}$ and, hence, voltage drops $\underline{U}_1 - \underline{U}'_2 = \underline{U}_{\mathrm{r}} + \underline{U}_{\sigma} = (R + \mathrm{j}X_{\sigma})\underline{I}_1$.

This equation corresponds to the following phasor diagram, where $\underline{U}_1$ is the imposed voltage and the load $\underline{Z}_{\mathrm{L}} = R_{\mathrm{L}} + \mathrm{j}X_{\mathrm{L}}$.

$\underline{U}_1 - \underline{U}'_2$ is the hypotenuse of a rectangular − **Kappa triangle.**

Energy balance

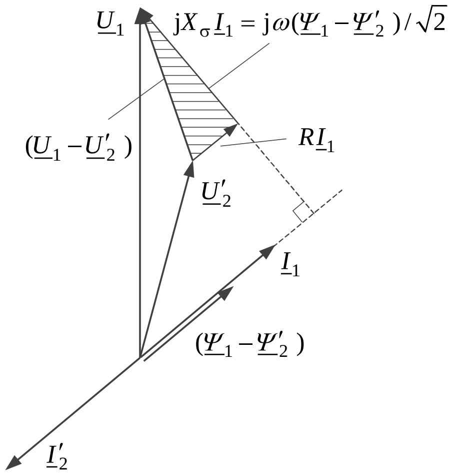
If there is $\underline{I}_1 = -\underline{I}'_2$ and $\underline{U}'_2 = \underline{U}'_\mathrm{L}$, it will be:

$$P_1 = \mathrm{Re}\left(\underline{U}_1 \underline{I}_1^*\right) = \mathrm{Re}\left(\underline{U}'_\mathrm{L} \underline{I}'^*_\mathrm{L} + (R_1 + R'_2)\underline{I}_1 \underline{I}_1^*\right) =$$

$$= P_\mathrm{L} + R_1 I_1^2 + R_2 I_2^2 ,$$

$$P_1 = P_\mathrm{L} + P_\mathrm{Cu} .$$

The received power is used for the power at the consumer and the losses in the transformer winding.



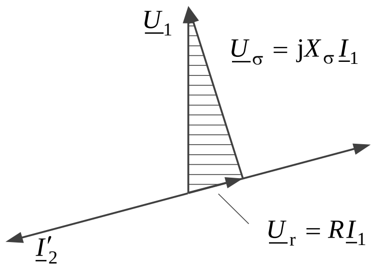
In a short-circuit current-ideal and voltage-real transformer at the imposed grid voltage

$\underline{U}_1 = \underline{U}_\mathrm{grid}$ and $\underline{U}_2 = 0$ will be:

$$\underline{U}_1 = \underline{U}_\mathrm{r} + \underline{U}_\sigma = (R + \mathrm{j}X_\sigma)\underline{I}_1 .$$

Short-circuit current: $\underline{I}_1 = -\underline{I}'_2 = \dfrac{\underline{U}_1}{\underline{Z}_\mathrm{sc}}$, $\underline{Z}_\mathrm{sc} = R + \mathrm{j}X_\sigma$,

$(R = R_\mathrm{sc} = R_1 + R'_2)$, $(X_\sigma = X_\mathrm{sc} = X_{\sigma 1} + X'_{\sigma 2})$.



Operation of a voltage-ideal current-real transformer

A real transformer is voltage-ideal for:

$\underline{U}_1 - \underline{U}'_2 = 0$ on condition

$\mu_\mathrm{air} = 0,$ $\gamma_\mathrm{c} = \infty$ (no voltage drops or losses P_Cu)

$\mu_\mathrm{Fe} \neq \infty,$ $\gamma_\mathrm{Fe} \neq 0$ (needs magnetizing current and has losses P_Fe)

Because $\mu_\mathrm{air} = 0$ will be: $\Psi_1 = N_1 \Phi$ and $\Psi_2 = N_2 \Phi$.

Voltage equations: $u_1 = N_1 \frac{\mathrm{d}\phi}{\mathrm{d}t}$

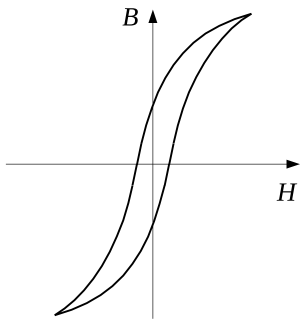
$$u_2 = N_2 \frac{\mathrm{d}\phi}{\mathrm{d}t}$$

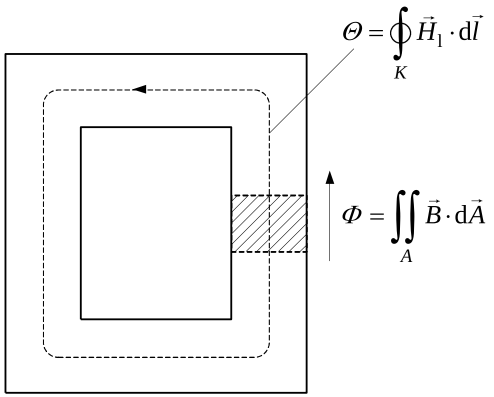
For a voltage of cosine (sine) form $u_1 = \hat{U}_1 \cos(\omega t + \varphi_{u1})$ to be applied, the flux must also be of sine (cosine) form:

$$\phi = \frac{\hat{U}_1 \cos(\omega t + \varphi_{u1} - \pi/2)}{\omega N_1} = \hat{\Phi} \sin(\omega t + \varphi_{u1}) \, .$$

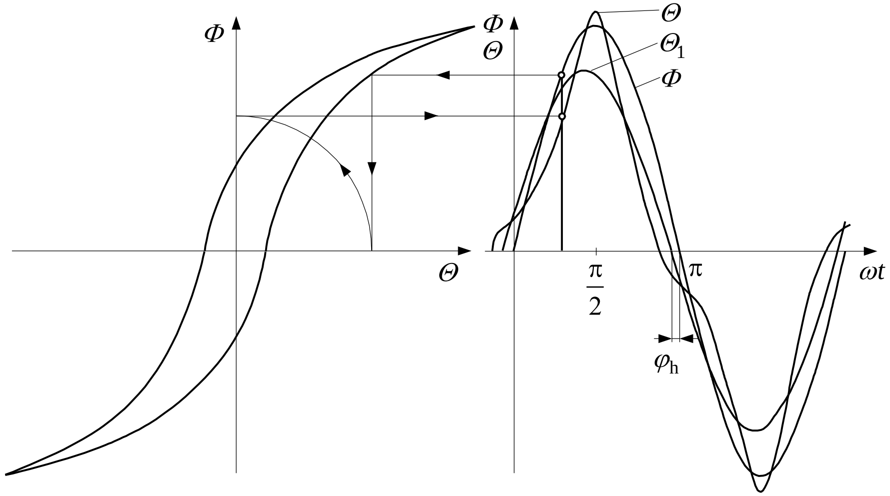
Magnetization phenomena in operation on a rigid grid

The magnetic curve of the core material $B = \mathrm{f}(H)$ with the dimensions of the core and the definitions of $\Phi = \iint_A \vec{B} \cdot \mathrm{d}\vec{A}$ and $\Theta = \oint_K \vec{H}_1 \cdot \mathrm{d}\vec{l}$ gives us the magnetization characteristic $\Phi = \mathrm{f}(\Theta)$ of the magnetic circuit.

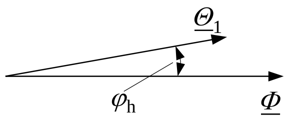


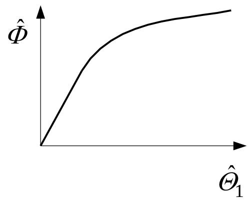


The sinusoidal voltage determines the sinusoidal flux with phase shift 90°. Taking into account the non-linear magnetization characteristic (hysteresis loop) $\Phi = \mathrm{f}(\Theta)$ results in excitation ampere-turns that are not sinusoidal in shape.



According to Fourier, a function $\Theta = \mathrm{f}(t)$ has odd-order components $(\nu = 3, 5, 7, ...)$ in addition to the fundamental harmonic component $(\nu = 1)$. Θ_1 overtakes the flux Φ by an angle of φ_h. Between $\hat{\Phi}$ and $\hat{\Theta}_1$ a non-linear dependence applies.





Losses in the core

They are divided into hysteresis losses (due to the alternating magnetization) and eddy current losses (due to the eddy current in the core lamellae).

Hysteresis losses

$$P_\mathrm{Feh} = \frac{1}{T} \int_t^{t+T} \Theta \,\mathrm{d}\Phi = f \int_t^{t+T} \Theta \,\mathrm{d}\Phi = \mathrm{f}(f, \Phi)$$. Approximately valid $P_\mathrm{Feh} \propto f\, \hat{\Phi}^2$.

Eddy current losses

In a short-circuit loop (core lamella) the equation applies:

$0 = R_e \underline{I}_e - \underline{E}_e$. Because it is $\underline{E}_e = -j\omega \dfrac{\underline{\Phi}}{\sqrt{2}}$, it will be:

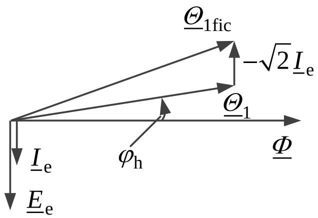
$$\underline{I}_e = \frac{\underline{E}_e}{R_e} = -j\frac{\omega}{R_e}\,\frac{\underline{\Phi}}{\sqrt{2}}$$

The eddy current contributes to the ampere-turns on the integration path through the magnetic core ($N_e = 1$):

$$\underline{\Theta}_1 = (\underline{I}_1 N_1 + \underline{I}_2 N_2 + \underline{I}_e)\sqrt{2}\ .$$

Combining $\underline{\Theta}_1$ and $\underline{I}_e$ gives

the total (fictitious) ampere-turns:

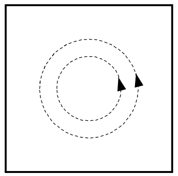


$$\underline{\Theta}_{1fic} = \underline{\Theta}_1 - \sqrt{2}\,\underline{I}_e = \sqrt{2}\,\underline{I}_1 N_1 + \sqrt{2}\,\underline{I}_2 N_2\ .$$

Eddy current losses in the core

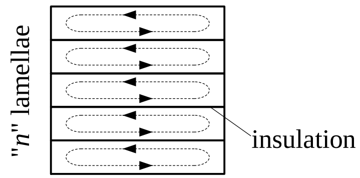
$$P_{Fee} = I_e^2 R_e = \left(\frac{\omega\hat{\Phi}}{R_e\sqrt{2}}\right)^2 R_e = \frac{\omega^2}{2R_e}\,\hat{\Phi}^2 = f(f,\hat{\Phi})^2$$

massive core



$\Phi,\ R_{Fe},\ E_e$

laminated core



$\Phi / n,\ nR_{Fe},\ E_e / n$

The eddy current losses in the massive core are: $P_{\mathrm{Fee}} = E_{\mathrm{e}} I_{\mathrm{e}}$, if the current is

$$I_{\mathrm{e}} = -\frac{\mathrm{d}\Phi}{\mathrm{d}t}\frac{1}{R_{\mathrm{Fe}}} \text{ and } I_{\mathrm{en}} = -\frac{\mathrm{d}(\Phi / n)}{\mathrm{d}t}\frac{1}{nR_{\mathrm{Fe}}} = -\frac{1}{n^2}\frac{\mathrm{d}\Phi}{\mathrm{d}t}\frac{1}{R_{\mathrm{Fe}}}.$$

For an n-times laminated core thickness d, there are eddy current losses:

$$P_{\mathrm{Fe(en)}} = n\frac{E_{\mathrm{e}}}{n}\frac{I_{\mathrm{e}}}{n^2} = \frac{P_{\mathrm{Fee}}}{n^2} \rightarrow P_{\mathrm{Fe(en)}} = \mathrm{f}\left(\frac{1}{n^2}\right) = \mathrm{f}(d^2).$$

Total magnetization losses in the core:

$$P_{\mathrm{Fe}} = P_{\mathrm{Feh}} + P_{\mathrm{Fee}} = \mathrm{f}(f, \hat{\Phi}).$$

The losses in the iron core are given as specific losses per kilogram of mass for a given thickness of steel:

$$p_{\mathrm{Fe}} = k_{\mathrm{h}} f B^x + k_{\mathrm{e}} f^2 B^2 \quad (\mathrm{W/kg}).$$

The exponent for hysteresis losses $x \approx 1,6 \div 2,8$ and depends on the value of B.

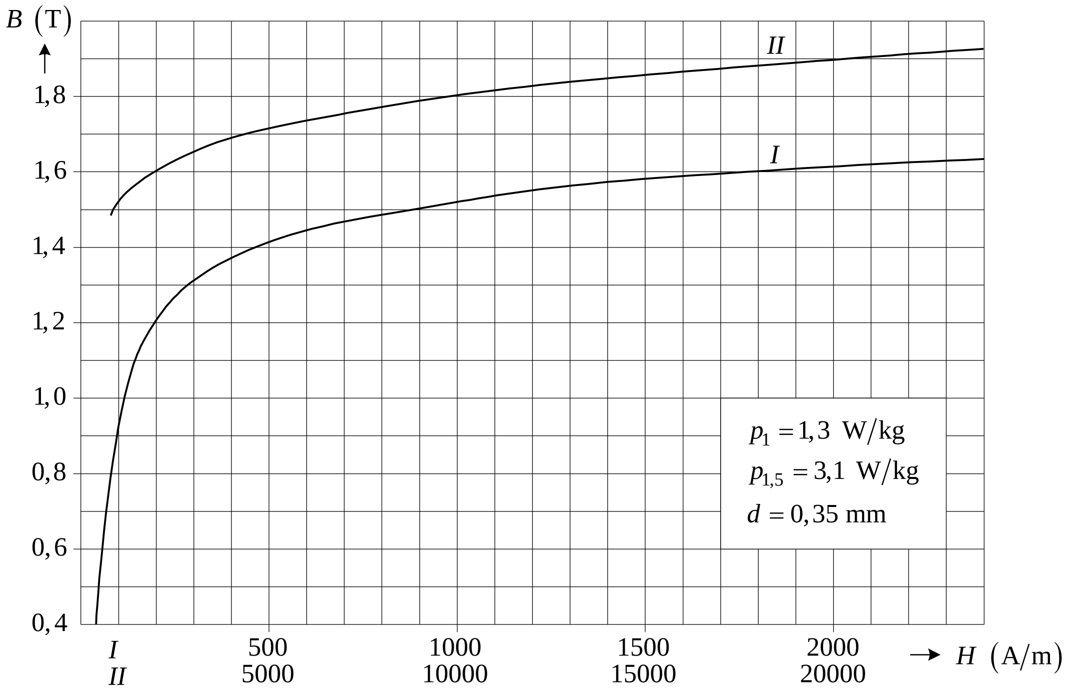
Electrical steel manufacturers give the losses as curves $p_{\mathrm{Fe}} = \mathrm{f}(B)$ for $f = 50\ \mathrm{Hz}$, or at magnetic flux densities of 1 T and 1,5 T.

The specific losses for any one B_{x} are calculated using the equation:

$$p_{\mathrm{Fex}} \approx p_{\mathrm{Fe}}\left(\frac{B_{\mathrm{x}}}{B}\right)^2.$$

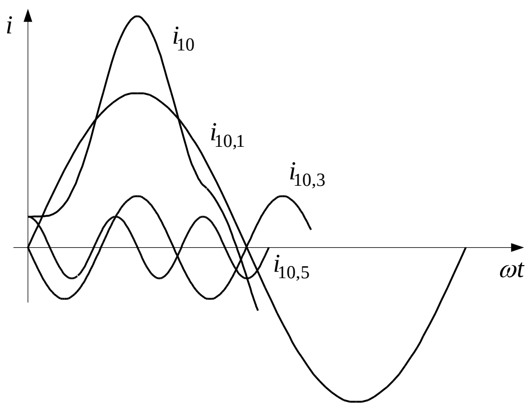
Example of a curve $B = \mathrm{f}(H)$ with losses data

The curve for the magnetic flux density is given as a function of the magnetic field intensity in two different scales, marked I and II. Steel sheet thickness $d = 0,35\ \mathrm{mm}$. The value for H can be given in RMS values and measured, e.g., at 50 Hz, (AC curve), or in peak values when magnetizing with a direct current (DC curve).



No-load current

The no-load current i_{10} is calculated from θ, which contains higher harmonic components.



$i_{10,1} = \dfrac{\theta_1}{N_1} - \dfrac{i_e}{N_1}$ (fundamental harmonic of the no-load current)

$$i_{10} = \left((\theta_1 - i_\nu) + \theta_3 + \theta_5 + \ldots + \theta_\nu\right) / N_1$$

$$i_{10} = i_{10,1} + i_{10,3} + i_{10,5} + \ldots + i_{10,\nu}$$

Fourier analysis gives the amplitude of the currents $\hat{I}_{10\nu}$ and, hence, $I_{10\nu} = \dfrac{\hat{I}_{10\nu}}{\sqrt{2}}$.

The common no-load current will be:

$$I_{10} = \sqrt{I_{10,1}^2 + I_{10,3}^2 + I_{10,5}^2 + \ldots + I_{10,\nu}^2} \;.$$

If the higher harmonic components are neglected, only the fundamental frequency magnitudes remain. Therefore, a complex notation of the equations is used.

$$\underline{U}_1 = -\underline{E}_1 = \mathrm{j}\omega \frac{\underline{\Psi}_1}{\sqrt{2}} = \mathrm{j}\omega N_1 \frac{\underline{\Phi}}{\sqrt{2}}$$

$$\underline{U}_2 = -\underline{E}_2 = \mathrm{j}\omega \frac{\underline{\Psi}_2}{\sqrt{2}} = \mathrm{j}\omega N_2 \frac{\underline{\Phi}}{\sqrt{2}}$$

Using transformed values, it is valid:

$$\underline{U}'_2 = -\underline{E}'_2 = \mathrm{j}\omega \frac{\underline{\Psi}'_2}{\sqrt{2}} = \mathrm{j}\omega N_1 \frac{\underline{\Phi}}{\sqrt{2}}$$

and follows $\underline{U}_1 - \underline{U}'_2 = 0 \rightarrow \underline{\Psi}_1 - \underline{\Psi}'_2 = 0$ (no leakage magnetic fields).

The equation of equilibrium of ampere-turns:

$$\underline{I}_1 N_1 + \underline{I}_2 N_2 = \frac{\underline{\Theta}_1}{\sqrt{2}} - \underline{I}_{\mathrm{e}}$$

and with transformed values

$$\underline{I}_1 + \underline{I}'_2 = \frac{1}{N_1}\left(\frac{\underline{\Theta}_1}{\sqrt{2}} - \underline{I}_{\mathrm{e}}\right) = \frac{\underline{\Theta}_{1\mathrm{fic}}}{\sqrt{2}\,N_1} \approx \underline{I}_{10} \text{ (the higher harmonic components are neglected).}$$

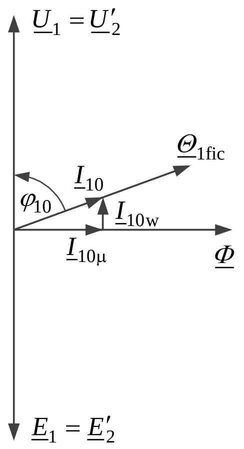
The deviation from ideal conditions is represented by the ampere-turns Θ_1 required for excitation and reverse action I_{e} in the core.

No-load voltage-ideal current-real transformer

$\underline{I}_2 = 0 \;\rightarrow$ the voltage on the primary side is equal to the grid voltage.

$$\underline{U}_1 = \underline{U}_{\text{grid}} \rightarrow \underline{\Phi} = \frac{\sqrt{2}\underline{U}_1}{\mathrm{j}\omega N_1}$$

$$\underline{U}'_2 = \underline{U}_1,\ \underline{I}_1 \equiv \underline{I}_{10} = \frac{\underline{\Theta}_{1\text{fic}}}{\sqrt{2}\, N_1}$$



The energy balance:

$$P_{10} = P_{\text{mag}} = P_{\text{Fe}} = P_{\text{Feh}} + P_{\text{Fee}}$$

$$P_{10} = \text{Re}\left(\underline{U}_1 \underline{I}^*_{10}\right)$$

$$P_{10} = U_1 I_{10} \cos\varphi_{10}$$

Load of a voltage-ideal current-real transformer

$\underline{U}_1$, $\underline{Z}_{\text{L}}$ = load impedance

$$\underline{I}_2 = -\underline{I}_{\text{L}} = -\frac{\underline{U}_{\text{L}}}{\underline{Z}_{\text{L}}} = -\frac{\underline{U}_2}{\underline{Z}_{\text{L}}} = -\frac{N_2}{N_1}\frac{\underline{U}_1}{\underline{Z}_{\text{L}}}$$

Even under load, the imposed (applied) voltage $\underline{U}_1$ dictates the magnitude and phase of the flux $\underline{\Phi} = \sqrt{2}\underline{U}_1/(\mathrm{j}\omega N_1)$. So the $\underline{\Theta}_{1\text{fic}}$ stays the same as in no-load.

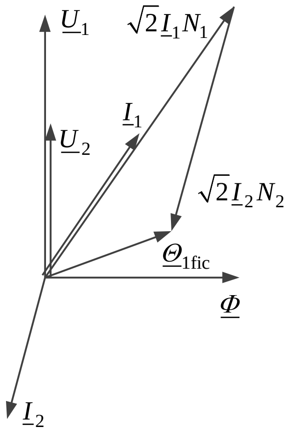
Load with $\underline{I}_2 \rightarrow \underline{I}_2 N_2 \rightarrow \underline{I}_1 \rightarrow \underline{I}_1 N_1 + \underline{I}_2 N_2 = \dfrac{\underline{\Theta}_{1\text{fic}}}{\sqrt{2}}$

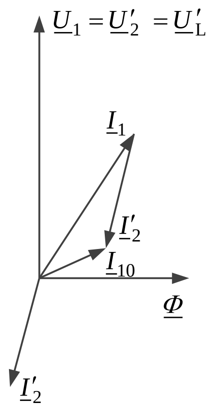
$$\underline{\Theta}_{1\text{fic}} \rightarrow \underline{\Phi} \rightarrow \underline{E}_1 \rightarrow \underline{U}_1 + \underline{E}_1 = 0$$

By introducing transformed secondary values on the primary side, it is valid:

$$\underline{I}_1 + \underline{I}'_2 = \frac{\underline{\Theta}_{1\text{fic}}}{\sqrt{2}\, N_1} \rightarrow \underline{I}_1 + \underline{I}'_2 = \underline{I}_{10}\,.$$

Every change $\underline{I}_2$ causes a change $\underline{I}_1$, so that $\underline{I}_{10} = \text{const.}$





Energy balance

$$P_1 = \mathrm{Re}\left(\underline{U}_1 \underline{I}_1^*\right) = \mathrm{Re}\left(\underline{U}_L \underline{I}_L^* + \underline{U}_1 \underline{I}_{10}^*\right) = P_L + P_{mag} = P_L + P_{Fe}$$

The accepted power P_1 covers the power of the consumer and the losses in the iron, but not the losses in the transformer windings.

Analytical treatment

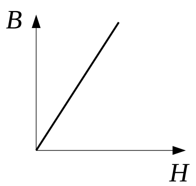
General equations for stationary operation

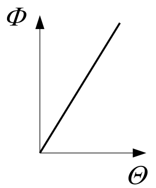
The derivation will be carried out for linear transformer theory, where:

$$\mu_{Fe} = \text{const.}, \quad \rightarrow \quad \text{no hysteresis loop and } \gamma_{Fe} = 0 \rightarrow \text{hence } P_{Fe} = 0$$

$$\mu_{air} = \mu_0 = \text{const.} \quad \text{and } \gamma_c = \text{const.}$$

The linear theory implies linear relationships between the flux linkage and the currents.





$$\underline{\Psi}_1 = L_1 \sqrt{2}\,\underline{I}_1 + L_{12} \sqrt{2}\,\underline{I}_2$$

$$\underline{\Psi}_2 = L_{21} \sqrt{2}\,\underline{I}_1 + L_2 \sqrt{2}\,\underline{I}_2$$

It is also true that $L_{12} = L_{21}$. Now write the two voltage equations:

$$\underline{U}_1 = R_1\underline{I}_1 + \mathrm{j}\omega\frac{\underline{\Psi}_1}{\sqrt{2}} = R_1\underline{I}_1 + \mathrm{j}X_1\underline{I}_1 + \mathrm{j}X_{12}\underline{I}_2,$$

$$\underline{U}_2 = R_2\underline{I}_2 + \mathrm{j}\omega\frac{\underline{\Psi}_2}{\sqrt{2}} = R_2\underline{I}_2 + \mathrm{j}X_{21}\underline{I}_1 + \mathrm{j}X_2\underline{I}_2.$$

By introducing transformed quantities for winding 2 with already known relationships

$U_2' = K_\mathrm{U}U_2,\qquad I_2' = K_\mathrm{I}I_2,\qquad \Psi_2' = K_\mathrm{U}\Psi_2$, continue to

$X_{12}' = X_{21}' = K_\mathrm{U}X_{12},\quad X_2' = K_\mathrm{U}^2X_2$ and $R_2' = K_\mathrm{U}^2R_2$

it is: $\underline{U}_1 = R_1\underline{I}_1 + \mathrm{j}\omega\frac{\underline{\Psi}_1}{\sqrt{2}} = R_1\underline{I}_1 + \mathrm{j}X_1\underline{I}_1 + \mathrm{j}X_{12}'\underline{I}_2'$,

$$\underline{U}'_2 = R_2'\underline{I}'_2 + \mathrm{j}\omega\frac{\underline{\Psi}'_2}{\sqrt{2}} = R_2'\underline{I}'_2 + \mathrm{j}X_{21}'\underline{I}_1 + \mathrm{j}X_2'\underline{I}'_2.$$

The difference of the two voltage equations gives the deviation from the ideal situation.

$$\underline{U}_1 - \underline{U}'_2 = R_1\underline{I}_1 - R_2'\underline{I}'_2 + \mathrm{j}\omega(\underline{\Psi}_1 - \underline{\Psi}'_2)/\sqrt{2}$$

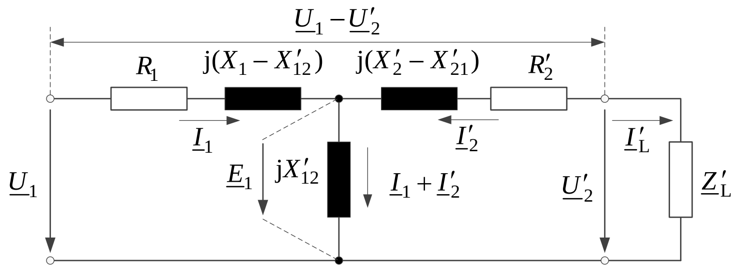
Transformer equivalent circuit

A transformer with two electrically isolated windings, i.e., isolated circuits, is converted into a circuit with electrically coupled circuits. This is achieved by adding the first voltage equation $\pm\mathrm{j}X_{12}'\underline{I}_1$ and the second equation $\pm\mathrm{j}X_{21}'\underline{I}'_2$, and combining the individual terms in a meaningful way.

$$\underline{U}_1 = R_1\underline{I}_1 + \mathrm{j}(X_1 - X_{12}')\underline{I}_1 + \mathrm{j}X_{12}'(\underline{I}_1 + \underline{I}'_2) = R_1\underline{I}_1 + \mathrm{j}(X_1 - X_{12}')\underline{I}_1 - \underline{E}_1$$

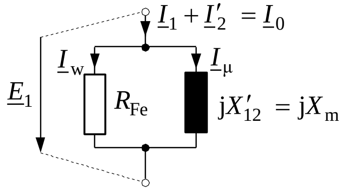
$$\underline{U}'_2 = R_2'\underline{I}'_2 + \mathrm{j}(X_2' - X_{21}')\underline{I}'_2 + \mathrm{j}X_{21}'(\underline{I}_1 + \underline{I}'_2) = R_2'\underline{I}'_2 + \mathrm{j}(X_2' - X_{21}')\underline{I}'_2 - \underline{E}'_2$$

The following circuit corresponds to these two equations.

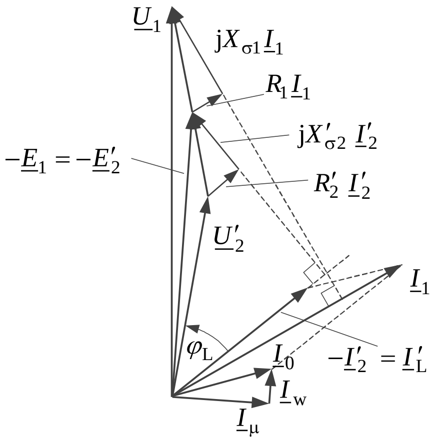


Deviation from voltage-ideal transformer $\rightarrow$ voltage drop $\underline{U}_1 - \underline{U}'_2$ across longitudinal links, deviation from current-ideal transformer $\rightarrow$ magnetizing current $\underline{I}_\mu = \underline{I}_1 + \underline{I}'_2$ flowing across the transverse link.

Further, $(X_1 - X'_{12})$ is the total leakage reactance between the two windings (no-load winding of side 2) for windings 1 and 2 on the same column, and, analogously, $(X'_2 - X'_{21})$ is the total leakage reactance (no-load winding of side 1). If the two windings are separated, then: $X_1 - X'_{12} = X_{\sigma 1}$ and $X'_2 - X'_{21} = X'_{\sigma 2}$. Taking into account the losses in the iron $P_{10} = P_{Fe}$ $(\underline{I}_0 = \underline{I}_1 + \underline{I}'_2)$, the equivalent resistance for the losses in the iron R_{Fe} is added in the equivalent circuit in parallel to the magnetizing (main) reactance $X'_{12} = X_m$.



Now it is possible to draw a complete phasor diagram of a loaded transformer with impedance $\underline{Z}_L = R_L + \mathrm{j}X_L$.



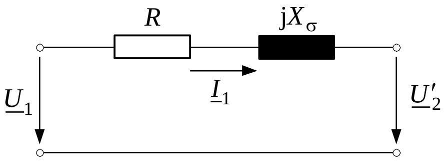
Approximate situation

If the transformer is excluded from the observation during no-load and at low loads, it will be $\underline{I}_1 + \underline{I}'_2 = 0 \rightarrow \underline{I}'_2 = -\underline{I}_1$ and the difference of the voltage equations:

$$\underline{U}_1 - \underline{U}'_2 = (R_1 + R'_2)\underline{I}_1 + \mathrm{j}\big((X_1 - X'_{12}) + (X'_2 - X'_{21})\big)\underline{I}_1 ,$$

$$\underline{U}_1 - \underline{U}'_2 = (R + \mathrm{j}X_\sigma)\underline{I}_1 .$$

The equation $\underline{U}_1 - \underline{U}'_2$ is illustrated by a simplified equivalent circuit.



Transformer tests

They are carried out at no-load and in a permanent short-circuit. They are used to determine the actual operating conditions of the transformer.

No-load test

For an $I_2 = 0 \rightarrow I_1 = I_{10} << I_{1\mathrm{N}} \rightarrow$ transformer is the voltage-ideal. The measurements will give:

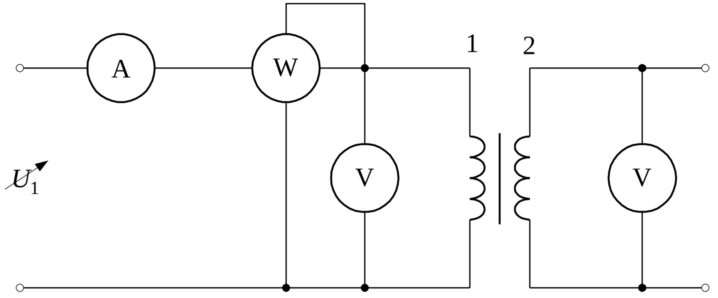
1) nominal voltage ratio $K_\mathrm{U} = \dfrac{U_1}{U_2} \approx \dfrac{E_1}{E_2} = \dfrac{N_1}{N_2}$ and

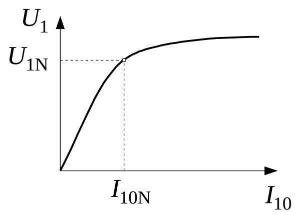
2) the no-load characteristic.

The sinusoidal voltage $\underline{U}_1$ is said to dictate the sinusoidal flux, because $\underline{U}_1 + \underline{E}_1 = 0$,

$\rightarrow \hat{\Phi} = \dfrac{\sqrt{2}U_1}{\omega N_1}$ generated by $\hat{\Theta}_{1\mathrm{fic}} = \sqrt{2} I_{10} N_1$.

Therefore, due to $E_1 \approx U_1 = \mathrm{f}(\hat{\Phi})$, $\hat{\Phi} = \mathrm{f}(\hat{\Theta})$ and $\hat{\Theta}_{1\mathrm{fic}} = \mathrm{f}(I_{10})$, the no-load characteristic $U_1 = \mathrm{f}(I_{10})$ is similar to the magnetizing characteristic or the magnetic curve, if the air gap can be neglected.



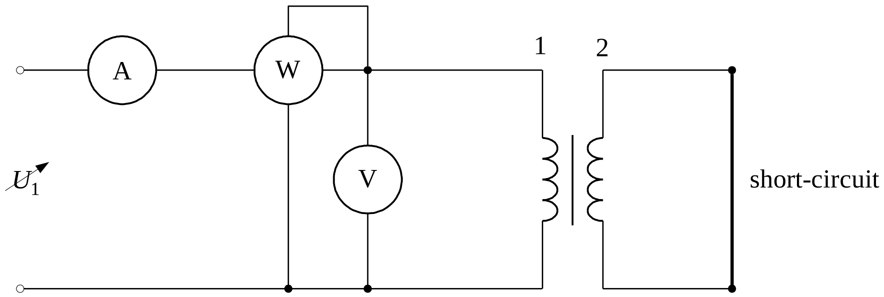


3) Iron losses

Since $I_{10} << I_{\mathrm{N}}$, the losses in the primary winding are negligible ($I_{10} \approx 0{,}01 \div 0{,}02 I_{\mathrm{N}}$). Measurement: $P_{\mathrm{w}} \approx P_{\mathrm{Fe0}}$ (at $U = U_{\mathrm{N}}$) gives the magnetization losses. From the losses it is possible to calculate the equivalent resistance R_{Fe}, $I_{0\mathrm{w}}$, from I_0 still $I_{0\mu}$ and $X'_{12} = X_{\mathrm{m}}$.

Short circuit test

The switching scheme measures the current $I_{1\mathrm{sc}}$ and the power $P_{1\mathrm{sc}}$ at the variable voltage at the transformer terminals.



The measurements give:

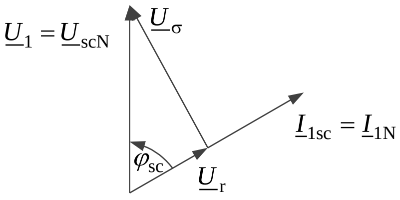
1) the short-circuit voltage at which the rated current flows through the transformer windings.

Relative value for the short-circuit voltage:

$$u_{sc}^{*} = \frac{U_{sc}}{U_N} \text{ or } u_{sc} = \frac{U_{sc}}{U_N}100 .$$

In the short circuit, the hypotenuse of the Kappa triangle $U_{scN} = U_1$ and the

$$U_r = U_{scN} \cos\varphi_{sc} \text{ and } U_\sigma = U_{scN} \sin\varphi_{sc} .$$



Permanent short-circuit current

$$I_{sc} = \frac{U_N}{Z_{sc}}, \text{ if } Z_{sc} = \sqrt{R^2 + X_\sigma^2} .$$

Relative value of the permanent short-circuit current

$$i_{sc}^{*} = \frac{I_{sc}}{I_N} = \frac{U_N}{Z_{sc} I_N} = \frac{U_N}{U_{sc}} = \frac{1}{u_{sc}^{*}} \rightarrow I_{sc} = \frac{1}{u_{sc}^{*}} I_N$$

Relative value for the ohmic and inductive voltage drop:

$$u_r^{*} = \frac{R\,I_N}{U_N} = \frac{R}{Z_N} = r \text{ , where } \frac{U_N}{I_N} = Z_N - \text{is the rated impedance.}$$

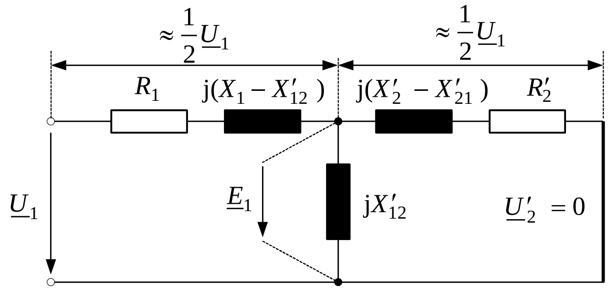
Also applies $u_r^{*} = r = \frac{R\,I_N}{U_N} = \frac{R\,I_N^2}{U_N I_N} = \frac{P_{CuN}}{S_N} = p_{CuN}^{*}$ (relative losses in the windings), and

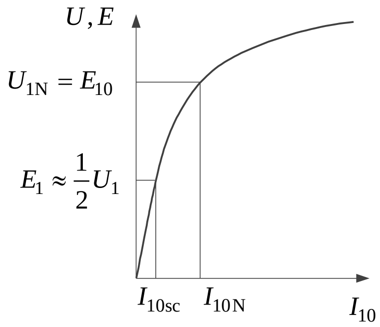
$$u_\sigma^{*} = x_\sigma = \frac{X_\sigma I_N}{U_N} = \frac{X_\sigma I_N^2}{U_N I_N} = \frac{Q_{\sigma N}}{S_N} = q_{\sigma N}^{*} \text{ (the relative reactive power of the leakage field).}$$

2) Losses in the windings

From the short-circuit equivalent circuit it can be seen that the induced voltage is approximately half of the applied voltage:

$$E_1 \approx \frac{U_1}{2} \;\rightarrow\; \Phi_{scN} \approx \frac{\Phi_N}{2} \;\rightarrow\; I_{10sc} < I_{10N}\,.$$





In a short circuit, when the voltage $U_{sc} = (4 \div 16\ \%)U_N$ is reduced by $I_{1sc} = I_{1N}$ and $I_{2sc} = I_{2N}$ for: $\Phi_{sc} << \Phi_N \;\rightarrow\; B_{sc} << B_N \;\rightarrow\; P_{Fesc} \approx 0\,;\; P_w = (P_{1sc})_{I_1 = I_N} \approx P_{CuN} =$

$= I_{1N}^2 (R_1 + R_2')$ (rated losses in the windings).

A wattmeter shall be used to measure the total short-circuit losses in the windings, i.e., joule losses and eddy current losses in the case of massive conductors.

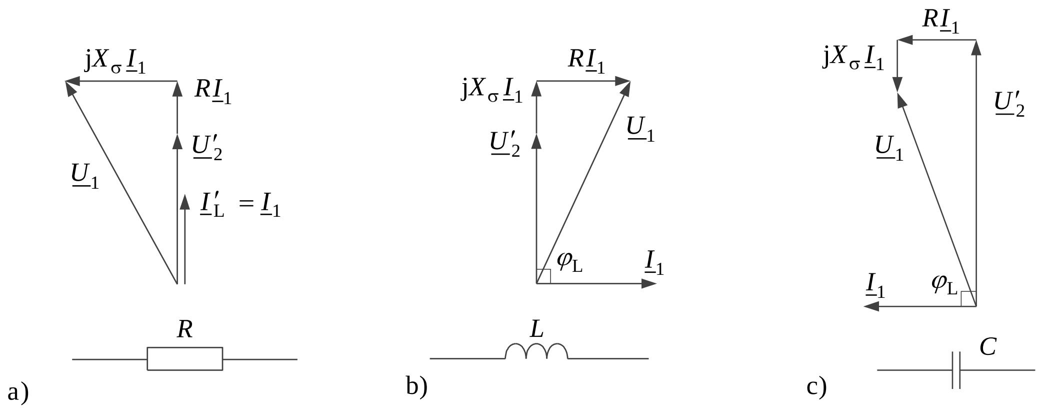
Since $\Phi_{sc} \approx \dfrac{\Phi_N}{2} \;\rightarrow\; I_\mu \approx 0 \;\rightarrow\; X_{12}' \approx \infty$ a simple short-circuit equivalent circuit is valid, drawn for approximate conditions (p. 44). The short-circuit impedance can also be obtained from U_{sc} and I_{sc}:

$$\underline{Z}_{sc} = R_{sc} + jX_{sc} = \frac{\underline{U}_{sc}}{\underline{I}_{sc}}. \qquad (R_{sc} = R_1 + R_2' \text{ and } X_{sc} \approx X_\sigma)$$

Transformer operation on a rigid grid

For a rigid grid: $U_1 = \text{const.}$

U_2 or U'_2 is a function of the size and type (character) of the load Z_L, as shown in the Figures below.



Kappa diagram

For the Kappa diagram it is not derived from $U_1 = \text{const.}$, but $\underline{I}_1 = \text{const.}$, $\underline{U}'_2 = \mathrm{f}(\varphi_\mathrm{L})$, and, here, the position of the Kappa triangle is unchanged. From the equation

$\underline{U}_1 - \underline{U}'_2 = (R + \mathrm{j}X_\sigma)\underline{I}_1$ express $\underline{U}'_2$:

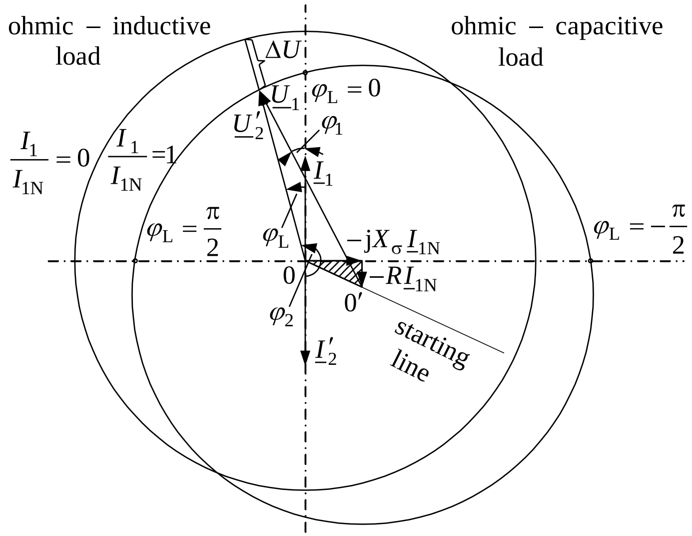
$\underline{U}'_2 = -(R + \mathrm{j}X_\sigma)\underline{I}_1 + \underline{U}_1$.

In the complex plane, draw:

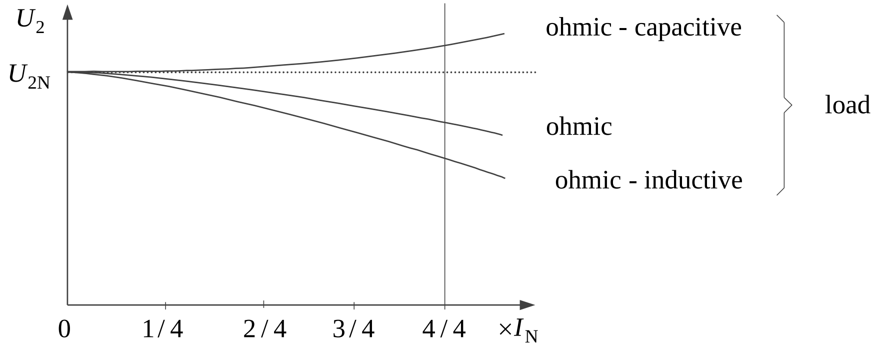
1) $-(R + \mathrm{j}X_\sigma)\underline{I}_1$,

2) circles of radius U_1 for different I_1, e.g., $I_1 = I_{1\mathrm{N}}$ and $I_1 = 0$ on the starting line.

For any phase angle φ_L is obtained ΔU respectively $\underline{U}'_2$.



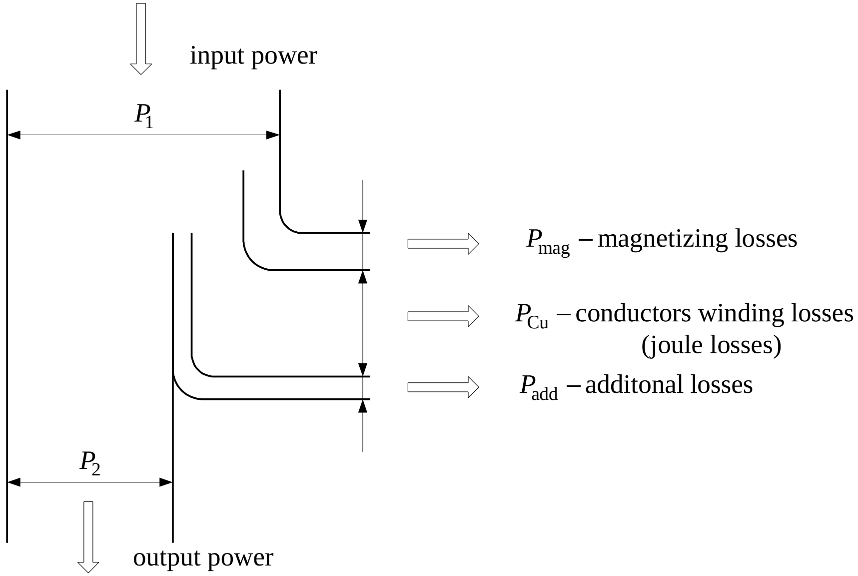
External characteristic $U_2 = \mathrm{f}(I_2)$



The Kappa diagram gives for arbitrary values of current I_2, e.g. $I_2 / I_{2N} = 1/4,\ 1/2 \ldots$ and $\varphi_L = \text{const.}$, external (load) characteristics $U_2 = \mathrm{f}(I_2)$ at $U_1 = \text{const.}$

Energy balance, losses and transformer efficiency

Energy balance - power flow diagram



Losses

Total losses are:

$$P_l = P_{\mathrm{Fe}} + P_{\mathrm{Cu}} + P_{\mathrm{add}} .$$

The additional losses are divided into: those in the iron − voltage-dependent − which are captured in the no-load losses P_0, and those in the conductors − load-current-dependent − (eddy-current losses in massive conductors in higher-power transformers), and are captured in the short-circuit losses P_{sc}.

For any load, i.e., $y = I / I_{\mathrm{N}}$, the losses are:

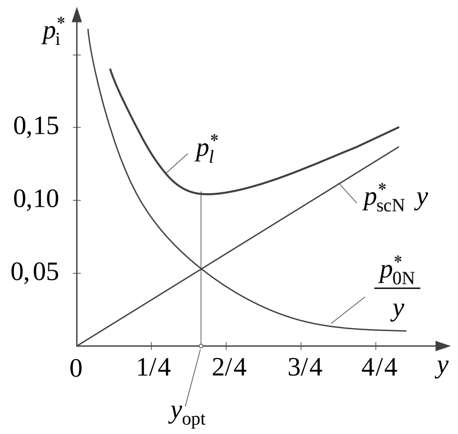
$$P_l = P_{0\mathrm{N}} + P_{\mathrm{scN}} \left(\frac{I}{I_{\mathrm{N}}} \right)^2 = P_{0\mathrm{N}} + y^2 P_{\mathrm{scN}} \quad \text{for } y = \frac{I}{I_{\mathrm{N}}} = \frac{S}{S_{\mathrm{N}}} ,$$

where S is the apparent transformer power.

Introduce relative losses:

$$p_l^* = \frac{P_l}{S} = \frac{P_{0\mathrm{N}}}{S_{\mathrm{N}}} \frac{S_{\mathrm{N}}}{S} + \frac{P_{\mathrm{scN}}}{S_{\mathrm{N}}} \frac{S_{\mathrm{N}}}{S} y^2 = \frac{p_{0\mathrm{N}}^*}{y} + p_{\mathrm{scN}}^* y .$$

At a given load (a given current $I < I_{\mathrm{N}}$), for $y = y_{\mathrm{opt}}$ the total losses (p_l^*) are the lowest ($p_{0\mathrm{N}}^* / y = p_{\mathrm{scN}}^* y$) and the efficiency is the highest.



Efficiency

It is defined as: $\eta = \dfrac{P_2}{P_1}$ or $\eta = \dfrac{P_1 - P_l}{P_1}$.

For $P_2 = y\, S_N \cos\varphi$ and $P_1 = P_2 + P_l = yS_N \cos\varphi + P_{0N} + y^2 P_{scN}$ it will be:

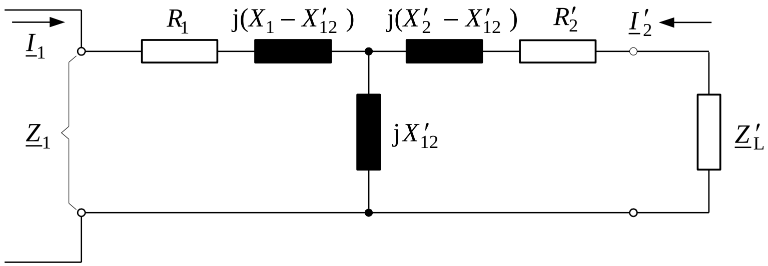
$$\eta = \frac{P_2}{P_1} = \frac{y\, S_N \cos\varphi}{y\, S_N \cos\varphi + P_{0N} + y^2 P_{scN}} \rightarrow \eta = f(y,\ \cos\varphi)\,.$$

Example: $S_N = 20$ kVA, $y = 1$, $\cos\varphi = 1{,}0 \rightarrow \eta = 0{,}97$

$$\cos\varphi = 0{,}8 \rightarrow \eta = 0{,}963$$

Current transformer

It operates at an imposed current $\underline{I}_1$ which is supposed to be constant. Of course, it is not possible to assume that the current in a circuit will remain unchanged if a current transformer is connected to it in series. It is subject to the same laws as a voltage transformer, i.e., the same equivalent circuit and the same phasor diagram.



Ideal current transformer

Valid: $\underline{I}_1 N_1 + \underline{I}_2 N_2 = 0 \rightarrow \underline{I}_2 = -\frac{N_1}{N_2}\underline{I}_1 = -\frac{\underline{I}_1}{K_I}$ or $\underline{I}_1 + \underline{I}'_2 = 0 \rightarrow \underline{I}'_2 = -\underline{I}_1$

The load current $\underline{I}_L = -\underline{I}_2$ causes a voltage drop across the complex resistance $\underline{Z}_L$ of the two-terminal $\underline{U}_L = \underline{Z}_L \underline{I}_L = -\underline{Z}_L \underline{I}_2 = \underline{U}_2$.

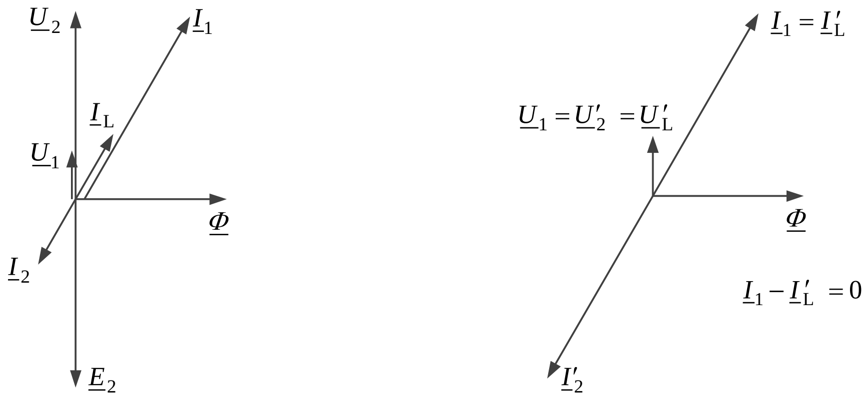
The voltage drops across a current transformer feed back to the circuit.

As it is considered to be the voltage of an ideal transformer $\underline{U}_1 = K_\mathrm{U}\underline{U}_2$ and $\underline{Z}'_\mathrm{L} = K_\mathrm{U}^2\underline{Z}_\mathrm{L}$,

it will be: $\underline{U}_1 = K_\mathrm{U}\underline{U}_2 = K_\mathrm{U}(-\underline{Z}_\mathrm{L}\underline{I}_2) = K_\mathrm{U}\underline{Z}_\mathrm{L}K_\mathrm{U}\underline{I}_1 = K_\mathrm{U}^2\underline{Z}_\mathrm{L}\underline{I}_1 = \underline{Z}'_\mathrm{L}\underline{I}_1$.

$\underline{U}_1$ is the voltage on the primary side, and, at the same time, the voltage drops across the transformer. Therefore, the impedance $\underline{Z}_\mathrm{L}$ must be small relative to the impedance of circuit 1, which determines the current $\underline{I}_1$.

Phasor diagram of a loaded ideal current transformer



Analytical treatment of a real transformer

From the voltage drop across the secondary winding (p. 42)

$$\underline{U}_2 = R_2\underline{I}_2 + \mathrm{j}X_{21}\underline{I}_1 + \mathrm{j}X_2\underline{I}_2$$

for $\underline{U}_2 = -\underline{Z}_\mathrm{L}\underline{I}_2 = -(R_\mathrm{L} + \mathrm{j}X_\mathrm{L})\underline{I}_2$ is the current:

$$\underline{I}_2 = -\frac{\mathrm{j}X_{21}}{R_2 + \mathrm{j}X_2 + \underline{Z}_\mathrm{L}}\underline{I}_1;\ \underline{I}_2 \propto \underline{I}_1 \text{ and are no longer displaced by } 180^\circ .$$

Voltage drops across the primary winding:

$$\underline{U}_1 = R_1\underline{I}_1 + \mathrm{j}X_1\underline{I}_1 + \mathrm{j}X_{12}\underline{I}_2 = \left(R_1 + \mathrm{j}X_1 + \frac{X_{12}^2}{R_2 + \mathrm{j}X_2 + \underline{Z}_\mathrm{L}}\right)\underline{I}_1 = \underline{Z}_1\underline{I}_1 .$$

The complex resistance $\underline{Z}_1$ is, according to the alternative circuit in the Figure, the resistance between the input terminals on the primary side of the current transformer.

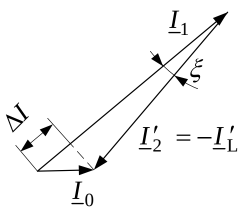
Since $U_1 = \mathrm{f}(I_1)$ will be $I_1 = 0,\ \ U_1 = 0$ and $U_2 = 0.$ Therefore, the current transformer is not used as a voltage source, but for measurements.

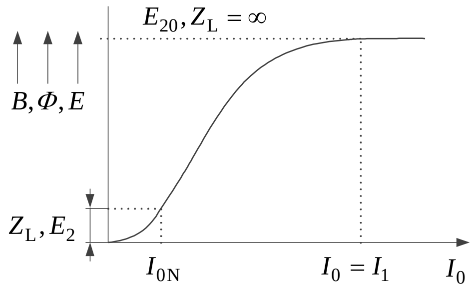
Measuring current transformer

For a real transformer, it is:

$$\underline{I}_1 + \underline{I}'_2 = \underline{I}_0 > 0 \ \text{ or } \ \underline{I}_1 - \underline{I}'_\mathrm{L} = \underline{I}_0 \,.$$

Deviations from the ideal conditions result in ratio error and angle error.





Ratio error: $e_{\mathrm{CT}} = \dfrac{\Delta I}{I_1} 100$ (%)

Angle error: $\xi \ \rightarrow$ in degrees or minutes

To minimize these two errors, the current transformer operates in the linear part of the magnetization characteristic $(B = 0{,}08 \div 1\ \mathrm{T})$.

Accuracy class (0,1 – 0,2 – 0,5 – 1 – 3 – 5) defines both errors,

e.g., the class 1 $\rightarrow$ $e_{\mathrm{CT}} = \pm 1\ \%$ and $\xi = \pm 1^\circ = \pm\, 60'$.

Since the ratio error changes as the load increases above the nominal value, the overcurrent number is also a known quantity.

The overcurrent number is that multiple of the rated current at which the ratio error reaches the value –10 %.

Danger of open secondary terminals

It's normal: $\underline{I}_1 + \underline{I}'_2 = \underline{I}_{0N}$.

For $\underline{Z}_L = \infty \rightarrow \underline{I}_2 = 0, \quad \underline{I}_1 \equiv \underline{I}_0 \rightarrow \Phi_0 = x\Phi \ \rightarrow \ B_0 = xB$ and

$P_{Fe} = k\left(B_0 / B\right)^2 = x^2 P_{FeN} \ \rightarrow$ core heating. Due to $\Phi_0 = x\Phi \ \rightarrow \ E_{20} = xE_2$ (increase of voltage at the terminals). Therefore, after the load is removed, the secondary terminals must be short circuited.

Three-phase transformation

The three-phase system is the most widespread in the world. One reason for this is that the power in a symmetrical three-phase system does not have an AC share, as is the case in a single-phase system for example. The three-phase system also makes it possible to take advantage of the phenomenon of rotating magnetic fields in rotating machines.

Three-phase voltage transformers connect individual three-phase systems of different voltage levels.

Options: a) three single-phase transformers b) three-phase transformer

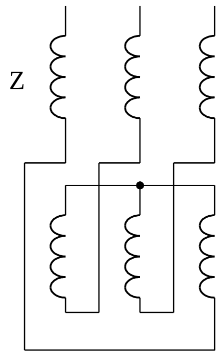
Basic winding connections

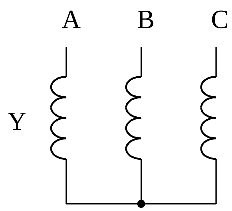
There are three typical connections: a) star Y

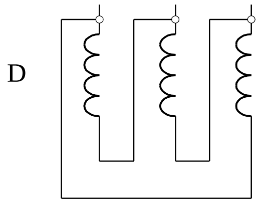
b) delta D

c) zigzag Z

For star and zigzag connection $\sum I = 0$, if the winding has no zero conductor. In a delta connection, the phase and line currents can be asymmetrical.







Phase shift

This occurs between the primary and secondary actual or imagined value of a phase voltage.

The phase shift is: $n \times 30^{\circ}$ electric ($n = 0 \div 12$). Typical shifts are: $n = 0,\ 5,\ 6,\ 11$.

It is given as the phase lag of the lower voltage phasor against the higher voltage phasor. The phase shift is identical to the displacement of the hour hands by whole hours.

Vector group

Example of marking: Dy5

Capital letter:

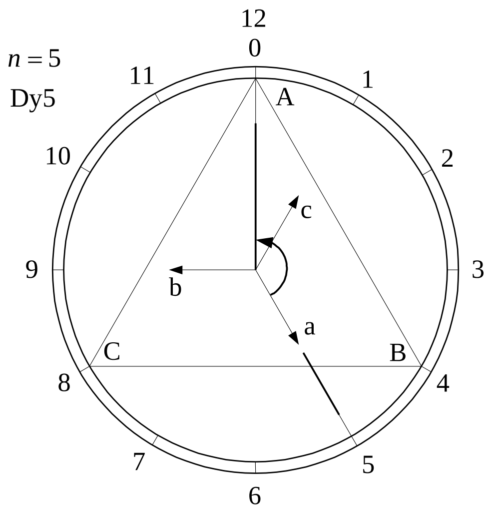
– delta connection of the primary side;

lowercase letter:

– star connection on the secondary side;

Number:

– phase shift ($5 \times 30^{\circ} = 150^{\circ}$).



The direction of the secondary phase voltage (U_{a}) is equal to the direction of the primary line voltage (U_{AB}) between phase A and phase B.

The Dy5 vector group can be changed to Dy11, and vice versa, by cyclic permutation of the two terminals on the primary side and the next two terminals on the secondary side.

The Yy0 vector group can be changed to Yy6, and vice versa. This is done by swapping the starts and ends of the windings that are connected to the neutral terminal of the star on the primary or secondary side.

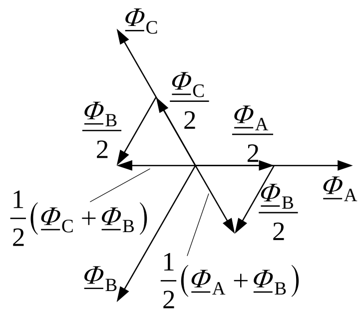
Example of use: generator transformer Dy5

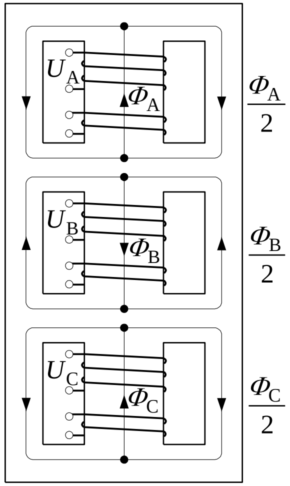
distribution transformer Yy0, Yd5

local grid Dy5, Yz5

Types of cores

Three-phase shell transformer
It is obtained by adding three
single-phase transformers.



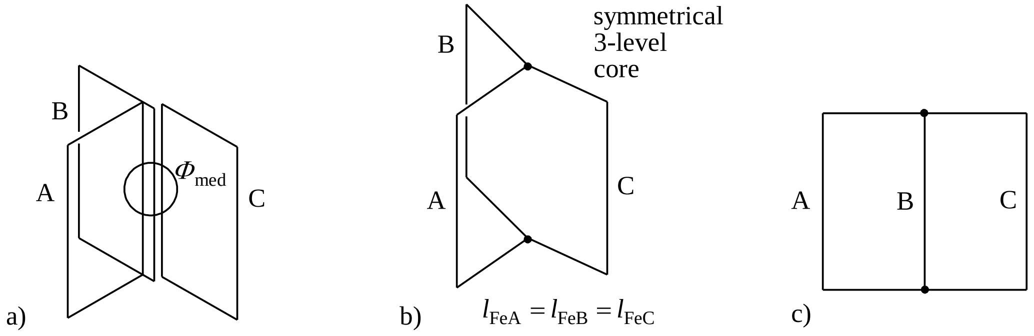


The yoke of a shell transformer has half the cross-section of the column if the winding on the middle column is wound in the opposite direction to the outermost two, or the winding connection sequence on the middle column (phase) is reversed.

Valid $\underline{\Phi}_y = \underline{\Phi}_A / 2 + \underline{\Phi}_B / 2$ or $\underline{\Phi}_C / 2 + \underline{\Phi}_B / 2$.

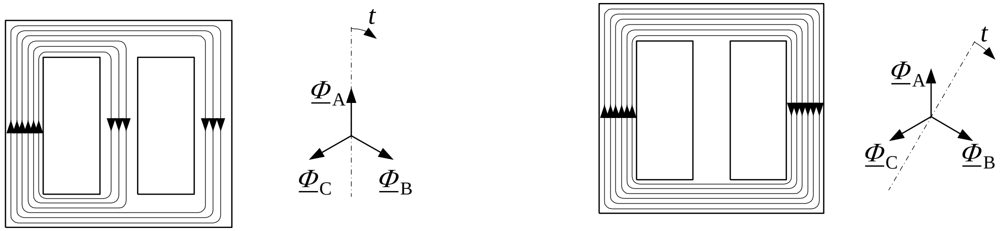
Three-phase core transformer

It is made up of three single-phase transformers and is technically usable as an asymmetrical design (Figure c).



It has two magnetic nodes in which the condition $\underline{\Phi}_{\mathrm{A}} + \underline{\Phi}_{\mathrm{B}} + \underline{\Phi}_{\mathrm{C}} = 0$.

The Figures show the flux waveform for $\underline{\Phi}_{\mathrm{A}} = \hat{\Phi}_{\mathrm{A}}$ and $\underline{\Phi}_{\mathrm{B}} = 0$.

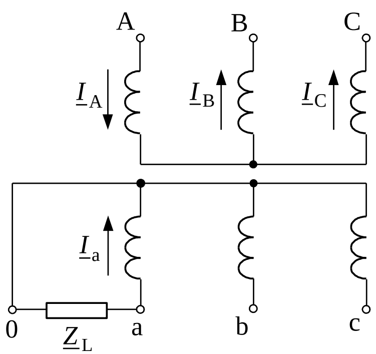


Asymmetrical load

a) There are no problems in the YNyn connection with zero conductors on the primary and secondary sides.

b) In the Yyn connection with a zero conductor only on the secondary side, asymmetry occurs.

Worst case: pure single-phase load.



Because it is $\underline{\Phi}_{\mathrm{A}} + \underline{\Phi}_{\mathrm{B}} + \underline{\Phi}_{\mathrm{C}} = 0$ and $|\underline{\Phi}_{\mathrm{A}}| = |\underline{\Phi}_{\mathrm{B}}| = |\underline{\Phi}_{\mathrm{C}}|$, shall be $|\underline{\Theta}_{\mathrm{A}}| = |\underline{\Theta}_{\mathrm{B}}| = |\underline{\Theta}_{\mathrm{C}}|$.

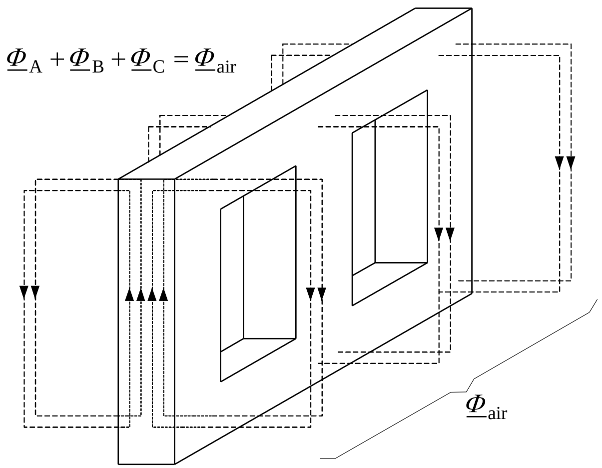
With asymmetrical loading, the balance of ampere-turns on the columns will be:

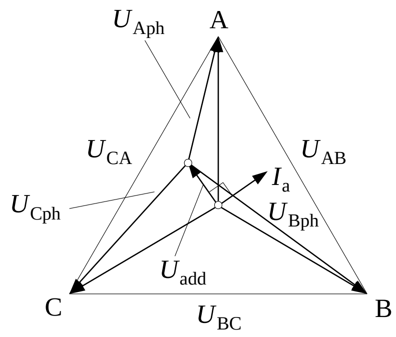
$$\left.\begin{array}{lll} \underline{I}_{\mathrm{a}} N_2 + \underline{I}_{\mathrm{A}} N_1 = \underline{I}_{\mathrm{x}} N_1 & & \\ 0 \quad + \underline{I}_{\mathrm{B}} N_1 = \underline{I}_{\mathrm{x}} N_1 & \rightarrow & \underline{I}_{\mathrm{B}} = \frac{1}{3} \underline{I}'_{\mathrm{a}} \\ 0 \quad + \underline{I}_{\mathrm{C}} N_1 = \underline{I}_{\mathrm{x}} N_1 & \rightarrow & \underline{I}_{\mathrm{C}} = \frac{1}{3} \underline{I}'_{\mathrm{a}} \end{array}\right\} \underline{I}_{\mathrm{A}} = -(\underline{I}_{\mathrm{B}} + \underline{I}_{\mathrm{C}}) = -\frac{2}{3} \underline{I}'_{\mathrm{a}}$$

$\underline{I}_{\mathrm{a}} N_2 + (\underline{I}_{\mathrm{A}} + \underline{I}_{\mathrm{B}} + \underline{I}_{\mathrm{C}}) N_1 = 3 \underline{I}_{\mathrm{x}} N_1 = 3 \underline{\Theta}_{\mathrm{air}} / \sqrt{2}$, because it is in the star $\underline{I}_{\mathrm{A}} + \underline{I}_{\mathrm{B}} + \underline{I}_{\mathrm{C}} = 0$. In-phase excitation

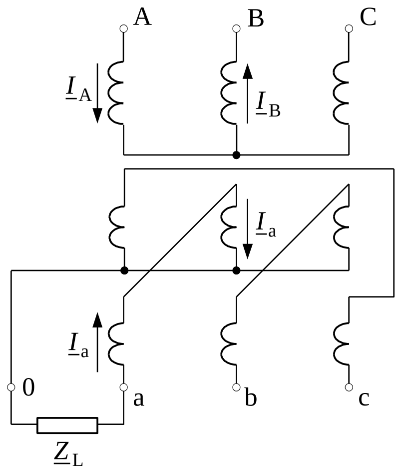
$$\underline{\Theta}_{\mathrm{air}} = \sqrt{2} \underline{I}_{\mathrm{x}} N_1 = \frac{1}{3} \sqrt{2} \underline{I}_{\mathrm{a}} N_2 \rightarrow \underline{I}_{\mathrm{x}} = \frac{1}{3} \underline{I}_{\mathrm{a}} \frac{N_2}{N_1} = \frac{1}{3} \underline{I}'_{\mathrm{a}}$$

$\Theta_{\mathrm{air}} \rightarrow \Phi_{\mathrm{air}} \rightarrow E_{\mathrm{air}} \rightarrow U_{\mathrm{add}}$ (additional) $\rightarrow$ zero shift





Yzn connection



c) In the Yzn connection with no zero conductor on the primary side there is no problem (only two phases are loaded).

d) There is also no problem in the Dyn connection, because the load current on the primary side flows only in one phase.

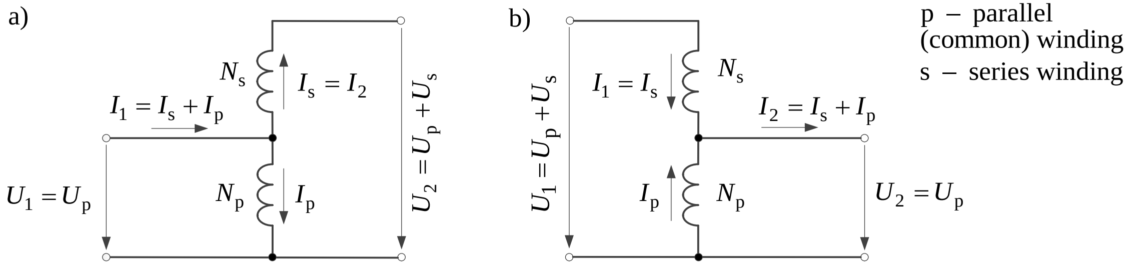
Autotransformer

If the two windings of a normal transformer are combined, part of the winding will be common to both voltage sides.

Common – parallel winding $\rightarrow$ difference (differential) currents

Series winding $\rightarrow$ load current of primary or secondary

Use $\rightarrow$ to save material (iron and conductors). This saving is maximum for the case of $K_U \approx 1$. In the three-phase version, the Y connection is used.



For an ideal single-phase autotransformer (connection a): $I_s N_s - I_p N_p = 0 \rightarrow$

$$\rightarrow \; I_s = \frac{N_p}{N_s} I_p \quad \text{and} \quad \frac{U_s}{U_p} = \frac{U_2 - U_1}{U_1} = \frac{N_s}{N_p}.$$

The transformer's inherent or typical power, i.e., the power for which the transformer is built, is: $S_t = U_p I_p = U_s I_s = (U_2 - U_1) I_2$ or for connection b) $S_t = (U_1 - U_2) I_1$.

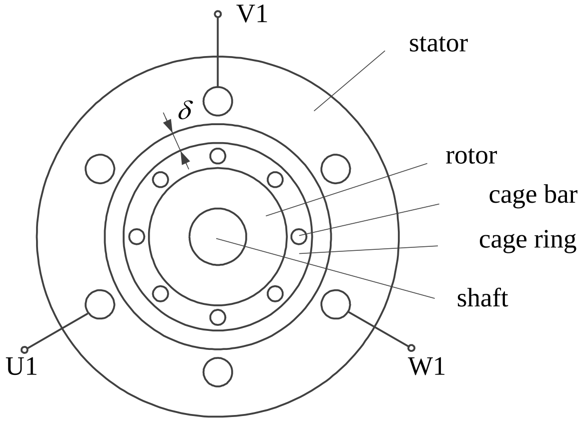
Passing power: $S_{pas} = U_1 I_1 = U_2 I_2$

The equation for the typical power is: $S_t = U_2 I_2 \left(1 - \frac{U_1}{U_2}\right) = S_{pas} \left(1 - \frac{U_1}{U_2}\right)$. ($S_t < S_{pas}$)

The reverse would be true for the connection b): $S_t = U_1 I_1 \left(1 - \frac{U_2}{U_1}\right) = S_{pas} \left(1 - \frac{U_2}{U_1}\right)$.

INDUCTION MACHINE

An induction machine (IM) has some similarities with a transformer.



Primary winding - on the stator

Secondary winding - on the rotor

There is an air gap δ between them.

Number of phases on the stator m_s

Number of phases on the rotor m_r
$(m_r \geq m_s)$

The stator currents induce rotating ampere-turns, which excite a rotating flux that induces a voltage in both windings. The frequency of the induced voltage in the stator $f_s = f$. Frequency of induced voltage in the rotor $f_r \leq f_s$ or $f_r > f_s$. The rotor at standstill operation is equivalent to a short-circuit of the transformer; the rotor winding is short-circuited.

The resultant rotating magnetic field and currents in the short-circuited rotor winding produce a force that rotates the rotor. The rotating magnetic field rotates with synchronous speed n_s, the rotor with speed $n \gtrless n_s$ – hence asynchronously.

Description of construction

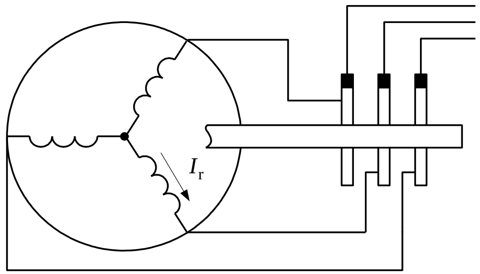
Stator – laminated sheet, with slots in which the m_s phase winding is located.

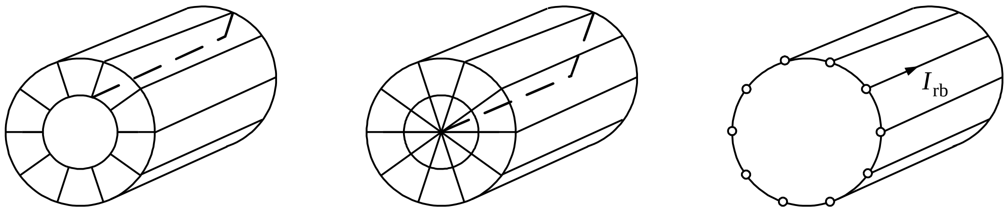
Rotor – laminated sheet with slots.

We distinguish between:

a) a wound rotor

b) a rotor with a short-circuit squirrel cage





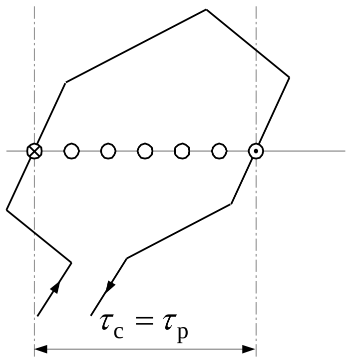
c) massive iron rotor

The stator and rotor can also be cylindrical or disc-shaped.

Induction machine windings

Three-phase belt windings: are single, double, and combined layer windings.

Single-layer winding:
diameter winding $(\tau_c = \tau_p)$



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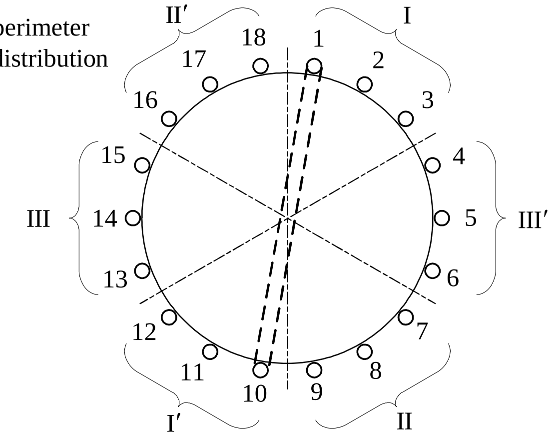


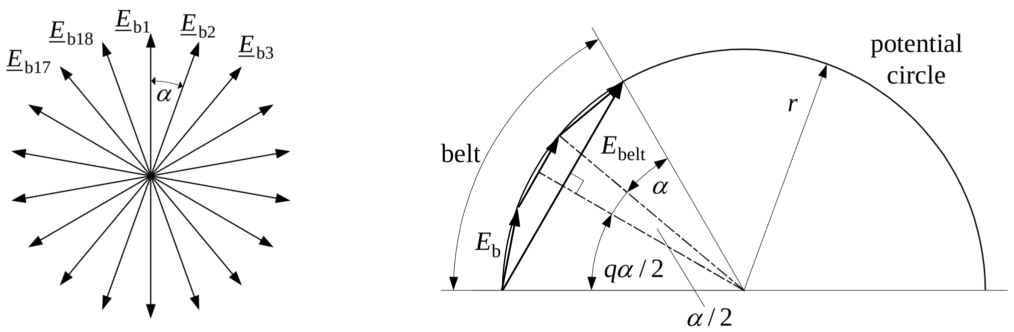
Each phase has a belt: $\alpha_b = \frac{360^\circ}{2m} = \frac{360^\circ}{6} = 60^\circ$ and there are q slots in it.

For Q slots on the perimeter of the machine: $Q_p = Q/(2p)$ the number of slots per pole, and $2p = 2$ the number of slots per pole and phase. The pole-arc is $Q = 18$ slots, or the width of the coil expressed by the number of slots. The voltages of the individual bars (or conductors) of the slots (or their phasors) are offset from each other by an electrical angle

$\alpha = p\alpha_Q = p\frac{360^\circ}{Q}$ and form a slot star.

Three-phase single-layer winding: calculation of the data for $2p = 2$ and $Q = 18$ slots.

The number of slots per pole is $Q_p = \frac{Q}{2p} = \frac{18}{2} = 9$, the number of slots per pole and phase $q = \frac{Q_p}{m} = \frac{9}{3} = 3$, the electrical angle between the slots $\alpha = p\frac{360^\circ}{Q} = 1\cdot\frac{360^\circ}{18} = 20^\circ$.

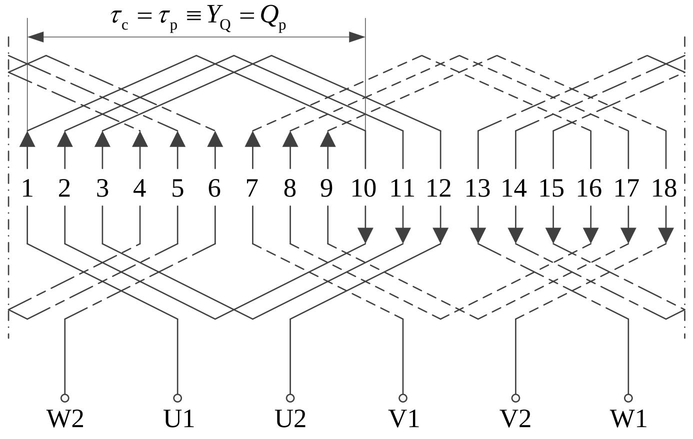


1) Distribution factor

$$f_d = \frac{\sum\limits_{geom.} E_b}{\sum\limits_{arit.} E_b} = \frac{2r\sin\left(q\frac{\alpha}{2}\right)}{q\,2r\sin\left(\frac{\alpha}{2}\right)} = \frac{\sin\left(q\frac{\alpha}{2}\right)}{q\sin\left(\frac{\alpha}{2}\right)}$$

For our example, the distribution factor: $f_\mathrm{d} = \dfrac{\sin\left(q\dfrac{\alpha}{2}\right)}{q\sin\left(\dfrac{\alpha}{2}\right)} = \dfrac{\sin\left(3\cdot 10^\circ\right)}{3\sin\left(10^\circ\right)} = \dfrac{0,5}{3\cdot 0,17365} = 0,96$.

Diameter winding: the width of the Y_Q coils expressed in the number of slots $Y_\mathrm{Q} = Q_\mathrm{p} = 9$. Next, determine the winding pitch. The latter is defined as the expression: $\text{pitch} = 1 - (1 + Y_\mathrm{Q})$, i.e., for the slots from the initial (first) slot onwards. The pitch is: $1-(1+Y_\mathrm{Q}) = 1-(1+9) = = 1-10$, i.e., from the first to the tenth slot.



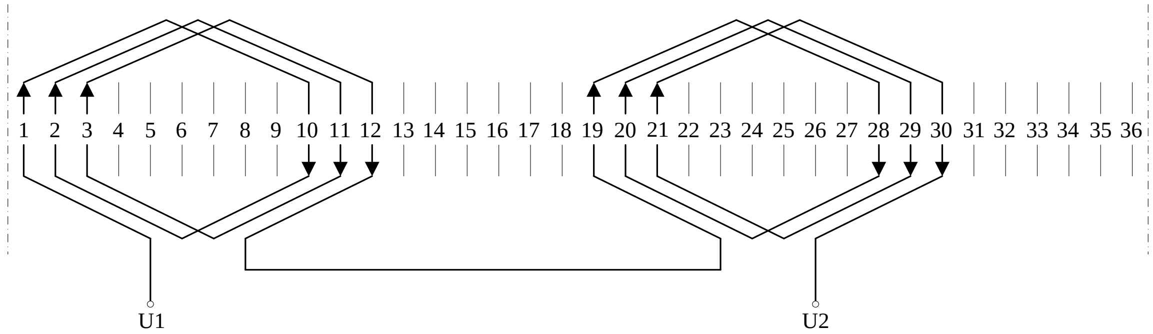
The start of the first phase is marked U1 and the end U2. In the Figure we see that the start of the second phase is in slot 7, which is shifted by $6\cdot 20^\circ = 120^\circ$ (electrical) with respect to slot 1, and analogously, the start of the third phase is in slot 13 by $12\cdot 20^\circ = 240^\circ$. The phase shifts correspond to the condition for the generation of a circular rotating field. The direction of the current in the slots of the first phase changes twice. This corresponds to two poles. The same argument also applies to the three phases taken together. The directions of the currents in the Figure correspond to the position of the timeline $\omega t = 60^\circ$, when the currents of the first and second phases are positive and of half amplitude, and the current of the third phase is negative and maximum.

In the case of twice as many poles $(2p = 4)$, the number of slots is also twice as large, i.e., $Q = 36$.

The number of slots / pole is $Q_p = 36/4 = 9$, the diameter width of the coils $Y_Q = Q_p = 9$, the number of slot / pole / phase $q = 3$, the electrical angle between the slot

$$\alpha = p\frac{360^\circ}{Q} = 2\cdot\frac{360^\circ}{36} = 20^\circ.$$

Thus, practically all the data are the same as for the two poles on 18 slots. The winding pitch (of the coils) will also be the same $1-(1+Y_Q) = 1-(1+9) = 1-10$. According to the Figure below (drawn for the first phase only), it can be seen that the group of the first three coils and the other three coils are connected in series, i.e., the end in the 12th slot with the beginning of the second group of coils in the 19th slot. The 19th slot is shifted with respect to the 1st slot by 360° (electrical).



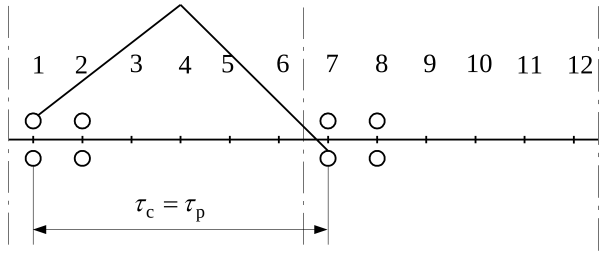
2) Pitch factor

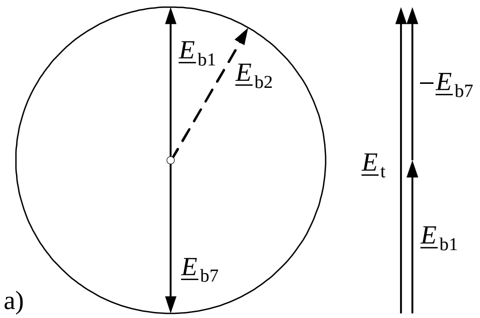
In practice, it proves useful to shorten the winding, i.e., to shorten the pitch (fractional pitch winding), thereby shortening the end winding connecting the individual slots. This also saves on material (copper). In the end winding, which are outside the slots of the stator pack (in the air), no voltage is induced due to the weak field.

Only a two-layer winding can be shortened by x slots $(x \geq 1 \rightarrow \tau_c < \tau_p$ or $Y_Q < Q_p)$ so that the width of the coils is $Y_Q = Q_p - x$. A two-layer winding has two coils in the slots with half the number of turns. Therefore, there are twice as many coils as in a single-layer winding.

Example for a two-layer winding: $Q=12,\ 2p=2,\ m=3,\ q=2,\ \alpha=30^\circ,\ Q_\mathrm{p}=12/2=6$, $Y_\mathrm{Q}=Q_\mathrm{p}=6$ and the winding pitch $1-(1+Y_\mathrm{Q})=1-(1+6)=1-7$.

a) Diameter pitch $1-7$

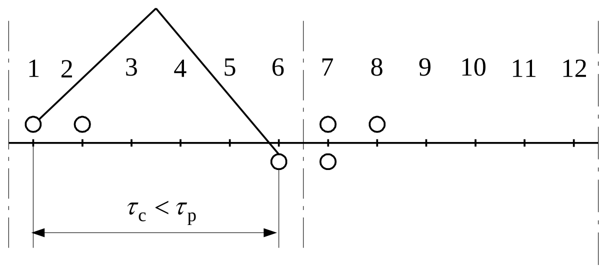


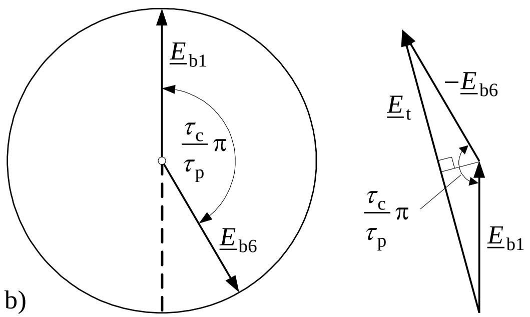


Turn (loop) voltage $\underline{E}_\mathrm{t}=\underline{E}_\mathrm{b1}-\underline{E}_\mathrm{b7}$.

a)

b) Shorted pitch $1-6$ $(Y_\mathrm{Q}=Q_\mathrm{p}-1=6-1=5)$





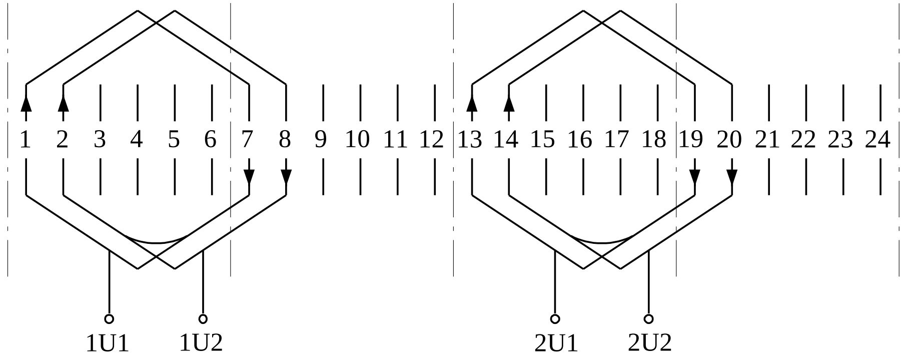
b)

Turn voltage $\underline{E}_\mathrm{t}=\underline{E}_\mathrm{b1}-\underline{E}_\mathrm{b6}$.

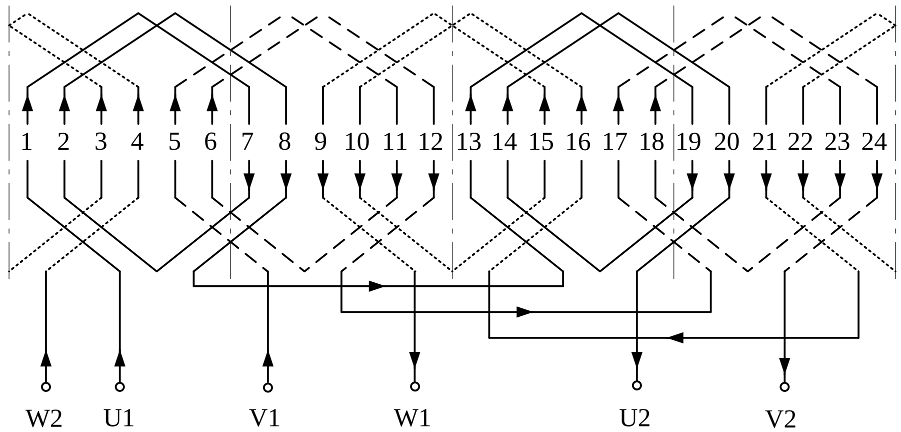
pitch factor: $$f_\mathrm{p}=\frac{\sum\limits_{\text{geom.}}E_\mathrm{t}}{\sum\limits_{\text{arit.}}E_\mathrm{t}}=\frac{2E_\mathrm{b}\sin\left(\frac{1}{2}\frac{\tau_\mathrm{c}}{\tau_\mathrm{p}}\pi\right)}{2E_\mathrm{b}}=\sin\left(\frac{\tau_\mathrm{c}}{\tau_\mathrm{p}}\frac{\pi}{2}\right)=\sin\left(\frac{Y_\mathrm{Q}}{Q_\mathrm{p}}\frac{\pi}{2}\right)$$

<u>winding factor</u>: $f_\mathrm{w}=f_\mathrm{d}\,f_\mathrm{p}\qquad (f_\mathrm{w}\le 1)$

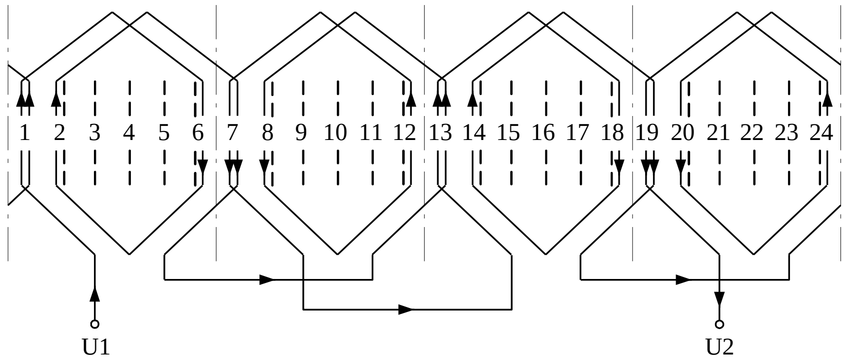
Example: $Q=24,\ 2p=4,\ m=3,\ q=2,\ \alpha=30^\circ$ (electric angle), $Y_\mathrm{Q}=Q_\mathrm{p}=24/4=6$



a) Single-layer winding: pitch $1-7$, drawn for the first phase



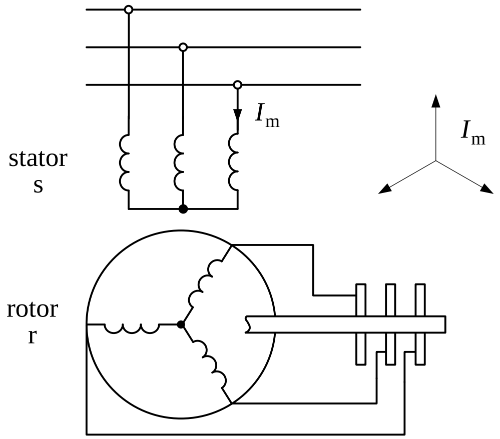
b) Single-layer winding: pitch $1-7$ (The directions of the currents are drawn for $\omega t = 60^{\circ}$.)



c) Two-layer winding with shortened pitch: pitch 1−6, drawn for the first phase.

Operation mode

a) Rotor winding − open (transformer no-load)



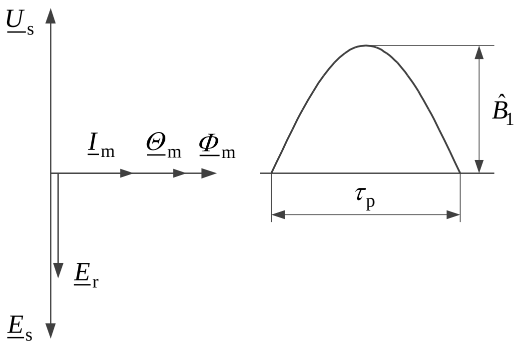
Amplitude of the stator *MMF* caused by the magnetization current:

$$\hat{\Theta}_s = \frac{m_s}{2}\frac{4}{\pi}\frac{N_s\, f_{ws}}{2p}\sqrt{2}I_m\,,$$

generates a rotating (main) flux

$$\hat{\Phi}_m = \frac{2}{\pi}\hat{B}_1 A_\delta = \frac{2}{\pi}\hat{B}_1 \tau_p l\,.$$

$2/\pi$ is the factor of mean value for a sinusoidal magnetic field.



The rotating flux induces voltages:

$$E_s = 4,44\, f\, N_s\, f_{ws}\hat{\Phi}_m\,,$$

$$E_r = 4,44\, f\, N_r\, f_{wr}\hat{\Phi}_m\,,$$

frequencies $f = f_s = f_r$. When the rotor is at a standstill, $E_{ro} = E_r$ is the induced voltage of the open terminals of the (wound) rotor.

The reduction factor or voltage ratio is: $K_U = \dfrac{E_s}{E_r} = \dfrac{N_s\, f_{ws}}{N_r\, f_{wr}}$.

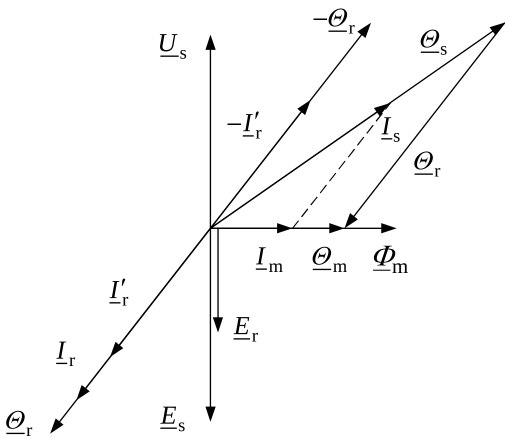
b) Rotor winding − short-circuit

The amplitude of the rotor's rotational *MMF* is: $\hat{\Theta}_r = \dfrac{m_r}{2}\dfrac{4}{\pi}\dfrac{N_r\, f_{wr}}{2p}\sqrt{2}\, I_r$.

Due to the equilibrium condition, $U_s = -E_s$, to remain the same Φ_m as at no-load, the current in the stator winding must increase from I_m to I_s.

$$\underline{\Theta}_s + \underline{\Theta}_r = \underline{\Theta}_m$$

$$\underline{I}_s + \underline{I}'_r = \underline{I}_m$$



Due to the compensation of the ampere-turns:

$$\hat{\Theta}'_r = \frac{m_s}{2}\frac{4}{\pi}\frac{N_s f_{ws}}{2p}\sqrt{2} I'_r = \hat{\Theta}_r .$$

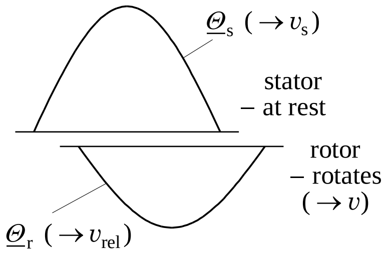
Current ratio

$$K_I = \frac{I'_r}{I_r} = \frac{m_r N_r f_{wr}}{m_s N_s f_{ws}} = \frac{m_r}{m_s}\frac{1}{K_U}$$

Rotating machine operation

The notion of a slip

$$s = \frac{n_{rel}}{n_s} = \frac{n_s - n}{n_s} \quad \rightarrow n = n_s(1-s) = \frac{f}{p}(1-s)$$



1) $n = 0 \div n_s$ induction motor $(s = 1 \div 0)$,

2) $n = n_s \div +\infty$ induction generator $(s = 0 \div -\infty)$,

3) $n = 0 \div -\infty$ brake, the rotor rotates against the rotating magnetic field $(s = 1 \div +\infty)$.

Slip is the difference between the synchronous speed of the MMF n_s and the rotor speed n.

Frequencies of the induced voltages:

synchronous speed of the rotating magnetic field $\upsilon_s = D\pi n_s$

rotor speed $\upsilon = D\pi n$

relative speed $\upsilon_{rel} = \upsilon_s - \upsilon = D\pi(n_s - n) = D\pi n_{rel} = D\pi n_s s = \upsilon_s s$

$e_{sb} = Blv_s$ (induced stator voltage − in the bar or conductor)

$e_{rb} = Blv_{rel} = Blv_s s = e_{sb} s$ (induced rotor voltage − in the bar)

$f = f_s = pn_s$ (frequency of the induced stator voltage)

$f_r = pn_{rel} = pn_s s = s f_s = s f$ (frequency of the induced rotor voltage)

$X_{sr} = 2\pi f_r L_{sr} = 2\pi s f_s L_{sr} = s X_{sr0}$ (change of the mutual reactance)

Force and torque of a short-circuit rotor winding

The occurrence of a force on a short-circuited rotor winding (squirrel cage):

rotating magnetic field of the air gap $B_1 \rightarrow E_{rb} = E_r \rightarrow I_{rb}$.

A force is acting on a single bar (cage) in a rotating magnetic field on a current-carrying conductor according to Lorenz's law:

$$F_b = I_{rb} \frac{\hat{B}_1}{\sqrt{2}} l ,$$

where I_{rb} is the current in each rotor bar, and l is the length of the conductor.

In the equation for force, we have to account for the RMS value of the fundamental harmonic of the flux density. Considering the equation for the frequency in the rotor, the equation for the induced voltage in each rotor bar is $z_r = 1$ (half turn $N_r = z_r / 2 = 1/2$):

$$E_r = E_{rb} = sE_{sb} = s\, 2\pi\ f N_r f_{wr} \frac{\hat{\Phi}_m}{\sqrt{2}} = \frac{s\omega r}{p} \frac{\hat{B}_1}{\sqrt{2}} l .$$

The winding factor of each rotor bar is taken into account $f_{wr} = 1$, $\hat{\Phi}_m = (2/\pi)\hat{B}_1 \tau_p l$ and pole pitch $\tau_p = D\pi/(2p) = r\pi/p$. Express $B_1 = \mathrm{f}(E_r)$ and the equation for the force takes a new form:

$$F_b = \frac{p E_r I_{rb}}{s\omega r} = \frac{E_r I_{rb}}{s\Omega_{ms} r} = \frac{E_r I_{rb}}{s v_s} ,$$

where ω is the electrical angular frequency, ω_s the angular frequency of the stator MMF $(\omega_s = \omega)$ and $\Omega_{ms} = \omega_s / p = 2\pi f_s / p$ is the mechanical synchronous angular velocity.

The force is applied to each bar of the rotor with radius $r = D/2$. For the Q_r bars in the rotor, we obtain the final expression for the torque from the internal power (P_{int}):

$$M = Q_r F_b r = \frac{Q_r E_r I_r}{s\,\Omega_{ms}} = \frac{m_r\, E_r\, I_{rb}}{s\,\Omega_{ms}} = \frac{m_s\, E_r\, I_r}{s\,\Omega_{ms}} = \frac{P_{int}}{\Omega_{ms}},$$

if $I_r = \dfrac{m_r}{m_s} I_{rb} = \dfrac{Q_r}{m_s} I_{rb}$. (Transforming multiphase bar current to a three-phase rotor current.)

In the case of a squirrel cage, the number of phases m_r is equal to the number of slots in the rotor Q_r**,** or equal to the number of bars $(Q_r = m_r)$. The equation for the torque fails when $s = 0$. (For $s = 0 \rightarrow E_r = 0,\ I_r = 0$ and get $0/0$.)

Analytical treatment

Voltage equations:

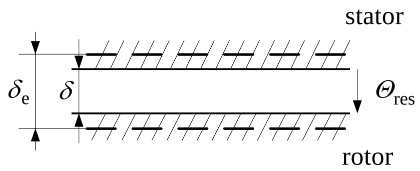
$$\underline{U}_s = R_s \underline{I}_s + jX_{\sigma s} \underline{I}_s + j\omega (N_s\, f_{ws}) \frac{\underline{\Phi}_m}{\sqrt{2}} = R_s \underline{I}_s + jX_{\sigma s} \underline{I}_s - \underline{E}_s$$

$$\underline{U}_r = R_r \underline{I}_r + js X_{\sigma r} \underline{I}_r + js\omega (N_r\, f_{wr}) \frac{\underline{\Phi}_m}{\sqrt{2}} = R_r \underline{I}_r + js X_{\sigma r} \underline{I}_r - s\underline{E}_r$$

$\underline{\Phi}_m = \hat{\Phi}_m e^{j\varphi_0}$ generate the resulting ampere-turns: $\underline{\Theta}_{res} = \underline{\Theta}_s + \underline{\Theta}_r$.

The magnetic flux density is given by:

$$b_{res}(x,t) = \hat{B}_{res} \cos\Big((x_s / \tau_p)\pi - \omega t - \varphi_0 \Big).$$



For $\mu_{Fe} \rightarrow \infty$, an equivalent (smooth) air gap is considered: $\hat{B}_{res} = \dfrac{\mu_0}{\delta_e} \hat{\Theta}_{res}$.

For $x_s = 0$, i.e., in the winding symmetry of phase "a", the resulting *MMF* is given by:

$$\theta_{res} = \hat{\Theta}_s \cos(\omega t + \varphi_{is}) + \hat{\Theta}_r \cos(\omega t + \varphi_{ir}) .$$

$$\underline{\Phi}_m = \frac{2}{\pi}\tau_p l \underline{B}_{res} = \frac{2}{\pi}\tau_p l \frac{\mu_0}{\delta_e}\frac{3}{2}\frac{4}{\pi}\frac{1}{2p}\left((N_s f_{ws})\underline{I}_s + (N_r f_{wr})\underline{I}_r\right)\sqrt{2}$$

Now insert the equation for the main flux on the left-hand side of the two equations for the stator and rotor voltages on the previous page, and they take the following form:

$$\underline{U}_s = R_s \underline{I}_s + jX_{\sigma s}\underline{I}_s + j\omega(N_s f_{ws})\frac{2}{\pi}\tau_p l \frac{\mu_0}{\delta_e}\frac{3}{2}\frac{4}{\pi}\left(\frac{N_s f_{ws}}{2p}\underline{I}_s + \frac{N_r f_{wr}}{2p}\underline{I}_r\right),$$

$$\underline{U}_r = R_r \underline{I}_r + jsX_{\sigma r}\underline{I}_r + js\omega(N_r f_{wr})\frac{2}{\pi}\tau_p l \frac{\mu_0}{\delta_e}\frac{3}{2}\frac{4}{\pi}\left(\frac{N_s f_{ws}}{2p}\underline{I}_s + \frac{N_r f_{wr}}{2p}\underline{I}_r\right).$$

By introducing magnetic or air gap reactance

$X_m = \omega L_m = \omega \frac{2}{\pi}\tau_p l \frac{\mu_0}{\delta_e}\frac{3}{2}\frac{4}{\pi}\frac{(N_s f_{ws})^2}{2p}$ are the rearranged voltage equations:

$$\underline{U}_r = R_r \underline{I}_r + jsX_{\sigma r}\underline{I}_r + jsX_m \frac{N_r f_{wr}}{N_s f_{ws}}\underline{I}_s + jsX_m\left(\frac{N_r f_{wr}}{N_s f_{ws}}\right)^2 \underline{I}_r .$$

The stator's own reactance is introduced: $X_s = X_{\sigma s} + X_m$.

In the voltage equations, the term $X_m \frac{N_r f_{wr}}{N_s f_{ws}} = \frac{X_m}{K_U} = X_{sr} = X_{rs}$ is equal to the mutual or

magnetizing reactance between the stator and the rotor: $X_m = X_{sr} K_U = X'_{sr}$

Introduce the rotor's own reactance: $X_r = X_{\sigma r} + X_m\left(\frac{N_r f_{wr}}{N_s f_{ws}}\right)^2 = X_{\sigma r} + \frac{X_m}{K_U^2} = X_{\sigma r} + \frac{X_{sr}}{K_U}$.

The result is two basic voltage equations: $\begin{cases} \underline{U}_s = R_s \underline{I}_s + jX_s \underline{I}_s + jX_{sr} \underline{I}_r \\ \underline{U}_r = R_r \underline{I}_r + jsX_{rs} \underline{I}_s + jsX_r \underline{I}_r \end{cases}$

Induction machine equivalent circuit

1) By introducing reduced values and for the same number of phases, $m_s = m_r$ is:

$$\underline{U}'_r = \underline{U}_r K_U, \ \ \underline{I}'_r = \frac{\underline{I}_r}{K_U}, \ \ R'_r = R_r K_U^2, \ \ X'_{\sigma r} = X_{\sigma r} K_U^2 \ \text{and} \ X'_{sr} = X_{sr} K_U = X_m .$$

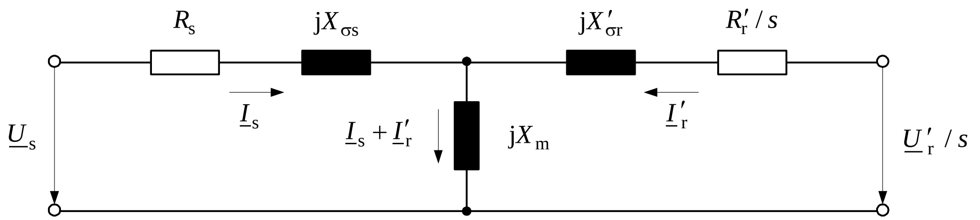
2) Add the first equation for voltage $\pm jX'_{sr} \underline{I}_s$ and the second equation $\pm jX'_{rs} \underline{I}'_r$.

3) The equation for the rotor voltage is divided by the slip to give:

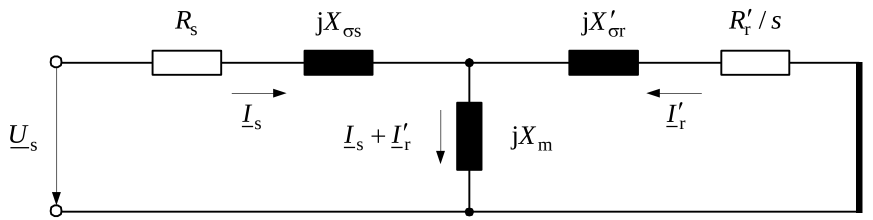
$$\underline{U}_s = (R_s + jX_{\sigma s}) \underline{I}_s + jX_m (\underline{I}_s + \underline{I}'_r) ,$$

$$\underline{U}'_r / s = (R'_r / s + jX'_{\sigma r}) \underline{I}'_r + jX_m (\underline{I}_s + \underline{I}'_r) ,$$

where $jX_m (\underline{I}_s + \underline{I}'_r) = \underline{E}_s = \underline{E}'_r$. These equations are matched by an equivalent circuit.



For a squirrel cage rotor winding $U_r = 0$, respectively $U'_r = 0$, the equivalent circuit applies:

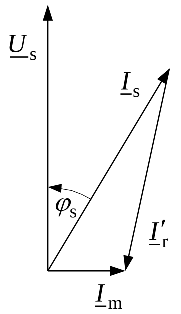


Approximate situation

a) No-load run

The assumption $(R_\mathrm{s} + \mathrm{j}X_{\sigma\mathrm{s}})\underline{I}_\mathrm{m} = 0$ gives an expression for $\underline{U}_\mathrm{s}$:

$$\underline{U}_\mathrm{s} = \mathrm{j}X_\mathrm{m}(\underline{I}_\mathrm{s} + \underline{I}'_\mathrm{r}) = \mathrm{j}X_\mathrm{m}\underline{I}_\mathrm{m} \rightarrow \underline{I}_\mathrm{m} = \frac{\underline{U}_\mathrm{s}}{\mathrm{j}X_\mathrm{m}}.$$



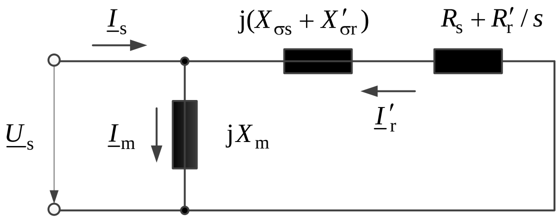
b) Load

The voltage drop due to magnetizing current is neglected, $\underline{I}_\mathrm{s} = -\underline{I}'_\mathrm{r}$ and $\underline{U}'_\mathrm{r} / s = 0$ are taken.

The voltage equations take the form:

$$\underline{U}_\mathrm{s} = -(R_\mathrm{s} + \mathrm{j}X_{\sigma\mathrm{s}})\underline{I}'_\mathrm{r} + \mathrm{j}X_\mathrm{m}(\underline{I}_\mathrm{s} + \underline{I}'_\mathrm{r}),$$

$$0 = (R'_\mathrm{r} / s + \mathrm{j}X'_{\sigma\mathrm{r}})\underline{I}'_\mathrm{r} + \mathrm{j}X_\mathrm{m}(\underline{I}_\mathrm{s} + \underline{I}'_\mathrm{r}).$$



Subtract the two equations to get:

$$\underline{U}_\mathrm{s} = -\Big((R_\mathrm{s} + R'_\mathrm{r} / s) + \mathrm{j}(X_{\sigma\mathrm{s}} + X'_{\sigma\mathrm{r}})\Big)\underline{I}'_\mathrm{r}.$$

This equation corresponds to the simplified equivalent circuit above. For the slip $s = 1$, the simplification applies: $R_\mathrm{sc} = R_\mathrm{s} + R'_\mathrm{r}$ and $X_\mathrm{sc} = X_{\sigma\mathrm{i}} = X_{\sigma\mathrm{s}} + X'_{\sigma\mathrm{r}}$.

Operating an induction machine on a rigid grid

The rotor winding is short-circuited $(\underline{U}_\mathrm{r} = 0)$.

The second basic voltage equation gives:

$\underline{I}_\mathrm{r}(R_\mathrm{r} + \mathrm{j}sX_\mathrm{r}) = -\mathrm{j}sX_\mathrm{sr}\underline{I}_\mathrm{s}$, divided by slip "$s$"

$$\underline{I}_\mathrm{r} = \frac{-\mathrm{j}X_\mathrm{sr}}{R_\mathrm{r} / s + \mathrm{j}X_\mathrm{r}}\underline{I}_\mathrm{s}$$ and inserted into the first voltage equation

$$\underline{U}_\mathrm{s} = U_\mathrm{s} = \left(R_\mathrm{s} + \mathrm{j}\,X_\mathrm{s} + \frac{X_\mathrm{sr}^2}{R_\mathrm{r}\,/\,s + \mathrm{j}\,X_\mathrm{r}} \right) \underline{I}_\mathrm{s} = \underline{Z}_\mathrm{s}\,\underline{I}_\mathrm{s}\ .$$

Calculate the current in the stator winding: $\underline{I}_\mathrm{s} = \dfrac{U_\mathrm{s}}{\underline{Z}_\mathrm{s}} = \dfrac{U_\mathrm{s}}{R_\mathrm{s} + \mathrm{j}\,X_\mathrm{s} + \dfrac{X_\mathrm{sr}^2}{R_\mathrm{r}\,/\,s + \mathrm{j}\,X_\mathrm{r}}}$.

The no-load current for $s = 0$: $\underline{I}_\mathrm{s0} = \dfrac{U_\mathrm{s}}{R_\mathrm{s} + \mathrm{j}\,X_\mathrm{s}}$.

In an ideal short circuit, the current for $s = \pm\infty$: $\underline{I}_\mathrm{ki} = \dfrac{U_\mathrm{s}}{R_\mathrm{s} + \mathrm{j}X_\mathrm{s}\left(1 - \dfrac{X_\mathrm{sr}^2}{X_\mathrm{s}X_\mathrm{r}}\right)} = \dfrac{U_\mathrm{s}}{R_\mathrm{s} + \mathrm{j}\sigma X_\mathrm{s}}$.

Stator current curve of IM

For an approximate situation: $\underline{I}_\mathrm{s} = \dfrac{U_\mathrm{s}}{\mathrm{j}X_\mathrm{m}} - \underline{I}'_\mathrm{r} = \underline{I}_\mathrm{m} - \underline{I}'_\mathrm{r}$.

$\underline{I}'_\mathrm{r} = -\dfrac{U_\mathrm{s}}{R_\mathrm{s} + R'_\mathrm{r}/s + \mathrm{j}\,X_{\sigma\mathrm{i}}}$, where $X_{\sigma\mathrm{i}}$ is the leakage reactance $(X_{\sigma\mathrm{i}} \approx X_{\sigma\mathrm{s}} + X'_{\sigma\mathrm{r}})$

for ideal short-circuit i.e., common leakage of the stator and rotor.

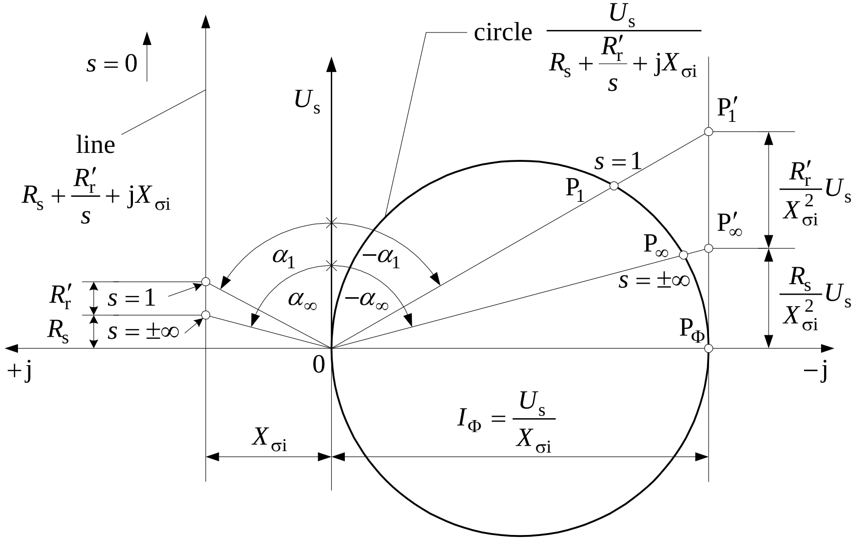
The stator current is now: $\underline{I}_\mathrm{s} = \dfrac{U_\mathrm{s}}{\mathrm{j}\,X_\mathrm{m}} + \dfrac{U_\mathrm{s}}{R_\mathrm{s} + R'_\mathrm{r}\,/\,s + \mathrm{j}\,X_{\sigma\mathrm{i}}}$.

We look at the characteristic values of the current for $s = 0,\ 1$ and $\pm\infty$.

For $R_\mathrm{s} + R'_\mathrm{r}\,/\,\mathrm{s} \to 0$, the current is purely inductive.

The second term in the equation for stator current represents a circle of diameter: $\underline{I}_\Phi = \dfrac{U_\mathrm{s}}{\mathrm{j}\,X_{\sigma\mathrm{i}}}$.

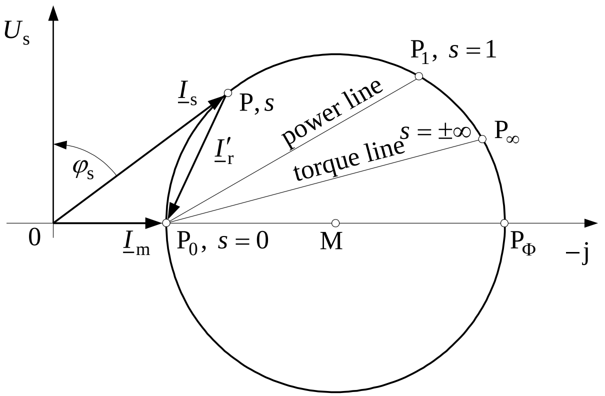
The circle is a graphical process, and is obtained by inversion of the complex quantities.

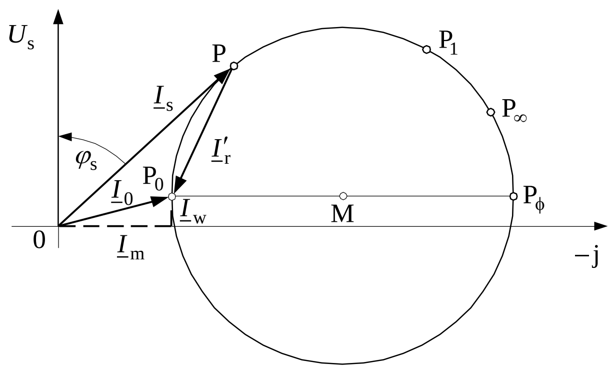


$$\overline{P_\Phi P'_\infty} = \text{ctg}\,\alpha_\infty I_\Phi = \frac{R_s}{X^2_{\sigma i}} U_s \text{ and } \overline{P'_\infty P'_1} = \text{ctg}\,\alpha_1 I_\Phi - \overline{P_\Phi P'_\infty} = \frac{R'_r}{X^2_{\sigma i}} U_s \text{ , if it is}$$

$$\text{ctg}\,\alpha_\infty = \frac{R_s}{X_{\sigma i}} \quad \text{and} \quad \text{ctg}\,\alpha_1 = \frac{R_s + R'_r}{X_{\sigma i}}.$$

Now move the circle from the coordinate origin for the magnitude of the magnetizing current $\underline{I}_m = -jU_s / X_m$ to the right to obtain the current curve $\underline{I}_s = f(s)$.





Taking into account the losses in the iron, the friction and windage losses, the no-load current has a watt component $\underline{I}_{\mathrm{w}}$.

Energy balance

Power flow diagram

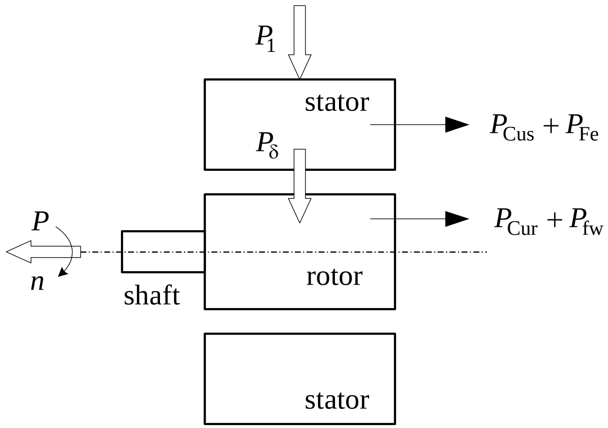
$$P_1 = P_{\mathrm{Cus}} + P_{\mathrm{Fe}} + P_\delta \quad (P_{\mathrm{Fe}} = P_{\mathrm{Fes}})$$

P_δ (air gap power)

$$P_\delta = P_{\mathrm{Cur}} + P_{\mathrm{fw}} + P \quad (P_{\mathrm{Fer}} = 0,\ \ f_{\mathrm{r}} \to 0)$$

$P = P_{\mathrm{m}}$ the power delivered or mechanical power at the motor shaft. Input stator power is:

$$P_1 = P_{\mathrm{Cus}} + P_{\mathrm{Fe}} + P_{\mathrm{Cur}} + P_{\mathrm{fw}} + P\,.$$



Power calculation for the m-phase stator $(m_{\mathrm{s}} = 3)$

Input electrical power delivered to the stator: $\quad P_1 = m_{\mathrm{s}} U_{\mathrm{s}} I_{\mathrm{s}} \cos\varphi_{\mathrm{s}}$

Joule losses in the stator winding: $\quad P_{\mathrm{Cus}} = m_{\mathrm{s}} I_{\mathrm{s}}^2 R_{\mathrm{s}}$

Power delivered to the rotor: $\quad P_\delta = m_{\mathrm{s}} I_{\mathrm{r}}'^2 \dfrac{R_{\mathrm{r}}'}{s} = \dfrac{P_{\mathrm{Cur}}}{s}$

Joule losses in the rotor winding: $\quad P_{\mathrm{Cur}} = m_{\mathrm{s}} \underline{I}_{\mathrm{r}}'^2 R_{\mathrm{r}}' = m_{\mathrm{r}} I_{\mathrm{r}}^2 R_{\mathrm{r}}$

Output mechanical power $(P_2 = P)$ at the shaft: $\quad P = P_\delta - P_{\mathrm{Cur}} = m_{\mathrm{s}} I_{\mathrm{r}}'^2 R_{\mathrm{r}}' \left(\dfrac{1}{s} - 1\right) =$

$$= m_{\mathrm{s}} I_{\mathrm{r}}'^2 R_{\mathrm{r}}' \left(\frac{1-s}{s}\right) = P_\delta (1-s)$$

Motor torque

At "n" rotor rotations $\Omega_{\mathrm{m}} = 2\pi n = 2\pi n_{\mathrm{s}}(1-s) = \Omega_{\mathrm{ms}}(1-s)$ and $P_\delta = \Omega_{\mathrm{ms}} M_\delta$

$$M = \frac{P}{\Omega_{m}} = \frac{P}{\Omega_{ms}(1-s)} = \frac{m_{s} I_{r}'^{2} R_{r}'}{\Omega_{ms}(1-s)} \frac{1-s}{s} = \frac{P_{\delta}}{\Omega_{ms}} = M_{\delta}$$

$M = M_{\delta}$ is valid only if the friction and windage losses are $P_{fw} = 0$ and $M_{fw} = 0$, respectively.

Operating areas on the circle diagram

Depending on the points on the circle, the following areas of operation are obtained for different slip values:

a) rotor lagging behind the rotating field s = 0 to 1, " **motor area** " ($n = n_{s} \div 0$);

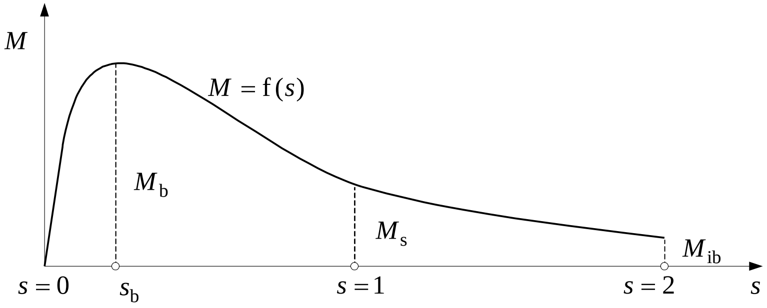
b) the rotor rotates against the rotating field s = 1 to $+\infty$, " **braking area** " ($n = 0 \div -\infty$);

c) the rotor rotates faster than the rotating field s = 0 to $-\infty$, " **generator area** " ($n = n_{s} \div +\infty$).

Torque

The torque $M = \mathrm{f}(s)$ is obtained graphically (from a circle diagram) or analytically.

a) Torque waveform $M = \mathrm{f}(s)$ for the slip area $s = 0 \div 2$



The labels in Figure $M = \mathrm{f}(s)$ have the following meanings:

M_{b} is the breakdown,

M_{s} is the starting (pull-up) torque,

M_{ib} is the initial braking torque.

b) Analytical derivation from power P_δ

$$M=\frac{P_\delta}{\Omega_{ms}}=\frac{m_s I_r'^2 R_r'}{\Omega_{ms} s} \qquad \text{for} \qquad I_r'=\frac{U_s}{\sqrt{(R_s+R_r'/s)^2+X_{\sigma i}^2}} \text{ (p. 74) is obtained:}$$

$$M=\frac{m_s U_s^2}{\Omega_{ms}}\,\frac{R_r'/s}{(R_s+R_r'/s)^2+X_{\sigma i}^2}=\frac{m_s U_s^2}{\Omega_{ms}}\,\frac{1}{(R_s^2+X_{\sigma i}^2)\,s/R_r'+R_r'/s+2R_s}\,.$$

The maximum (breakdown) torque is obtained from the condition:

$\frac{\partial M}{\partial s}=0$, which gives a value for $s_b=\pm\frac{R_r'}{\sqrt{R_s^2+X_{\sigma i}^2}}\approx\pm\frac{R_r'}{X_{\sigma i}}$ if ($R_s \ll X_{\sigma i}$), and

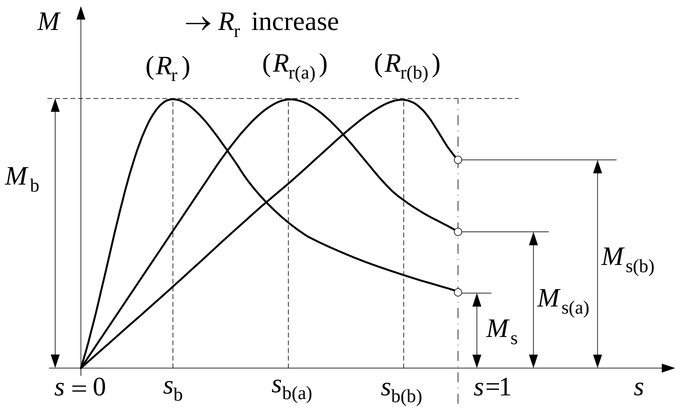
$$M_b=\frac{m_s U_s^2}{2\Omega_{ms}}\,\frac{1}{R_s+\sqrt{R_s^2+X_{\sigma i}^2}} \;\Rightarrow\; M_b\approx\frac{3}{2\Omega_{ms}}\,\frac{U_s^2}{X_{\sigma i}}\,.$$

Effect of changing R_r and $X_{\sigma i}$ on the torque curve

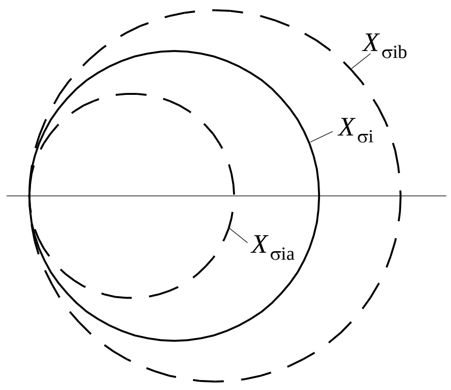
a) Increase R_r (R_r')

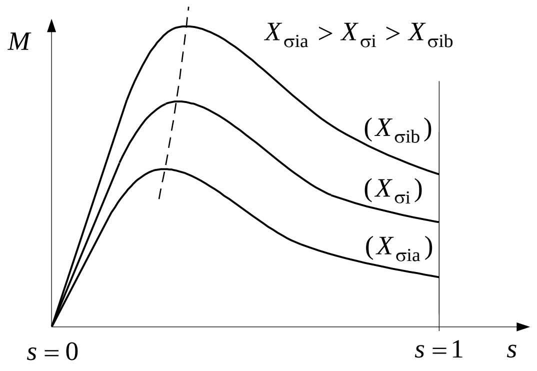
$$R_{r(b)}>R_{r(a)}>R_r$$

As R_r increases, the point s_b moves along the circle towards higher values of the slip. It is even possible that $s_b>1$.



b) Change $X_{\sigma i} = X_{\sigma s} + X'_{\sigma r}$





The Kloss equation

It represents the ratio $\dfrac{M}{M_b}$, which is obtained from the equation for M and M_b respectively:

$$\frac{M}{M_b} = \frac{2(1 + s_b R_s / R'_r)}{s / s_b + s_b / s + 2 s_b R_s / R'_r}.$$

If $R_s \approx R'_r$ and $s_b << 1$, it is obtained for $R_s \to 0$.

<u>The Kloss ratio</u>: $\dfrac{M}{M_b} = \dfrac{2}{s / s_b + s_b / s}$,

where the breakdown slip is $s_b \approx \dfrac{R'_r}{X_{\sigma i}}$ and $M_b \approx \dfrac{3}{2\Omega_{ms}} \dfrac{U_s^2}{X_{\sigma i}}$.

For $s \to 0$, applicable $s / s_b << s_b / s$ and $\dfrac{M}{M_b} = \dfrac{2s}{s_b} = k_1 s$ (equation of a line).

For $s \to 1$, applicable $s / s_b >> s_b / s$ and $\dfrac{M}{M_b} = \dfrac{2s_b}{s} = k_2 \dfrac{1}{s}$ (hyperbola).

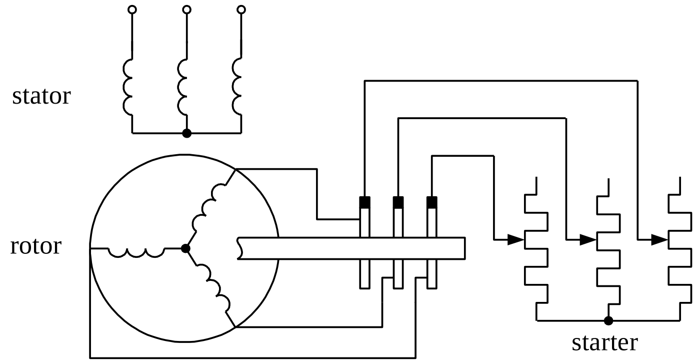
Starting an induction motor

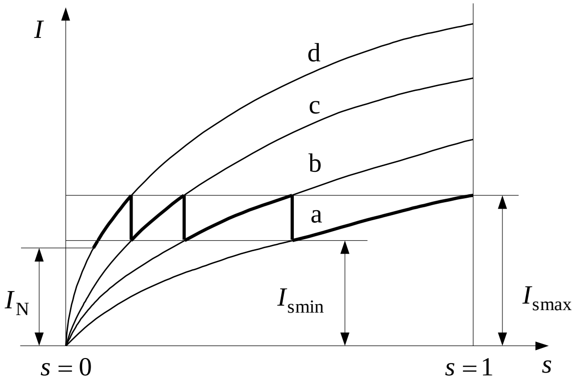
Induction motor starting is the process that takes a certain amount of time for the rotor to reach speed from $n = 0$ to n. Ideally: ratio M_s / M_N is as large as possible and I_s / I_N as small as possible.

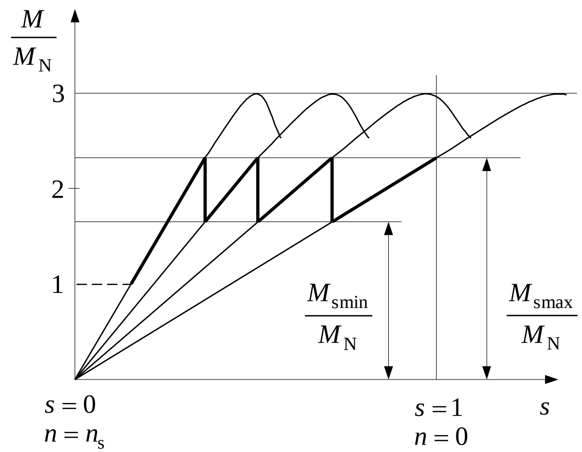
a) Starting the motor with a starter

During start-up, apply additional ohmic resistors via the slip rings (applies only to the wound rotor).

$$R_a + R_r > R_b + R_r >$$
$$> R_c + R_r > R_r$$







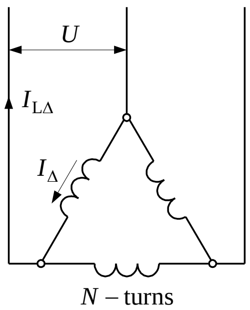
Maximum permissible starting current $I_{smax} < I_s$

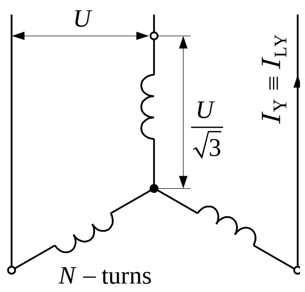
Minimum permissible starting current $I_{smin} \approx I_N$

b) Starting the motor with the switch Y / Δ

In a star connection, the voltage across the winding is: $U_Y = U / \sqrt{3} = U_s$.

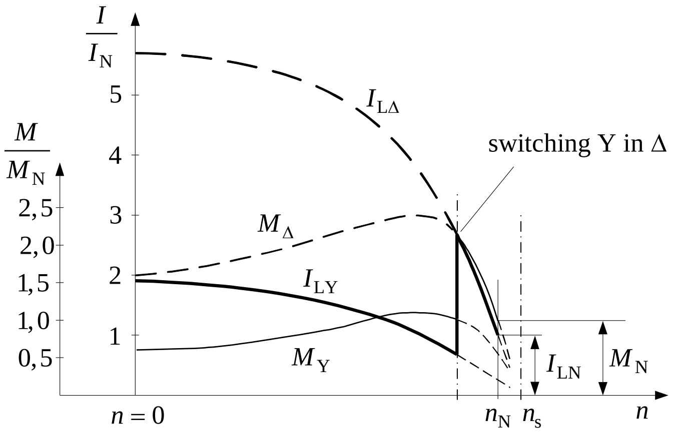
The current in the terminals for a star connection compared to a delta connection is:





$$I_{LY} = I_Y = \frac{I_\Delta}{\sqrt{3}} = \frac{I_{L\Delta}}{\sqrt{3}\cdot\sqrt{3}} = \frac{I_{L\Delta}}{3}.$$

Display current (and torque) curves by switching from Y in Δ



The winding voltage is $U/\sqrt{3} \rightarrow$

$$\Phi_Y = \frac{\Phi_\Delta}{\sqrt{3}} \quad \rightarrow \quad B_Y = \frac{B_\Delta}{\sqrt{3}}.$$

Starting torque

$$M_Y = k B_Y I_Y = k \frac{B_\Delta}{\sqrt{3}} \frac{I_\Delta}{\sqrt{3}}$$

$$M_Y = \frac{1}{3} M_\Delta$$

c) Start-up transformer (autotransformer)

Ratio $K_U = \frac{U_{1N}}{U_{1x}} \geq 1$ and $K_I = \frac{I_L}{I_s} = \frac{1}{K_U} \leq 1$, stator phase current of the motor $I_{sx} = K_I I_s$,

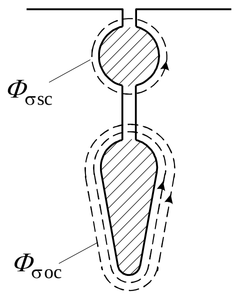
grid (line) current $I_{Lx} = K_I I_{sx} = K_I^2 I_s$ and $M_x = K_I^2 M$.

Special versions of squirrel cages

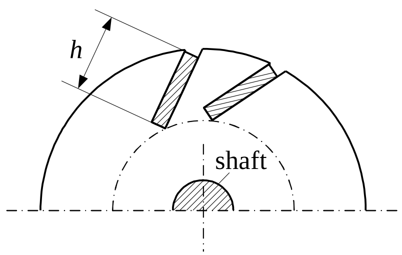
a) Double cage

Above is the starting cage (index "sc").

The lower one is the operating cage (index "oc").



$\Phi_{\sigma sc} > \Phi_{\sigma oc} \rightarrow X_{\sigma oc} > X_{\sigma sc}$; $X_{\sigma r} = s X_{\sigma r(ss)}$ (ss − standstill)



b) Deep slots

We take advantage of the impact of the ejection current.

The height or depth of the slot h is increased by the inclination of the slot.

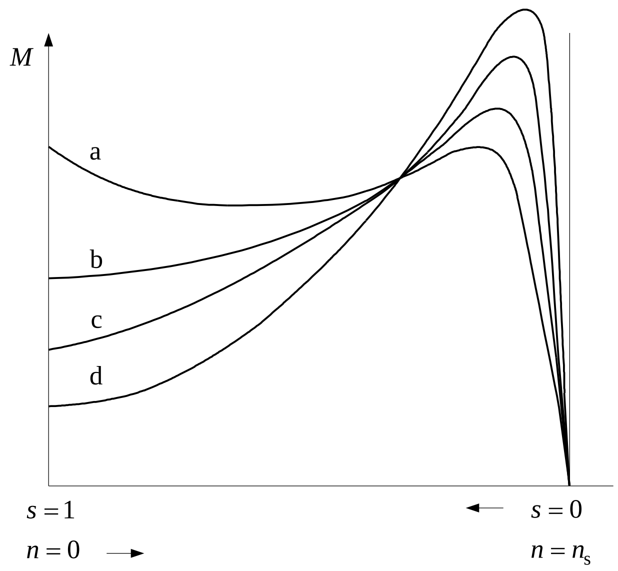
Shapes of torque curves of different rotor winding designs:

a) double cage,

b) deep slots,

c) normal cage,

d) wound rotor.



Options for changing the rotation speed

$$n = n_s(1-s) = \frac{f}{p}(1-s)$$

1) by changing the frequency,

2) by changing the number of pole pairs,

3) by changing the slip.

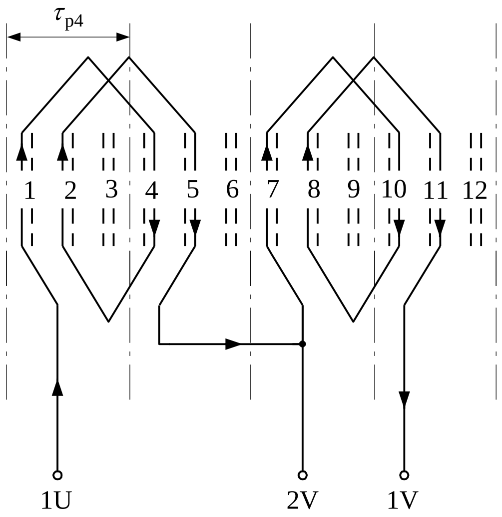
1) Frequency-changing

From $U = k f \Phi$ it applies: $\frac{U}{f} = \frac{U_x}{f_x} = k\Phi = \text{const.}$ (Applies up to the value $U_x \le U_N$.)

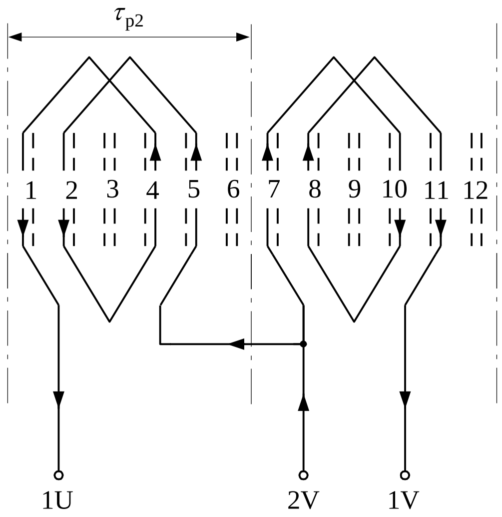
2) Pole pairs (poles) changing

a) With multiple windings for different numbers of poles

b) Dahlander connection ratio by 2:1. This uses a Δ / YY or Y/YY circuit combination. The first connection is used mostly, the second only to drive the fans.

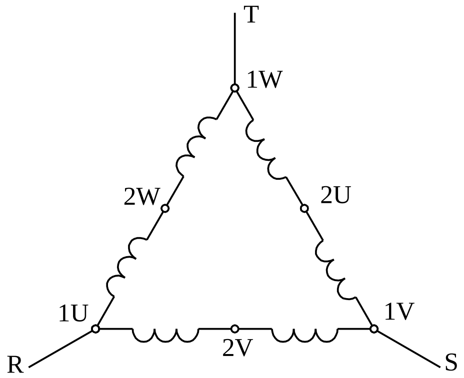


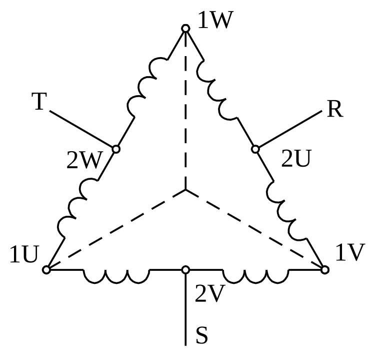
Directions of currents for $2p = 4$.



Directions of currents for $2p = 2$.

Circuit illustration for connection



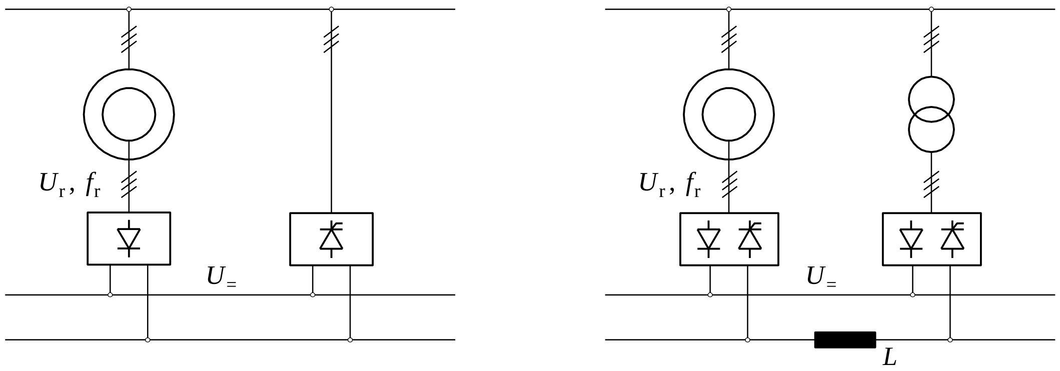


3) Slip-changing (for a motor with slip rings)

Since $M = \dfrac{P_\delta}{\Omega_{ms}}$, is $M = \text{const.}$ for $P_\delta = \text{const.}$ and with the resistance R_{add} in the rotor is:

$$P_\delta = m_s I_{rN}'^2 \frac{R_r'}{s_N} = m_s I_{rN}'^2 \frac{R_r' + R_{add}'}{s_x} \Rightarrow s_x = \frac{R_r' + R_{add}'}{R_r'} s_N .$$

With higher slip, the losses $P_{Cur} = P_\delta s_x$ in the rotor circuit increase. This is not economical. It is more economical to use a cascade, i.e., a rectifier − inverter (U_r, f_r, $U_=$ are variable) or converter cascades (U_r, f_r are variable, $U_= = \text{const.}$) in the rotor circuit.



Changing the torque

The torque is proportional to the power of the air gap or the power at the resistor according to the equivalent circuit R_r'/s:

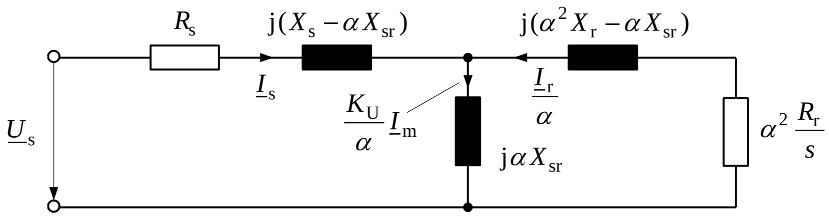
$$M = \frac{3}{\Omega_{ms}} I_r'^2 \frac{R_r'}{s} = \frac{3}{\Omega_{ms}} I_r^2 \frac{R_r}{s} ,$$

where the rotor current I_r' is reduced to the stator $I_r' = I_r K_I = \dfrac{I_r}{K_U}$ and $R_r' = K_U^2 R_r$ (for $m_s = m_r$).

Introduce a voltage drop across the rotor at the replacement resistor R_r / s $(E_{Rr} = I_r R_r / s)$. The rotor current $\underline{I}_r$ or $\underline{I}'_r$ is opposite to the stator current, and therefore the voltage drop $E'_{Rr} = \underline{I}'_r R'_r / s$ is of opposite magnitude to the induced voltage due to the rotation magnetic field $E_m = E_s = E'_r$.

$$M = \frac{3p}{\omega} E'_{Rr} I'_r = \frac{3p}{\omega} E_{Rr} I_r$$

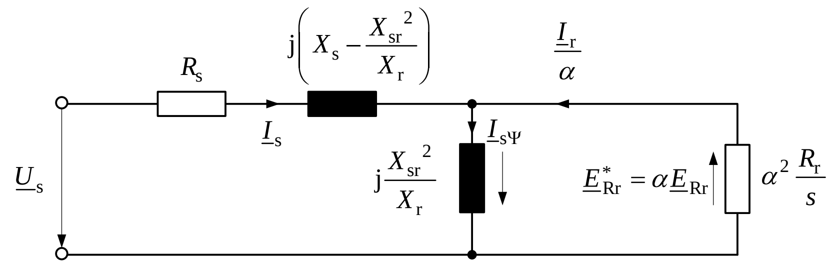
In an induction motor, torque variation (control) is achieved by varying the voltage E_{Rr}, i.e., by varying the current I_r. (The phase angle between E_{Rr} and I_r is zero.) For this purpose, we modify the usual equivalent circuit. The stator and rotor leakage reactance are expressed as the difference of the own and mutual reactance between the stator and rotor. The transformation constant α (in the circuit) can be chosen to be any value except zero. For the case $\alpha = K_U$, i.e., equal to the ratio of the effective stator and rotor turns, we obtain again the usual equivalent circuit.



From the condition $\alpha^2 X_r - \alpha X_{sr} = 0$ we obtain: $\alpha = X_{sr} / X_r$, i.e., the transformation constant at which the leakage reactance in the rotor circuit of the equivalent circuit vanishes.

The new rotor current is now $1/\alpha$-times the actual (three-phase) rotor current (I_r), and the new rotor voltage is α-times the original rotor voltage (voltage drop E_{Rr}) across the resistance. This new voltage (voltage drop E^*_{Rr}) is applied to the terminals of the new magnetic (main) reactance and is, thus, related directly to the flux of the rotor, which causes the voltage E_{Rr} $(E^*_{Rr} = \alpha E_{Rr})$.

By taking the product of $\alpha X_{\mathrm{sr}} = X_{\mathrm{sr}}^2 / X_{\mathrm{r}}$, we obtain a new equivalent circuit.



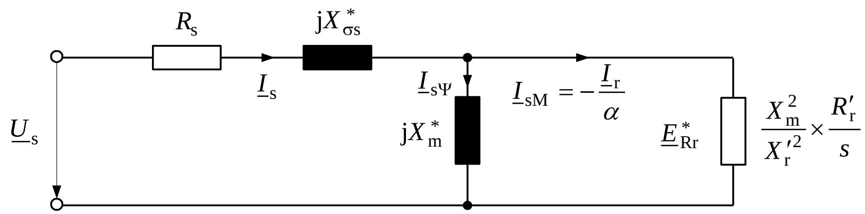
This equivalent circuit is reformulated by introducing a transient machine parameter. This is defined as the stator leakage reactance of the whole machine, and given by equation:

$$X_{\sigma\mathrm{s}}^* = X_{\mathrm{s}} - \frac{X_{\mathrm{sr}}^2}{X_{\mathrm{r}}} = X_{\mathrm{s}}\left(1 - \frac{X_{\mathrm{sr}}^2}{X_{\mathrm{s}} X_{\mathrm{r}}} \frac{K_{\mathrm{u}}^2}{K_{\mathrm{u}}^2}\right) = X_{\mathrm{s}}\left(1 - \frac{X_{\mathrm{m}}^2}{X_{\mathrm{s}} X_{\mathrm{r}}'}\right) = \sigma X_{\mathrm{s}}\,.$$

The asterisk index does not mean relative, but means the changed stator reactance. Similarly, an asterisk indicates the changed magnetizing reactance:

$$X_{\mathrm{m}}^* = \alpha X_{\mathrm{sr}} = \frac{X_{\mathrm{sr}}^2}{X_{\mathrm{r}}} \frac{K_{\mathrm{U}}^2}{K_{\mathrm{U}}^2} = \frac{X_{\mathrm{m}}^2}{X_{\mathrm{r}}'}\,.$$

Considering that: $\alpha = \frac{X_{\mathrm{sr}}}{X_{\mathrm{r}}} \frac{K_{\mathrm{U}}^2}{K_{\mathrm{U}}^2} = \frac{X_{\mathrm{m}}}{X_{\mathrm{r}}'} K_{\mathrm{U}}$ and $\alpha^2 \frac{R_{\mathrm{r}}}{s} = \frac{X_{\mathrm{m}}^2}{X_{\mathrm{r}}'^2} \times \frac{R_{\mathrm{r}}'}{s}$, we obtain a finitely transformed equivalent circuit in which all quantities are transformed to the stator.



The stator current is thus divided into two (mutually orthogonal) current components, namely the reactive component of the current $I_{\mathrm{s}\Psi} = I_{\mathrm{s}} \sin\gamma$ $(I_{\mathrm{s}\Psi} = (K_{\mathrm{U}} / \alpha) I_{\mathrm{m}}$, i.e., the excitation current) and the working component of the current $I_{sM} = I_s \cos\gamma$; this represents the torque. The working component may also be marked I_{sT}.

Varying the torque with the current $\underline{I}_{s\Psi}$ and $\underline{I}_{sM}$

The induced voltage, or the stator-reduced voltage drop E^*_{Rr} across the rotor substitution resistance $\alpha^2 R_r / s$, is equal to the time variation of the rotor magnetic linkage:

$$\underline{E}^*_{Rr} = -j\omega \underline{\Psi}^*_r / \sqrt{2} = -jX^*_m \underline{I}_{s\Psi} .$$

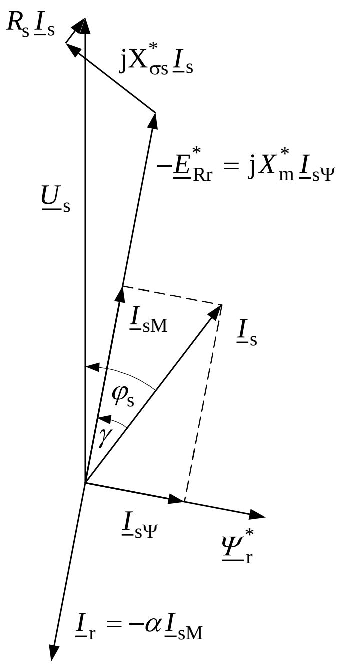
Reduced values can also be omitted from the equation, and it is:

$$\underline{E}_{Rr} = -j\omega \underline{\Psi}_r / \sqrt{2} .$$

According to a redesigned circuit, the reactive component of the excitation current $\underline{I}_{s\Psi}$ is given by equation:

$$\underline{I}_{s\Psi} = -\frac{\underline{E}^*_{Rr}}{jX^*_m} = -\frac{\alpha \underline{E}_{Rr}}{j\alpha X_{sr}} = -\frac{\underline{E}_{Rr}}{j\omega L_{sr}} .$$

From the last equation, the induced voltage is $E_{Rr} = \omega L_{sr} I_{s\Psi}$. Product $L_{sr} I_{s\Psi}$ in the induced voltage equation E_{Rr} represents a rotor magnetic linkage $\Psi_r = L_{sr} I_{s\Psi}$ (RMS value because the current is not multiplied by $\sqrt{2}$), and the reduced value of this is: $\Psi^*_r = \alpha \Psi_r = \alpha L_{sr} I_{s\Psi} = L^*_m I_{s\Psi}$.



The torque component of the current I_{sM} is obtained directly from the circuit as the expression:

$$I_{sM} = |\underline{I}_{sM}| = \frac{I_r}{\alpha} \rightarrow I_r = \alpha I_{sM} .$$

The final equation for the torque of the machine, if the expression for the voltage E_{Rr} and the current I_r (without the negative sign) is substituted in, reads:

$$M = \frac{3p}{\omega}(\omega L_{sr} I_{s\Psi})(\alpha I_{sM}) = 3p\,\alpha L_{sr} I_{s\Psi} I_{sM} = 3p\,L_m^* I_{s\Psi} I_{sM}\,.$$

The equation represents an expression that is a function of the two components of the current. The basic equation for the torque takes the form (with $\Psi_r^* = L_m^* I_{s\Psi}$ and $I_{sM} = I_s \cos\gamma$):

$$M = 3p\Psi_r^* I_s \cos\gamma = 3p\Psi_r^* I_{sM}\,,$$

i.e. the product of the rotor magnetic linkage and the working component of the stator current. Given an equivalent circuit, we obtain the working component of the stator current I_{sM}:

$$\underline{I}_{sM} = -\frac{\alpha \underline{E}_{Rr}}{\alpha^2 R_r / s} = -\frac{1}{\alpha}\frac{s\underline{E}_{Rr}}{R_r} = -\frac{X_r}{X_{sr}}\frac{s\underline{E}_{Rr}}{R_r} = -\frac{L_r}{L_{sr}}\frac{s\underline{E}_{Rr}}{R_r}\,.$$

Combining the last equation with the previous equation for $\underline{E}_{Rr}$ gives the relationship between the two stator current components I_{sM} and $I_{s\Psi}$:

$$\underline{I}_{sM} = \mathrm{j}\frac{L_r}{R_r} s\omega \underline{I}_{s\Psi}\,.$$

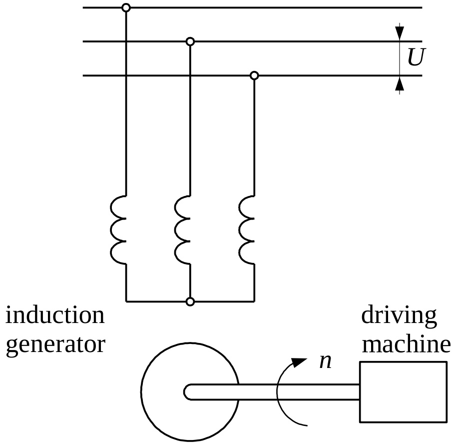
This connection is due to the fact that the voltages on the new magnetizing (mutual) reactance and on the replacement (fictitious) rotor resistance are the same. The slip angular frequency is also determined by the two current components. Rearranging the last equation gives the expression for the slip angular frequency:

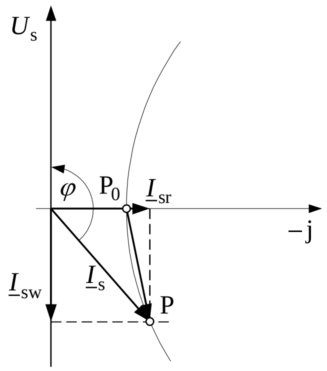
$$s\omega = \frac{R_r}{L_r}\frac{I_{sM}}{I_{s\Psi}} = \frac{R_r'}{L_r'}\frac{I_{sM}}{I_{s\Psi}} = \frac{1}{\tau_r}\frac{I_{sM}}{I_{s\Psi}}\,.$$

$\tau_r = L_r / R_r$ is the electrical time constant of the change in all rotor quantities. When both components of the current are selected (in stationary machine operation − motor), only one value of the slip in this equation gives us the corresponding rotor magnetic linkage Ψ_r and torque M.

Induction generator

Operating on a rigid grid



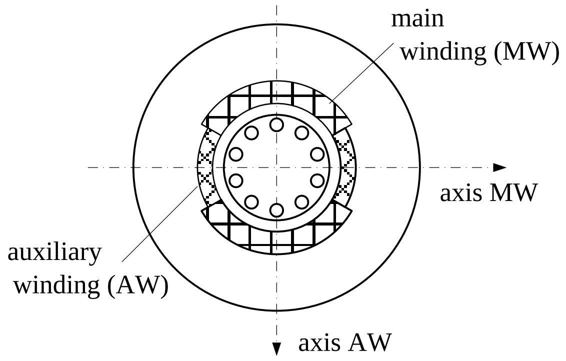


The grid provides the reactive power $(Q_{Gs} = 3U_s\, I_s \sin\varphi = 3U_s\, I_{sr})$ and the driving machine provides the working power. An IM works as a generator if it is a slip $s = (0 \div -1)$. Then the load drives the rotor faster than the speed of the rotating magnetic field and the generator emits electrical power. For the slip $s < -1$, the machine works in the generator braking area and does not emit any power.

Single-phase induction motor

They are building up to $2,2$ kW.

It has a main winding (usually occupying $2/3$ of the stator slots) and an auxiliary winding, the axis of which is shifted by 90 (electrical) degrees with respect to the axis of the main winding (usually occupying the remaining $1/3$ of the stator slots).

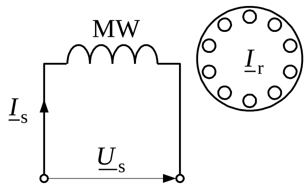


a) Operating mode if only the main winding is connected.

The sinusoidal voltage U_{s} dictates the sinusoidal flux Φ_{m}. The fundamental harmonic component of the magnetic flux density is a function of the local distribution in stator coordinates, or the circumferential angle $\vartheta_{\text{s}} = \left(x_{\text{s}} / \tau_{\text{p}} \right)\pi$ in the air gap, and time:

$$b_1 = \text{f}(x_{\text{s}}, t) = \hat{B}_1 \cos\vartheta_{\text{s}} \cos(\omega t) =$$

$$= \frac{\hat{B}_1}{2}\left(\cos\left(\vartheta_{\text{s}} - \omega t\right) + \cos\left(\vartheta_{\text{s}} + \omega t\right)\right).$$



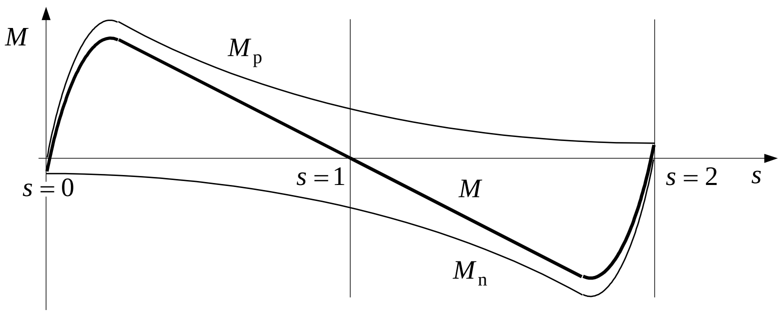
In the rotor coordinate system, the fundamental harmonic component of the field for

$\vartheta_{\text{s}} = \vartheta_{\text{r}} + p\Omega_{\text{m}} t = \vartheta_{\text{r}} + p(2\pi n)t$, taking into account $n = n_{\text{s}}(1-s) = \dfrac{f}{p}(1-s)$ and

$\vartheta_{\text{s}} = \vartheta_{\text{r}} + 2\pi pnt = \vartheta_{\text{r}} + 2\pi pn_{\text{s}}(1-s)t = \vartheta_{\text{r}} + \omega(1-s)t$, is:

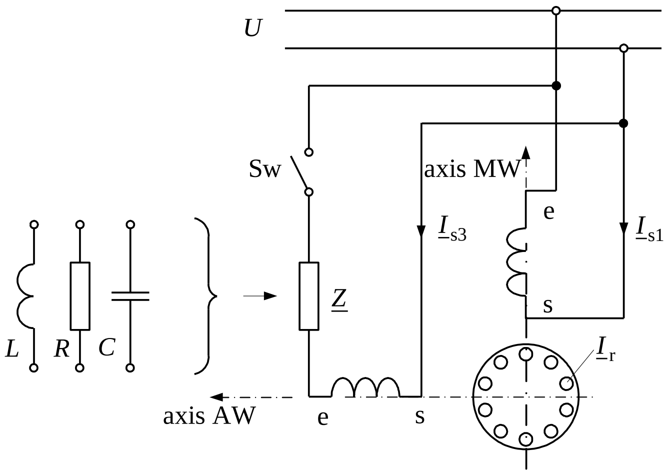
$$b_1 = \text{f}(x_{\text{r}}, t) = \frac{\hat{B}_1}{2}\cos\left(\vartheta_{\text{r}} - s\omega t\right) + \frac{\hat{B}_1}{2}\cos\left(\vartheta_{\text{r}} + (2-s)\omega t\right).$$

In the rotor, the induced voltage of frequency $s f$ and $(2-s) f$, i.e., the induced voltage is obtained of the positive and negative components of the field. At $s = 1$ the two rotating magnetic fields are equal and develop the same torque. Such a motor therefore does not start by itself.



b) Start

An auxiliary phase – winding (AW) is required to start the motor. The current in the auxiliary phase has to be shifted in time and an asymmetric (elliptical rotating magnetic field) is obtained. The capacitor gives the largest displacement.



Torque ratio:

1) $\underline{Z} = R \qquad M_s / M_N = 1 \div 1{,}3$

2) $\underline{Z} = jX_L \qquad M_s / M_N \approx 0{,}3$

3a) $\underline{Z} = jX_C \qquad M_s / M_N = 1{,}6 \div 2{,}1$

3b) $\underline{Z} = jX_C \qquad M_s / M_N \approx 0{,}5$

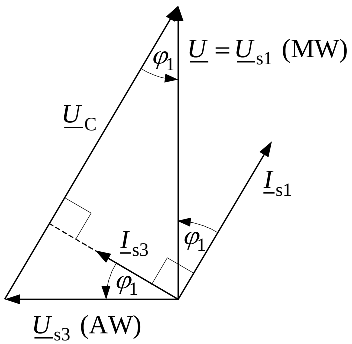
3a) starting capacitor,

b) operating capacitor,

c) combination 3a) + 3b).

Switch "Sw" switches off the start capacitor by $n \approx 0{,}75 n_s$.

Phasor diagram drawn for example 3b)

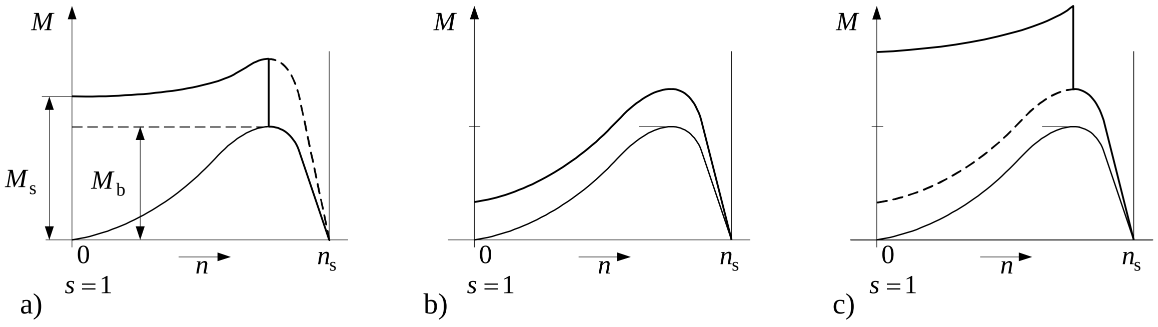


In this case, we have two-phase asymmetrical excitations $\Theta_{s1} \neq \Theta_{s3}$.

$$U_C > U_s$$
$$\cos\varphi \approx 1$$

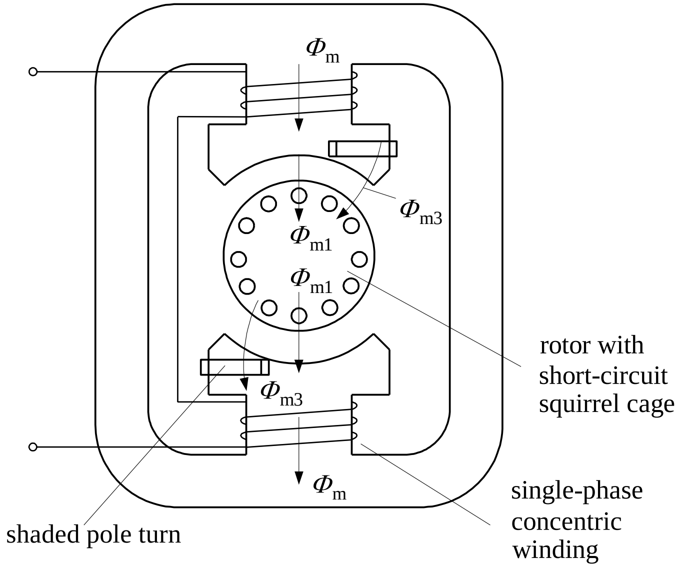
The capacitor motor rotates from the auxiliary to the main phase. To reverse the direction of rotation, the terminals, i.e., the start (s) and end (e) of the auxiliary or main phase, must be reversed.

Torque curves for starting with auxiliary winding for cases 3a), 3b) and 3c)



Shaded poles motor

These types of motors are built for low power (a few tens of watts). A short-circuit turn (ring) is mounted on part of the pole shoe.



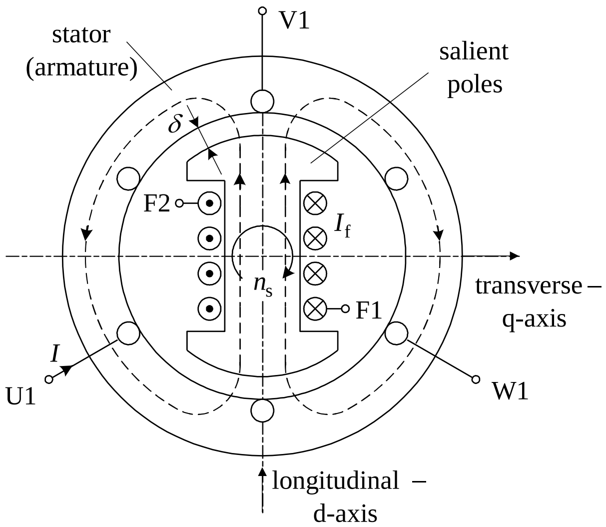
The main flux Φ_m is divided into a part going directly into the rotor Φ_{m1} and a shaded part Φ_{m3} passing through the short-circuit ring. The latter is time delayed towards Φ_{m1} and smaller. An elliptical rotating magnetic field is created. Due to the delay (Φ_{m3} relative to Φ_{m1}) the rotor will always rotate from the unshaded to the shaded part. To reverse the direction of rotation, the rotor must be reversed (reassembled).

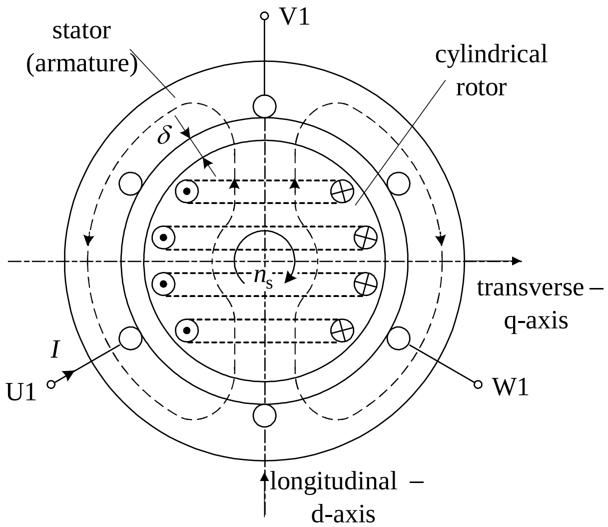
SYNCHRONOUS MACHINE

Description of construction

There are two designs, a machine with salient poles on the rotor, and a machine with non-salient poles or a machine with a cylindrical rotor. For both designs, the primary winding is on the rotor and the secondary winding is on the stator. The stator is identical to the IM stator.

For the salient poles, the air gap is $\delta = \mathrm{f}(x)$, non-salient pole $\delta = \mathrm{konst.}$





For the salient poles, the air gap "δ" in "d" and "q" axis is different (magnetic conductivity $\Lambda_\mathrm{d} \neq \Lambda_\mathrm{q}$). The excitation − field winding is wound concentric on the salient pole body, but belt (usually spread over 2/3 of the circumference) at a cylindrical rotor.

Operation mode

The rotor is supplied with a DC excitation current I_f or I_fd. To generate the excitation rotating ampere-turns (magnetomotive force − *MMF*) of the Θ_f, rotate it by $n_\mathrm{s} = f / p$ rotation. For a cylindrical rotor it is:

$$\hat{\Theta}_{\mathrm{f}} = \frac{4}{\pi}\,\frac{N_{\mathrm{f}}\ f_{\mathrm{wf}}}{2p}\,I_{\mathrm{f}}\,.$$

The salient pole winding factor $f_{\mathrm{wf}} = 1$, turns/pole $N_{\mathrm{f\,p}} = N_{\mathrm{f}}\,/\,(2p)$ and $\hat{\Theta}_{\mathrm{f}} = \frac{4}{\pi} N_{\mathrm{fp}} I_{\mathrm{f}}$.

No-load

$\hat{\Theta}_{\mathrm{f0}} \to \hat{B}_{\delta} = \frac{\mu_0\,\hat{\Theta}_{\mathrm{f0}}}{\delta_{\mathrm{e}}} = \hat{B}_1$ magnetic flux density of the fundamental harmonic component or the mean value of the main air gap flux $\hat{\Phi}_{\mathrm{m}} = \frac{2}{\pi}\hat{B}_1 A_{\delta} = \frac{2}{\pi}\hat{B}_1 \tau_{\mathrm{p}} l$

(τ_{p} is the pole pitch, l the length of the stack, δ_{e} the equivalent air gap of the machine).

At no-load, the current in the stator winding or armature winding is equal to $I_{\mathrm{s}} = I_{\mathrm{a}} = I = 0$. The main flux induces a no-load voltage in the stator winding which, when excited by $I_{\mathrm{f}} = I_{\mathrm{f\,0}}$, is equal to the grid voltage:

$$U = 4{,}44\, f\, N_{\mathrm{s}}\, f_{\mathrm{ws}} \hat{\Phi}_{\mathrm{m}}\,.$$

$\Phi_{\mathrm{m}} = \Phi_{\mathrm{m0}} = \Phi_{\mathrm{p0}}$ is the flux of the salient pole due to the field winding concentrically wound around the salient pole body, or arranged on a cylindrical rotor.

Load

When loaded, the stator current I generates stator ampere-turns Θ_{a} in the three-phase winding (reaction or armature ampere-turns):

$$\hat{\Theta}_{\mathrm{a}} = \frac{3}{2}\,\frac{4}{\pi}\,\frac{N_{\mathrm{s}}\, f_{\mathrm{ws}}}{2p}\sqrt{2}\,I\,.$$

Together with the field ampere-turns Θ_{f} (of the cylindrical rotor), the resultant ampere-turns are:

$$\underline{\Theta}_{\mathrm{res}} = \underline{\Theta}_{\mathrm{f}} + \underline{\Theta}_{\mathrm{a}}\,.$$

The ampere-turns of the reaction are converted to the excitation side of the machine from the condition of equality of the reduced and original ampere-turns with the current ratio K_{I}.

$$\hat{\Theta}_{\mathrm{fa}} = \frac{4}{\pi} \frac{N_{\mathrm{f}} f_{\mathrm{wf}}}{2p} I_{\mathrm{fa}} = \hat{\Theta}_{\mathrm{a}} \quad \Rightarrow \quad I_{\mathrm{fa}} = \frac{3}{2} \frac{N_{\mathrm{s}} f_{\mathrm{ws}}}{N_{\mathrm{f}} f_{\mathrm{wf}}} \sqrt{2} I = K_{\mathrm{I}} \sqrt{2} I$$ and applies to

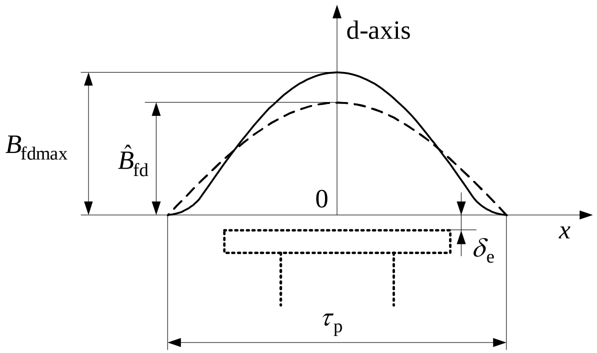
resultant excitation: $\underline{I}_{\mathrm{fres}} = \underline{I}_{\mathrm{f}} + \underline{I}_{\mathrm{fa}}$.

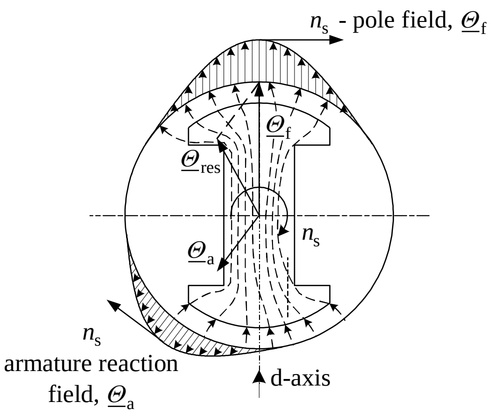
The effect of the armature reaction is captured in the magnetizing reactance of the machine $X_{\mathrm{m}} \propto 1/\delta_{\mathrm{e}}$ (p. 104), because the armature reaction acts through the air gap on the excitation side of the synchronous machine.

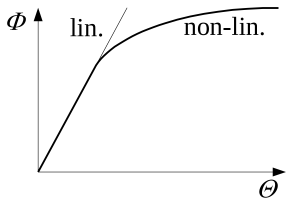
For $\Phi_{\mathrm{m}} = \Phi_{\mathrm{m0}} = \mathrm{const.}$ $(U = \mathrm{const.})$

we need to change the excitation that

will $\Theta_{\mathrm{res}} = \Theta_{\mathrm{f\,0}}$ or $\Theta_{\mathrm{f}} > \Theta_{\mathrm{f0}}$.







At no-load, the amplitude of the main flux $\hat{\Phi}_{\mathrm{m0}}$ lies in the d-axis of the pole, while, under load, the flux is displaced with respect to the d-axis of the pole. Its position then depends on the size and character of the load, i.e., the size and direction of the armature reaction.

The magnetizing characteristic is a non-linear curve in saturation and only in the initial (unsaturated) part is it linear and the fluxes can be summed geometrically.

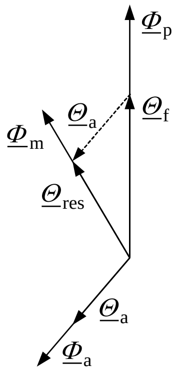
For a linear theory $(\mu_{\text{Fe}} = \text{const.})$, it is:

stator $\underline{\Theta}_{\text{a}} \rightarrow \underline{\Phi}_{\text{a}} = \hat{\Phi}_{\text{a}} e^{j\varphi_{\text{i}}}$ $\qquad \sqrt{2} I \rightarrow \hat{\Phi}_{\text{a}}$

rotor $\underline{\Theta}_{\text{f}} \rightarrow \underline{\Phi}_{\text{p}} = \hat{\Phi}_{\text{p}} e^{j\varphi_{0}}$ $\qquad I_{\text{f}} \rightarrow \hat{\Phi}_{\text{p}}$

result. $\underline{\Theta}_{\text{res}} \rightarrow \underline{\Phi}_{\text{m}} = \hat{\Phi}_{\text{m}} e^{j\varphi_{\text{res}}}$ $\qquad I_{\text{fres}} \rightarrow \hat{\Phi}_{\text{m}}$

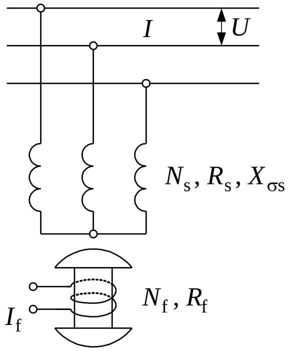
$$\underline{\Phi}_{\text{m}} = \hat{\Phi}_{\text{m}} e^{j\varphi_{\text{res}}} = \underline{\Phi}_{\text{p}} + \underline{\Phi}_{\text{a}} = \hat{\Phi}_{\text{p}} e^{j\varphi_{0}} + \hat{\Phi}_{\text{a}} e^{j\varphi_{\text{i}}}$$



φ_0 is the phase position of the pole flux for $\omega t = 0$, φ_{i} the phase position of the current and φ_{res} the phase position of the main flux.

Machine operation on a rigid grid (*U* = const., *f* = const.)

Three-phase machine (*m* = 3)



a) Induced voltages

$$\underline{E}_0 = -j\omega(N_{\text{s}} f_{\text{ws}}) \frac{\underline{\Phi}_{\text{m}}}{\sqrt{2}}$$

with angle $\varphi_0 = \varphi_{\text{res}} - \pi / 2$

and with flux components $\underline{\Phi}_{\text{p}}$ and $\underline{\Phi}_{\text{a}}$

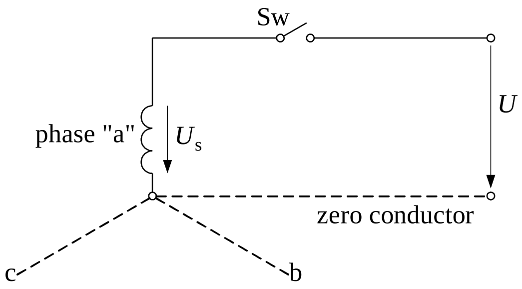
$$\underline{E}_0 = \underline{E}_{\text{p}} + \underline{E}_{\text{a}} .$$

For $R_{\text{s}} = 0$ and $X_{\sigma\text{s}} = 0$ it is valid $-\underline{E}_0 = \underline{U}_{\text{s}}$, or by the components of the pole wheel voltage U_{Ep} and the armature reaction voltage U_{a}.

$$\underline{U}_{\text{Ep}} = -\underline{E}_{\text{p}} \quad \text{and} \quad \underline{U}_{\text{a}} = -\underline{E}_{\text{a}}$$

$$\underline{U}_{\text{s}} = \underline{U}_{\text{Ep}} + \underline{U}_{\text{a}}, \qquad U_{\text{Ep}} \propto I_{\text{f}} / \sqrt{2} \quad \text{and} \quad U_{\text{a}} \propto I$$

b) Synchronization to the grid



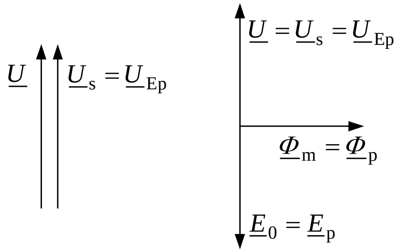
An excited synchronous machine (generator) with n_s revolutions is connected to the grid via the switch (Sw) if $\underline{U}_{Sw} = \underline{U} - \underline{U}_s = 0$. Therefore, $I = 0$, i.e., at $\underline{U}_s = \underline{U}$ and $\underline{U}_{Ep} = \underline{U}$.

An excited synchronous machine is therefore synchronized to the grid in an unloaded state.

The two conditions for synchronization are:

1) $U_s = U_{Ep} = U$ and $\varphi_{us} = \varphi_{up} = \varphi_u$,

2) $\omega_s = \omega$ (grid), $n = n_s = \dfrac{f_s}{p} = \dfrac{f}{p}$.



Taking on the load

1) Reactive load acceptance at $R_s = 0$ and $X_{\sigma s} = 0$

2) Working load acceptance at $R_s = 0$ and $X_{\sigma s} = 0$

It starts from synchronism $I = 0$. Under load, if the voltage drop is neglected $\underline{U}_s = \underline{U}$, the flux will be $\underline{\Phi}_m = \text{const.} = \underline{\Phi}_p + \underline{\Phi}_a$.

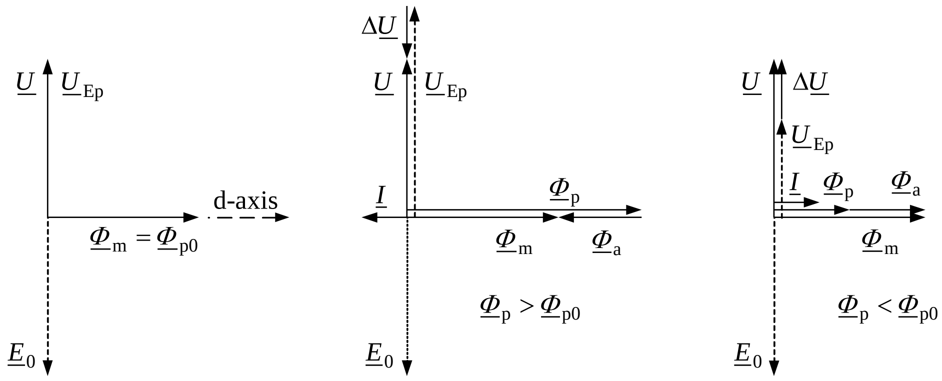
1) Reactive load acceptance

To change the reactive power input to a machine (generator or motor), change I_f with respect to the no-load value (I_{f0}) (assume here that $P_l = 0$, because is $R_s = 0$ and $R_f = 0$). The size of the excitation can be $I_f > I_{f0}$ or $I_f < I_{f0}$.

a) Synchronized $I = 0,\ I_{f0}$

b) Over-excitation $I > 0,\ I_f > I_{f0}$

c) Sub-excitation $I > 0,\ I_f < I_{f0}$



a) b) c)

b) I is capacitive for the grid, inductive for the machine.

c) I is inductive for the grid, capacitive for the machine.

2) Taking on the workload

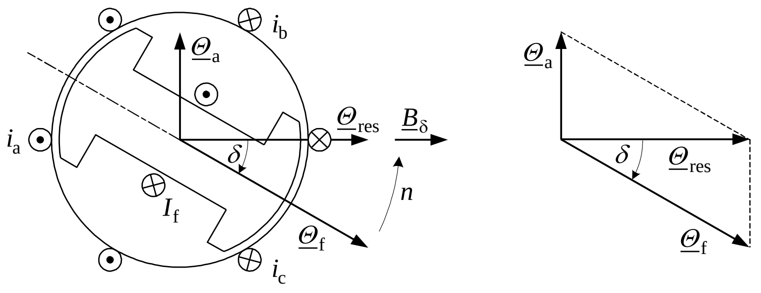
This is based on the assumption that $I_f = I_{f0}$, $\hat{\Phi}_{res} = \hat{\Phi}_{p0}$ and $\hat{U}_{Ep} = \hat{U}_{Ep0}$; force the generator to take up the power by connecting a mechanical load (the motor), or by connecting Z_L to the generator terminals. Let's leave it that $I_f = \text{const.}$

a) Synchronous motor

Mechanical load, $M_L = M_d < 0$, causing the rotor to lag behind the rotating magnetic field Φ_m. Therefore, the flux Φ_p lags behind the original position $\Phi_{p0} = \Phi_m$ and is $-\underline{E}_0 \neq \underline{U}$. The voltage difference $\underline{U} - \underline{U}_{Ep}$ drives a current $\underline{I}$ and this causes a reaction $\underline{\Theta}_a$ or flux $\underline{\Phi}_a$. The condition for the resultant magnetic field applies:

$$\underline{\Phi}_a + \underline{\Phi}_p = \underline{\Phi}_m = \frac{\sqrt{2}\,\underline{U}_s}{j\omega(N_s\, f_{ws})}.$$

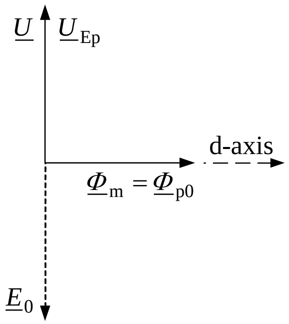
Space Figure and phasor diagram of motor *MMF* $(2p = 2)$



Phasor diagrams

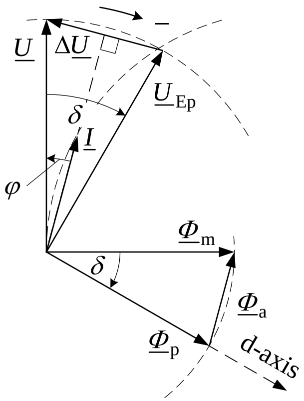
1) Synchronized machine

$I = 0, \quad n = n_s$



2) Loaded machine

$I > 0$, load angle $\delta < 0$



When a load is applied, a load angle δ occurs. The load angle δ is the angle between the rotor d-axis (flux Φ_p) and the resultant rotating field (flux Φ_m). The synchronous motor draws operating power from the grid $(I > 0,\ \delta < 0)$ and develops a torque +*M*:

$M = \dfrac{m}{\Omega_{ms}} U_s I \cos\varphi$. For number of phases $m = 3,\ \Omega_{ms} = \dfrac{\omega_s}{p},\ \omega_s = \omega$ (grid) will be:

$$M = \frac{3p}{\sqrt{2}} N_s\, f_{ws}\, \hat{\Phi}_m I \cos\varphi = c_M \hat{\Phi}_m I \cos\varphi \qquad \left(c_M = \frac{3p}{\sqrt{2}} N_s\, f_{ws}\right), \qquad (\varphi = \varphi_u - \varphi_i).$$

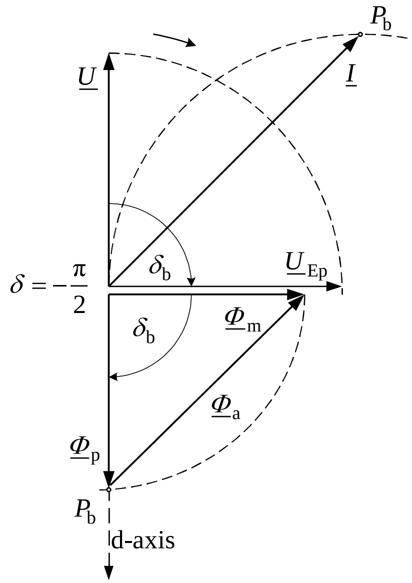
3) Stability limit $\delta_\mathrm{b} = -\frac{\pi}{2}$

In stationary operation:

$$M + M_\mathrm{L} = 0. \ \ (M_\mathrm{L} = M - \text{load})$$

The M of the motor will be maximum at $\delta_\mathrm{b} = -\frac{\pi}{2}$.

At $|\delta| > |\delta_\mathrm{b}|$, the machine falls out of synchronism.



b) Synchronous generator

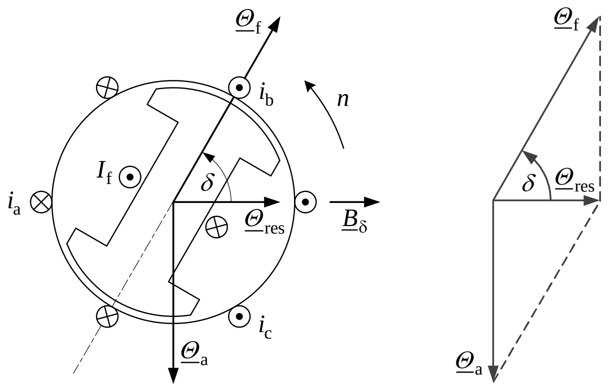
If the driving machine $M_\mathrm{d} > 0$ accelerates the rotor (pole wheel), $\underline{\Phi}_\mathrm{p}$ overtakes with respect to position $\underline{\Phi}_\mathrm{p0}$.

This makes a difference $\underline{U} - \underline{U}_\mathrm{Ep}$,

which caused the current $\underline{I} \rightarrow \underline{\Theta}_\mathrm{a} \rightarrow \underline{\Phi}_\mathrm{a}$.

The following condition applies:

$$\underline{\Phi}_\mathrm{a} + \underline{\Phi}_\mathrm{p} = \underline{\Phi}_\mathrm{m} = \frac{\sqrt{2}\underline{U}_\mathrm{s}}{\mathrm{j}\omega(N_\mathrm{s} f_\mathrm{ws})}.$$



The current increases by enough to keep it in stationary operation:

$$-M + M_\mathrm{d} = 0 \ .$$

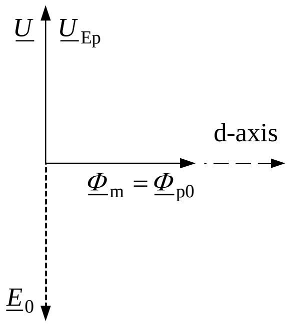
The current has a negative watt component.

$$-M = c_\mathrm{M}\, \Phi_\mathrm{m} I \cos\varphi$$

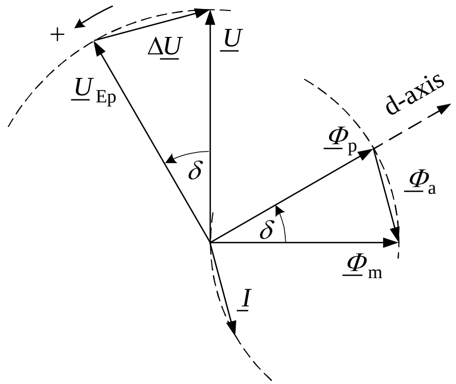
Supplying mechanical power to the shaft $\rightarrow$ delivered electrical power to the grid (load angle $\delta > 0$).

Phasor diagrams

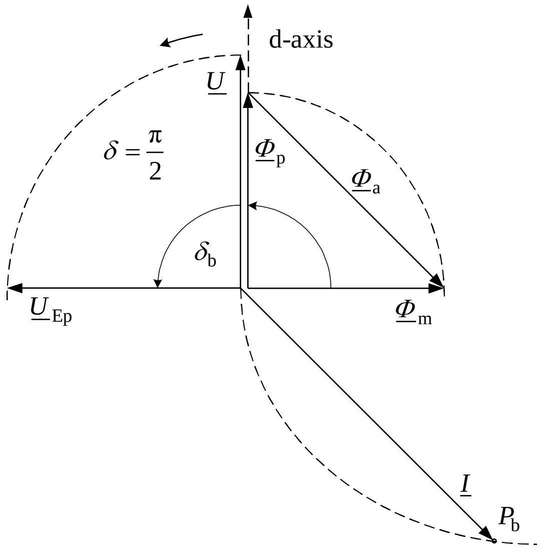
1) Synchronized machine



2) Loaded machine



3) Stability limit $\delta_{\mathrm{bd}} = +\frac{\pi}{2}$

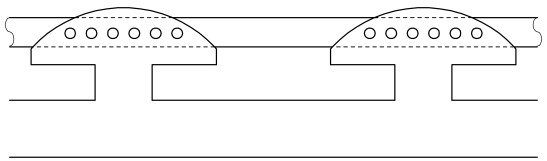


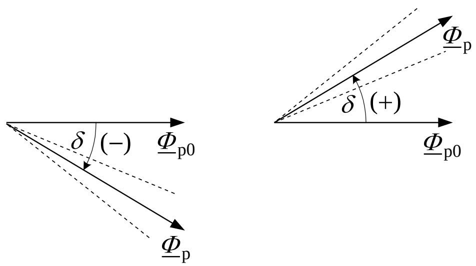
Before the load angle corresponding to a given load (angle δ) is established, the pole wheel, Φ_{p} and δ oscillate around position δ_0 − mechanical oscillation. That is why it needs to be throttled. This is achieved by a short-circuit cage at the rotor poles.

$n < n_{\mathrm{s}}$ induction motor

$n > n_{\mathrm{s}}$ induction generator

The cage is also used to start the motor.





Analytical treatment

For stationary operation, it is $n = n_\mathrm{s} = f \,/\, p \;\; (\mathrm{s}^{-1})$. The derivation is performed for non-salient poles. For the salient poles, we need to take into account $\Lambda_\mathrm{d} \neq \Lambda_\mathrm{q}$. The magnetizing characteristic of a synchronous machine is linear $(\mu_\mathrm{Fe} = \mathrm{const.})$.

Stator voltage equation: $\underline{U} = \underline{U}_\mathrm{s} = R_\mathrm{s}\underline{I} + \mathrm{j}\omega L_{\sigma\mathrm{s}}\underline{I} + \mathrm{j}\omega(N_\mathrm{s}\, f_\mathrm{ws})\dfrac{\underline{\Phi}_\mathrm{m}}{\sqrt{2}}$

a) $\underline{\Phi}_\mathrm{m}$ complex value of rotational (main) flux

Resulting ampere turns $(\underline{\Theta}_\mathrm{res} = \underline{\Theta}_\mathrm{f} + \underline{\Theta}_\mathrm{a})$ in the air gap $\hat{\Theta}_\mathrm{res} \to \hat{B}_\mathrm{res} \to \hat{\Phi}_\mathrm{m}$.

In stator coordinates, the main flux: $\phi_\mathrm{m} = \hat{\Phi}_\mathrm{m}\cos\left(\vartheta_\mathrm{s} - \omega t - \varphi_\mathrm{res}\right)$. In symmetry of the un-shortened coil $\left(x_\mathrm{s} = 0 \text{ or } \vartheta_\mathrm{s} = (x_\mathrm{s} / \tau_\mathrm{p})\pi = 0\right)$ the main flux is equal $\underline{\Phi}_\mathrm{m} = \hat{\Phi}_\mathrm{m}\mathrm{e}^{\mathrm{j}\varphi_\mathrm{res}}$.

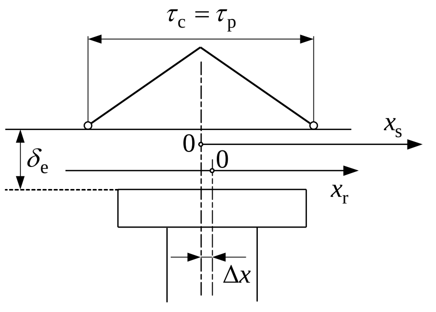
The main flux consists of the flux of the pole wheel and the flux of the armature reaction.

b) $\underline{\Phi}_\mathrm{p}$ pole wheel flux

MMF $\theta_\mathrm{f}(\vartheta_\mathrm{r}) = \hat{\Theta}_\mathrm{f}\cos\left(\vartheta_\mathrm{r}\right)$ causes a magnetic flux density in the air gap $b_\mathrm{f}(\vartheta_\mathrm{r}) = \hat{B}_\mathrm{f}\cos\vartheta_\mathrm{r}$, written in rotor coordinates $\left(\vartheta_\mathrm{r} = (x_\mathrm{r} / \tau_\mathrm{p})\pi\right)$

or pole wheel flux $\underline{\Phi}_\mathrm{p} = \hat{\Phi}_\mathrm{p}\mathrm{e}^{\mathrm{j}\vartheta_0}$

and this voltage $\underline{U}_\mathrm{Ep} = U_\mathrm{Ep}\mathrm{e}^{\mathrm{j}(\vartheta_0 + \pi/2)}$.



$\vartheta_0 = \dfrac{\Delta x_0}{\tau_\mathrm{p}}\pi$ is the rotor displacement of $\omega t = 0$ with respect to the armature winding symmetry (phase "a").

In stator coordinates, the excitation is $\theta_\mathrm{f} = \hat{\Theta}_\mathrm{f}\cos\left(\vartheta_\mathrm{s} - \omega t - \varphi_\mathrm{up} + \pi / 2\right)$.

The change in coordinates of the rotor rotating with the n_s revolutions is obtained from the following operations:

$$x_r = x_s - \Delta x = x_s - \upsilon t - \Delta x_0 .$$

Let's insert an expression for $\upsilon = r\Omega_{ms} = D\pi n = 2p\tau_p \frac{f}{p} = \frac{\tau_p}{\pi}\omega$ and derive:

$$x_r = x_s - \frac{\tau_p}{\pi}\omega t - \Delta x_0 \quad \Big| \times \frac{\pi}{\tau_p}, \qquad \left(\vartheta_0 = \varphi_{up} - \frac{\pi}{2} = \varphi_u + \delta - \frac{\pi}{2}\right)$$

$$\vartheta_r = \vartheta_s - \omega t - \varphi_{up} + \frac{\pi}{2}. \qquad \left(\vartheta_0 + \frac{\pi}{2} = \varphi_{up} = \varphi_u + \delta\right)$$

We usually take the voltage position $\varphi_u = 0$ $\left(\vartheta_0 + \frac{\pi}{2} = \delta\right)$, and it will be: $\underline{U}_{Ep} = U_{Ep} e^{j\delta}$.

The induced voltage is calculated using the well-known equation: $U_{Ep} = 4{,}44\, f\, N_s\, f_{ws}\, \hat{\Phi}_p$.

c) $\underline{\Phi}_a$ armature reaction flux

This flux is caused by load currents i_a, i_b and i_c. In phase "a", the moment value of the current is $i_a = \sqrt{2} I_a \cos(\omega t + \varphi_i)$ or complex $\underline{I}_a = \underline{I} = I\, e^{j\varphi_i}$.

In the stator coordinate system:

$\theta_a(x_s) = \hat{\Theta}_a \cos(\vartheta_s - \omega t - \varphi_i)$, which causes a magnetic flux density in the air gap

$$b_a = \hat{B}_a \cos(\vartheta_s - \omega t - \varphi_i).$$

In a rotary coordinate system, it will be:

$\theta_a(x_r) = \hat{\Theta}_a \cos\big(\vartheta_r + (\varphi + \delta - \pi/2)\big)$, if it is $\varphi = \varphi_u - \varphi_i$.

The position of the ampere-turns of the armature reaction θ_a is $f(\varphi, \delta)$, i.e., a function of the phase angle and the load angle of the machine.

Machine with cylindrical rotor

The excitation and reaction of the armature create a resultant excitation in the machine's air gap $(\underline{\Theta}_{\mathrm{res}} = \underline{\Theta}_{\mathrm{f}} + \underline{\Theta}_{\mathrm{a}})$. For $x_{\mathrm{s}} = 0$, i.e., in the coil symmetry of phase "a", there is a resultant excitation:

$$\theta_{\mathrm{res}} = \hat{\Theta}_{\mathrm{res}} \cos(\omega t + \varphi_{\mathrm{res}}) .$$

The resultant excitation creates a resultant magnetic flux density in the air gap:

$$b(x_{\mathrm{s}}) = b_{\mathrm{f}}(x_{\mathrm{s}}) + b_{\mathrm{a}}(x_{\mathrm{s}}) .$$

The amplitude of the magnetic flux density of the fundamental harmonic component in the air gap $(B_1 = B_\delta = B_{\mathrm{res}})$ is calculated from equation $\hat{B}_1 = \dfrac{\mu_0}{\delta_{\mathrm{e}}} \hat{\Theta}_{\mathrm{res}}$ and the peak-mean value of the pole flux $\hat{\Phi}_{\mathrm{m}} = \dfrac{2}{\pi} \hat{B}_1 \tau_{\mathrm{p}} l$. The complex flux value is: $\underline{\Phi}_{\mathrm{m}} = \dfrac{2}{\pi} \tau_{\mathrm{p}} l \dfrac{\mu_0}{\delta_{\mathrm{e}}} \underline{\Theta}_{\mathrm{res}}$.

The stator voltage equation will now be given by the equations for excitation amplitude $\hat{\Theta}_{\mathrm{f}}$ and armature reaction $\hat{\Theta}_{\mathrm{a}}$:

$$\underline{U}_{\mathrm{s}} = R_{\mathrm{s}} \underline{I} + \mathrm{j} X_{\sigma\mathrm{s}} \underline{I} + \mathrm{j}\omega (N_{\mathrm{s}} f_{\mathrm{ws}}) \frac{2}{\pi} \tau_{\mathrm{p}} l \frac{\mu_0}{\delta_{\mathrm{e}}} \frac{4}{\pi} \left(\frac{N_{\mathrm{f}} f_{\mathrm{wf}}}{2p} \frac{I_{\mathrm{f}}}{\sqrt{2}} \mathrm{e}^{\mathrm{j}(\varphi_{\mathrm{up}} - \pi/2)} + \frac{3}{2} \frac{N_{\mathrm{s}} f_{\mathrm{ws}}}{2p} \underline{I} \right) .$$

If the magnetizing (main) reactance is $X_{\mathrm{m}} = \omega L_{\mathrm{m}} = \omega \dfrac{2}{\pi} \tau_{\mathrm{p}} l \dfrac{\mu_0}{\delta_{\mathrm{e}}} \dfrac{3}{2} \dfrac{4}{\pi} \dfrac{(N_{\mathrm{s}} f_{\mathrm{ws}})^2}{2p}$ (i.e., the same as for the induction machine p. 71), will be:

$$\underline{U}_{\mathrm{s}} = R_{\mathrm{s}} \underline{I} + \mathrm{j} X_{\sigma\mathrm{s}} \underline{I} + \mathrm{j} X_{\mathrm{m}} \frac{2}{3} \frac{N_{\mathrm{f}} f_{\mathrm{wf}}}{N_{\mathrm{s}} f_{\mathrm{ws}}} \frac{I_{\mathrm{v}}}{\sqrt{2}} \mathrm{e}^{\mathrm{j}(\varphi_{\mathrm{up}} - \pi/2)} + \mathrm{j} X_{\mathrm{m}} \underline{I} \; .$$

Expression $X_{\mathrm{m}} \dfrac{2}{3} \dfrac{N_{\mathrm{f}} f_{\mathrm{wf}}}{N_{\mathrm{s}} f_{\mathrm{ws}}} = \dfrac{X_{\mathrm{m}}}{K_{\mathrm{I}}} = X_{\mathrm{avd}}$ is the mutual reactance between the stator and the rotor.

The stator voltage equation takes the following form when the order of the last two terms is reversed:

$$\underline{U}_{\mathrm{s}} = R_{\mathrm{s}}\underline{I} + \mathrm{j}X_{\sigma\mathrm{s}}\underline{I} + \mathrm{j}X_{\mathrm{m}}\underline{I} + X_{\mathrm{afd}}\frac{I_{\mathrm{f}}}{\sqrt{2}}\mathrm{e}^{\mathrm{j}\varphi_{\mathrm{up}}} .$$

Introduce synchronous reactance $X_{\mathrm{d}} = X_{\mathrm{m}} + X_{\sigma\mathrm{s}}$ ($X_{\mathrm{m}} = K_{\mathrm{I}}X_{\mathrm{afd}}$) and the voltage of the pole wheel $U_{\mathrm{Ep}} = X_{\mathrm{afd}}I_{\mathrm{f}}/\sqrt{2}$, to give the final form of the stator voltage equation:

$$\underline{U} = \underline{U}_{\mathrm{s}} = R_{\mathrm{s}}\underline{I} + \mathrm{j}X_{\mathrm{d}}\underline{I} + \underline{U}_{\mathrm{Ep}} .$$

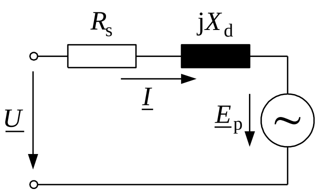
To this equation, we fit the equivalent circuit of a synchronous machine with a cylindrical rotor.

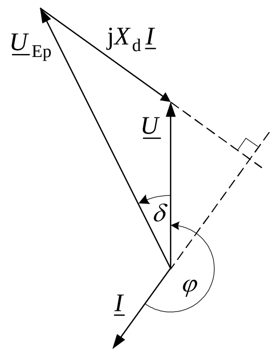
Pole wheel voltage $\hat{U}_{\mathrm{Ep}} = -\hat{E}_{\mathrm{p}} \propto I_{\mathrm{f}}$ and its position $\varphi_{\mathrm{up}} = \varphi_{\mathrm{u}} + \delta$.

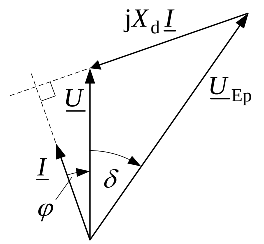
As it is $R_{\mathrm{s}} << X_{\mathrm{d}}$, take $R_{\mathrm{s}} = 0$ and get a phasor diagram of a generator or motor with a cylindrical rotor.

Synchronous reactance is usually given in the relative terms: $x_{\mathrm{d}} = \dfrac{X_{\mathrm{d}}}{Z_{\mathrm{N}}}$ and $Z_{\mathrm{N}} = \dfrac{U_{\mathrm{Nf}}}{I_{\mathrm{Nf}}}$.

$x_{\mathrm{d}} = 0{,}8 \div 2{,}5$







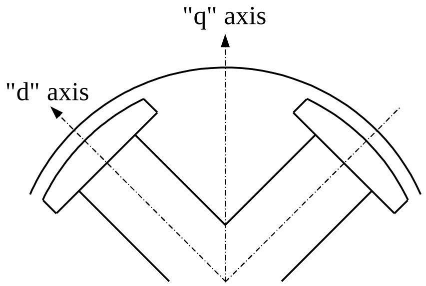
Machine with salient poles

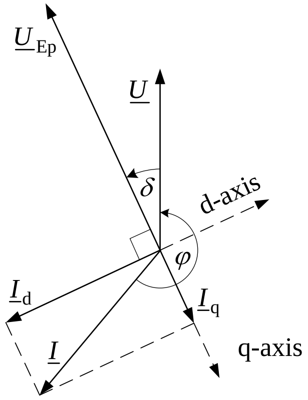
The longitudinal or d-axis and transverse or q-axis are distinguished. $X_{\mathrm{d}} \neq X_{\mathrm{q}}$ and $X_{\mathrm{d}} > X_{\mathrm{q}}$.

$X_{\mathrm{d}} = X_{\sigma\mathrm{s}} + X_{\mathrm{ad}}$ and $X_{\mathrm{ad}} = \beta_{\mathrm{ad}}X_{\mathrm{m}}$

$X_{\mathrm{q}} = X_{\sigma\mathrm{s}} + X_{\mathrm{aq}}$ and $X_{\mathrm{aq}} = \beta_{\mathrm{aq}}X_{\mathrm{m}}$

β_{ad} and β_{aq} are the pole shape factors in the d and q-axes.





The voltage equation applies:

$$\underline{U} = R_s \underline{I} + jX_d \underline{I}_d + jX_q \underline{I}_q + \underline{U}_{Ep}.$$

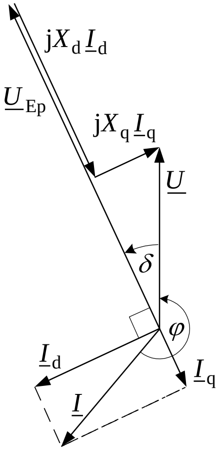
Machine phasor diagram with salient poles

Valid for $R_s = 0$.

To draw a phasor diagram, the following must be given:

$\underline{U}$, $\underline{U}_{Ep}$ and $\underline{I}$ or

$\underline{U}$, $\underline{I}$, X_d, X_q and load angle δ.



Permanent short circuit ($U = 0$)

For $R_s = 0$ will:

$$0 = jX_d \underline{I}_d + jX_q \underline{I}_q + \underline{U}_{Ep}.$$

The two components of the short-circuit current are:

$$jX_q \underline{I}_q = 0 \rightarrow I_q = 0 \text{ and}$$

$$\mathrm{j}X_\mathrm{d}\underline{I}_\mathrm{d}+\underline{U}_\mathrm{Ep}=0\rightarrow \underline{I}_\mathrm{k}=\underline{I}_\mathrm{dk}=-\frac{\underline{U}_\mathrm{Ep}}{\mathrm{j}X_\mathrm{d}}=\mathrm{j}\frac{\underline{U}_\mathrm{Ep}}{X_\mathrm{d}}.$$

The effective value of the permanent short-circuit current is: $I_\mathrm{k}=\dfrac{U_\mathrm{Ep}}{X_\mathrm{d}}$.

The value X_d is determined from the measurements:

a) of the no-load $U_\mathrm{Ep}=U$ no load voltage,

b) of the short-circuit I_k short circuit current.

$$X_\mathrm{d}=\frac{U_\mathrm{Ep}}{I_\mathrm{k}}=\frac{U}{I_\mathrm{k}}=\left(\frac{\text{no-load voltage}}{\text{short-circuit current}}\right)\text{ at the same excitation.}$$

Operating an unsaturated machine on a rigid grid

Derive the equations for a machine with salient poles.

$$\underline{I}=\underline{I}_\mathrm{d}+\underline{I}_\mathrm{q}, \qquad \underline{I}_\mathrm{d}=-\mathrm{j}I_\mathrm{d}\,\mathrm{e}^{\mathrm{j}\varphi_\mathrm{up}} \quad \text{and} \quad \underline{I}_\mathrm{q}=I_\mathrm{q}\mathrm{e}^{\mathrm{j}\varphi_\mathrm{up}}$$

$\varphi_\mathrm{up}=\varphi_\mathrm{u}+\delta$ for $R_\mathrm{s}=0$ and $\varphi_\mathrm{u}=0$ will be $\varphi_\mathrm{up}=\delta$ and obtained from the previous voltage equation:

$$U=X_\mathrm{d}I_\mathrm{d}\mathrm{e}^{\mathrm{j}\delta}+\mathrm{j}X_\mathrm{q}I_\mathrm{q}\mathrm{e}^{\mathrm{j}\delta}+U_\mathrm{Ep}\mathrm{e}^{\mathrm{j}\delta}.$$

Multiply the equation by $\mathrm{e}^{-\mathrm{j}\delta}$.

The real part of the equation:

$$U\cos\delta=X_\mathrm{d}I_\mathrm{d}+U_\mathrm{Ep}\rightarrow I_\mathrm{d}=\frac{U\cos\delta}{X_\mathrm{d}}-\frac{U_\mathrm{Ep}}{X_\mathrm{d}}$$

The imaginary part of the equation:

$$-U\sin\delta=X_\mathrm{q}I_\mathrm{q}\rightarrow I_\mathrm{q}=-\frac{U\sin\delta}{X_\mathrm{q}}$$

This is put in the initial current equation $\underline{I}=\underline{I}_\mathrm{d}+\underline{I}_\mathrm{q}$, and the solution for the armature current is:

$$\underline{I} = \mathrm{j}\frac{\underline{U}_{\mathrm{Ep}}}{X_{\mathrm{d}}} - \mathrm{j}\frac{\underline{U}}{X_{\mathrm{d}}}\mathrm{e}^{\mathrm{j}\delta}\cos\delta - \frac{\underline{U}}{X_{\mathrm{q}}}\mathrm{e}^{\mathrm{j}\delta}\sin\delta .$$

Taking into account $\cos\delta = \frac{1}{2}(\mathrm{e}^{\mathrm{j}\delta} + \mathrm{e}^{-\mathrm{j}\delta})$ and $\mathrm{j}\sin\delta = \frac{1}{2}(\mathrm{e}^{\mathrm{j}\delta} - \mathrm{e}^{-\mathrm{j}\delta})$ calculate the current:

$$\underline{I} = \mathrm{j}\frac{\underline{U}_{\mathrm{Ep}}}{X_{\mathrm{d}}} + \mathrm{j}\frac{\underline{U}}{2}\left(\frac{1}{X_{\mathrm{q}}} - \frac{1}{X_{\mathrm{d}}}\right)\mathrm{e}^{\mathrm{j}2\delta} - \mathrm{j}\frac{\underline{U}}{2}\left(\frac{1}{X_{\mathrm{d}}} + \frac{1}{X_{\mathrm{q}}}\right).$$

For a cylindrical rotor $X_{\mathrm{d}} = X_{\mathrm{q}}$: $\underline{I} = \mathrm{j}\frac{\underline{U}_{\mathrm{Ep}} - \underline{U}}{X_{\mathrm{d}}} = \frac{\underline{U} - \underline{U}_{\mathrm{Ep}}}{\mathrm{j}X_{\mathrm{d}}} = \frac{\Delta\underline{U}}{\mathrm{j}X_{\mathrm{d}}}$.

Torque:

$$M = \frac{P}{\Omega_{\mathrm{ms}}} \quad \to \quad P = \mathrm{Re}\left(\underline{U}\,\underline{I}^*\right) \text{ and for } m_{\mathrm{s}} = 3,\ \Omega_{\mathrm{ms}} = \frac{\omega}{p} \text{ we get:}$$

$$M = \frac{p\,P}{\omega} = \frac{3p}{\omega}\mathrm{Re}\left(\underline{U}\ \underline{I}^*\right) = \frac{3p}{\omega}U I_{\mathrm{w}} \quad \text{for } \underline{U} = U .$$

Using the equation for the current, we derive:

$$M = -\frac{3p}{\omega}\left(\frac{U U_{\mathrm{Ep}}}{X_{\mathrm{d}}}\sin\delta + \frac{U^2}{2}\left(\frac{1}{X_{\mathrm{q}}} - \frac{1}{X_{\mathrm{d}}}\right)\sin 2\delta\right).$$

synchronous torque — **reactance (reluctance) torque**

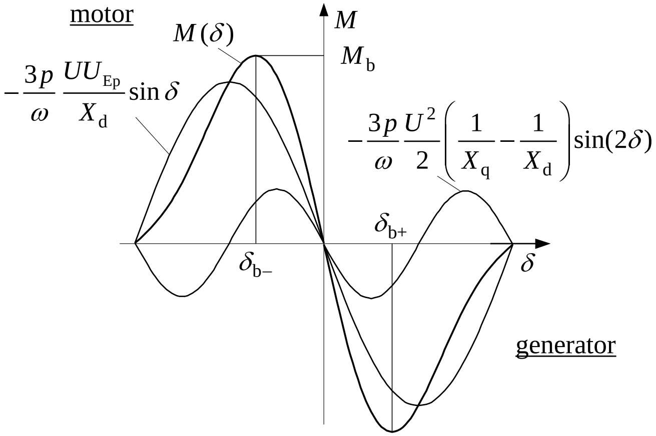
For a cylindrical (turbo) rotor $X_{\mathrm{d}} = X_{\mathrm{q}}$ and only synchronous torque is obtained:

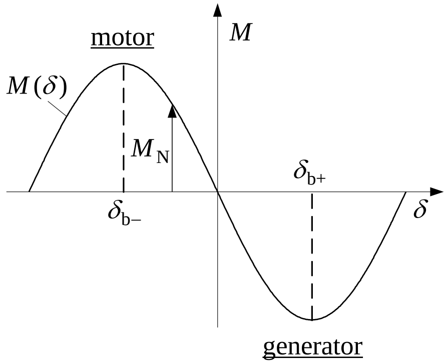
$$M = -\frac{3p}{\omega}\frac{U U_{\mathrm{Ep}}}{X_{\mathrm{d}}}\sin\delta, \qquad M = \mathrm{f}(I_{\mathrm{f}}, \delta).$$

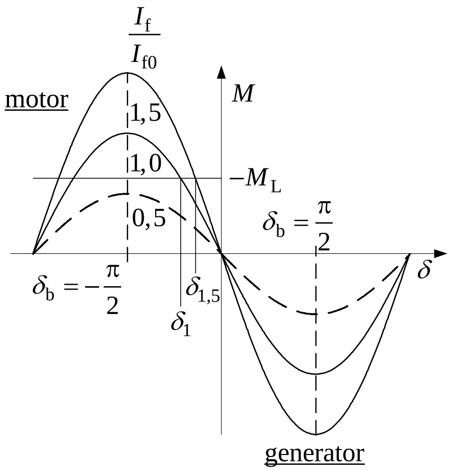
The maximum (breakdown) torque in both motor and generator operation is for a machine with salient poles at angle $|\delta_{\mathrm{b-}}| = |\delta_{\mathrm{b+}}| < \pi/2$ and for a cylindrical rotor $|\delta_{\mathrm{b-}}| = |\delta_{\mathrm{b+}}| = \pi/2$.

Torque Figures:

salient poles $|\delta_\mathrm{b}| < \frac{\pi}{2}$ cylindrical rotor $|\delta_\mathrm{b}| = \frac{\pi}{2}$







For a given torque, the smaller the excitation current, the greater the load angle δ. If the excitation current is too low (dashed curve), the motor falls out of synchronism or out of step. We are talking about static stability.

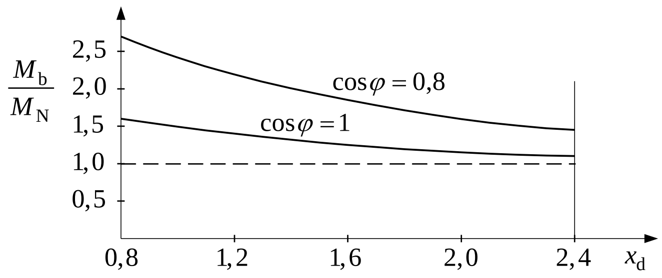
Static stability

The machine is able to take the load if it is loaded slowly. The limit of stability is determined by the breakdown torque. For a machine with a cylindrical rotor, the following applies:

$$|M_\mathrm{b}| = \frac{3p}{\omega}\,\frac{U_\mathrm{N}U_\mathrm{EpN}}{X_\mathrm{d}} \quad \text{and for} \quad M_\mathrm{N} = \frac{3p}{\omega} U_\mathrm{N} I_\mathrm{N} \cos\varphi$$

gives the ratio for the relative breakdown torque

$$\left|\frac{M_\mathrm{b}}{M_\mathrm{N}}\right| = \frac{U_\mathrm{EpN}}{X_\mathrm{d} I_\mathrm{N} \cos\varphi}\ .$$



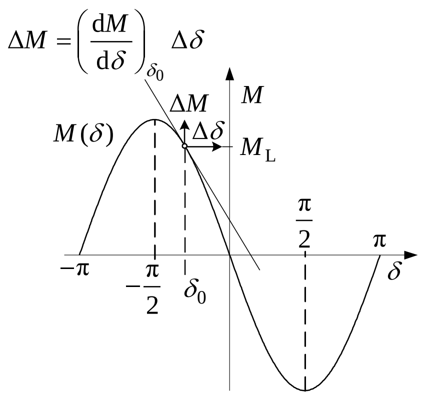
$U_\mathrm{Ep} = \mathrm{f}(X_\mathrm{d}, \cos\varphi)$, i.e., the voltage drops across X_d $(X_\mathrm{d} I)$ and $\cos\varphi$.

It is for various $\cos\varphi$: $\dfrac{M_\mathrm{b}}{M_\mathrm{N}} = \mathrm{f}(X_\mathrm{d})$.

For stable operation, e.g., for a motor loaded with M_L, the following applies:

$$M = -M_\mathrm{L} > 0 .$$

The increased load will be $\mathrm{d}M / \mathrm{d}\delta < 0$.



For small changes in angle δ, a linear relationship holds at the point of operation $M = \mathrm{f}(\delta)$.

$$\Delta M = \left(\frac{\mathrm{d}M}{\mathrm{d}\delta}\right)_{\delta_0} \Delta\delta ,$$

if the operating angle δ_0 is before the load change.

Varying the torque of a synchronous motor

A synchronous motor's torque is varied (controlled) by the armature voltage, or by excitation. In frequency inverter operation, the motor is varied simultaneously in voltage and frequency and thus in speed ($U = 0 \div U_N$, $f = 0 \div x f_N$, x is a multiple of f_N). If the torque is varied by excitation, only the load angle is varied. With permanent magnet excitation, the excitation cannot be corrected directly, but indirectly by varying the angle of the stator current.

Referring to the phasor diagram in Figure a) for a motor with a cylindrical rotor, we can introduce the internal angle γ, i.e., the angle between the pole wheel voltage U_{Ep} and the armature current I :

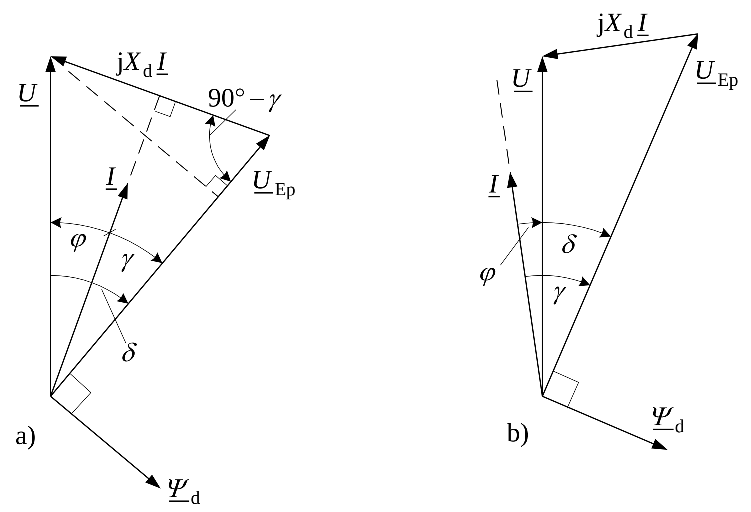
$$\gamma = \delta + \varphi = \varphi_{up} - \varphi_i .$$

The equation for the synchronous torque of the machine can be rearranged:

$$M = -\frac{3p}{\omega}\frac{UU_{Ep}}{X_d}\sin\delta = \frac{3p}{\omega} U_{Ep} I \cos\gamma = c_M \hat{\Phi}_m I \cos\gamma ,$$

where $|-U\sin\delta| = I X_d \sin(90-\gamma) = I X_d \cos\gamma$ and $U_{Ep} = \omega N_s f_{ws} \hat{\Phi}_m / \sqrt{2}$.

The torque is at its maximum when $\gamma = 0$, i.e., in the case of $\varphi = -\delta$. Then the armature current is in phase with the pole wheel voltage.



In the case of larger synchronous motors this is not very favorable, because the current lags behind the grid voltage (Figure a). In the case of larger motors, we usually want the current to overtake the grid voltage (reactive power generation). In this case, the angle is $\gamma = 40^\circ \div 60^\circ$ and the current overtakes the voltage (Figure b).

Stationary operation of a cylindrical rotor motor in a d-q model

The armature current can be decomposed into two components (d, q):

$$I_\mathrm{d} = I \sin\gamma \text{ and } I_\mathrm{q} = I \cos\gamma .$$

The angle γ is the space angle between the q-axis, where the voltage U_Ep is located, and the stator current. In conventional theory, γ is the (time) phase angle. The Figure shows a phasor diagram of a synchronous machine with a cylindrical rotor in d, q components.

The voltage of a pole wheel can also be expressed as a "voltage drop":

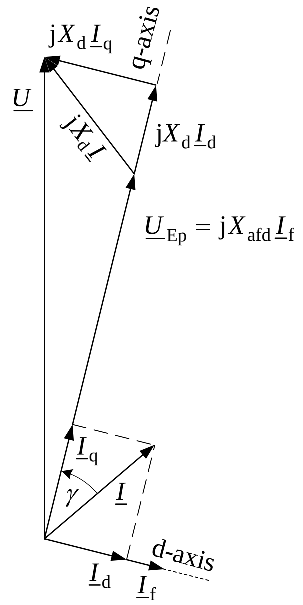
$$\underline{U}_\mathrm{Ep} = \mathrm{j}X_\mathrm{afd}\underline{I}_\mathrm{f} = \mathrm{j}\omega L_\mathrm{afd}\underline{I}_\mathrm{f} = \mathrm{j}\omega\frac{\underline{\Psi}_\mathrm{d}}{\sqrt{2}} .$$

$X_\mathrm{afd} = X_\mathrm{m} / K_\mathrm{I}$ is the mutual reactance between the armature winding with index "a" and the excitation winding in the d-axis, i.e., the magnetizing (main) reactance of the motor (stator) divided by the current ratio (p. 104); and $\hat{\Psi}_\mathrm{d} = L_\mathrm{afd} I_\mathrm{f}$ the value of the magnetic leakage in the d-axis of the rotor. Pole wheel voltage $U_\mathrm{Ep} = \mathrm{f}(f, I_\mathrm{f})$.

The equation for the synchronous torque can be transformed into the form:

$$M = \frac{3p}{\omega} U_\mathrm{Ep} I \cos\gamma = \frac{3p}{\omega} \frac{\omega L_\mathrm{afd} I_\mathrm{f}}{\sqrt{2}} I_\mathrm{q} = 3p\Psi_\mathrm{d} I_\mathrm{q} .$$

It can be seen that only the "q" component of the stator current, which is perpendicular to the excitation axis and in phase with the voltage U_Ep, contributes to the torque. In permanent magnet excitation ($\Psi_\mathrm{d} = \mathrm{const.}$), the torque varies only with the "q" component of the stator current, the "d" component of the stator current (in the axis of excitation) affects only the excitation (armature reaction), but also, indirectly, the magnitude of the torque. Therefore, if we want to vary the torque, we need to control the stator current in amplitude and phase. The stator current is increased by the voltage across the armature terminals, but this is only possible up to the nominal value. If the frequency f increases beyond the nominal value f_N, the input reactance of the machine also increases, and, therefore, at constant voltage, as the frequency increases, the armature current $I(I_q)$ decreases, and so does the torque. The situation is similar to the magnetic field weakening in an induction motor.

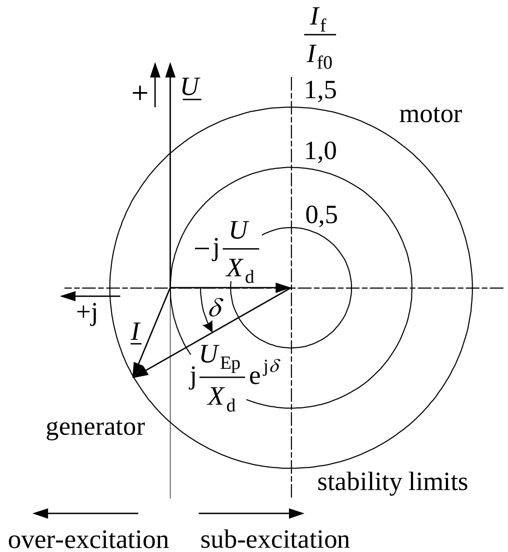


Current characteristics for a cylindrical rotor

For the armature current of a cylindrical rotor, the equation (p. 108) applies:

$$\underline{I} = -\mathrm{j}\frac{U}{X_d} + \mathrm{j}\frac{U_{Ep}\mathrm{e}^{\mathrm{j}\delta}}{X_d}.$$

The stator current characteristics $\underline{I}$ as a function of angle δ (circles in the complex plane) are obtained for $U_{Ep} = \text{const.}$ $(I_f = \text{const.})$ and load angle $\delta \neq \text{const.}$ The domain in which the motor or generator operates is the operating diagram. It covers only a part of the full-field domain.



Approximate treatment of a saturated machine

The superposition of magnetic field components in the air gap is no longer valid: $B_{\rm res} \neq B_{\rm f} + B_{\rm a}$. Synchronous reactance is important for stationary operation $(X_{\rm d} = X_{\sigma\rm s} + X_{\rm m})$, the main proportion of which is determined by the equivalent air gap $\delta_{\rm e}$ $(X_{\rm m} = {\rm f}(1/\delta_{\rm e}))$. The size of $\delta_{\rm e}$ is affected by saturation. In saturation, there is no longer any proportionality between the current and the magnetic field of the air gap. We will consider saturation for a machine operating at no-load and short-circuit roughly.

No-load and short-circuit

No-load: $I_{\rm f} = I_{\rm f0}$ and $I = 0$ or $I_{\rm f} = I_{\rm f\delta}$ and $I = 0$ for the air gap characteristic (AGC).

$$U = E_0 = \omega (N_{\rm s} f_{\rm ws}) \frac{\hat{\Phi}_{\rm m}}{\sqrt{2}}$$

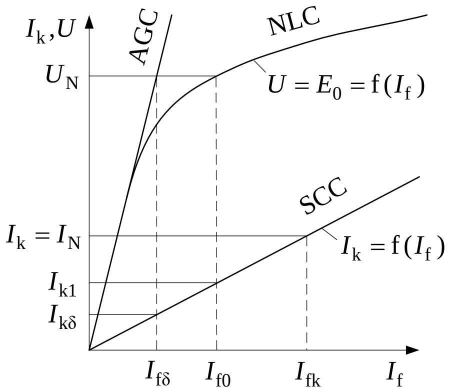
No-load characteristic (NLC): $U = E_0 = {\rm f}(I_{\rm f})$

measured at $n = {\rm const.}$

Short circuit: $E_{0\rm k} < E_0$

$\Phi_{\rm mk} < \Phi_{\rm m}$ due to armature reaction

Short circuit characteristic (SCC): $I_{\rm k} = {\rm f}(I_{\rm f})$



The excitation current $I_{\rm fk}$ is important. It is that excitation current at which, in a three-phase permanent short circuit, the armature current will be equal to the rated current $(I_{\rm k} = I_{\rm N})$.

The value of the unsaturated synchronous reactance is considered by IEC Standard 60034-4 to be the ratio of the no-load voltage $U = U_{\rm Ep}$ to the short-circuit current $I_{\rm k\delta}$:

$X_{\rm d} = \dfrac{U_{\rm N}}{I_{\rm k\delta}}$ with the same excitation $(I_{\rm f} = I_{\rm f\delta})$.

The relative value of the synchronous reactance is: $x_\mathrm{d} = \dfrac{X_\mathrm{d}}{Z_\mathrm{N}} = \dfrac{X_\mathrm{d} I_\mathrm{N}}{U_\mathrm{N}} = \dfrac{U_\mathrm{N} I_\mathrm{N}}{I_{\mathrm{k}\delta} U_\mathrm{N}} = \dfrac{I_\mathrm{N}}{I_{\mathrm{k}\delta}} = \dfrac{I_\mathrm{fk}}{I_{\mathrm{f}\delta}}$.

As a measure of armature reversibility, the IEC gives the ratio of the excitation current $I_{\mathrm{f}0}$ to I_fk, defined as the short-circuit ratio of the saturated machine $K_\mathrm{c} = \dfrac{I_\mathrm{f0}}{I_\mathrm{fk}}$.

A large ratio means that a larger short-circuit current I_k1 is required to compensate for the field of the excitation current $I_{\mathrm{f}0}$ (i.e., a small influence of the armature reaction).

a) Method for determining the excitation current of a saturated machine

For a given load at terminal voltage $\underline{U}$ and current $\underline{I}$, I_f and the load angle δ must be determined.

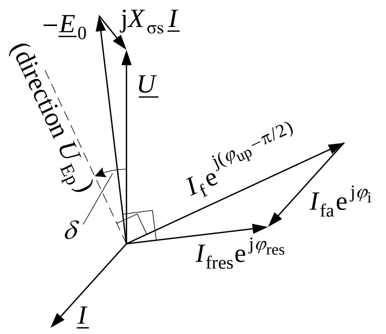
Mathematically, it is: $I_\mathrm{fres} \mathrm{e}^{\mathrm{j}\varphi_\mathrm{res}} = I_\mathrm{f} \mathrm{e}^{\mathrm{j}(\varphi_\mathrm{up} - \pi/2)} + I_\mathrm{fa} \mathrm{e}^{\mathrm{j}\varphi_\mathrm{i}}$. ($I_\mathrm{fa} = K_\mathrm{I} \sqrt{2} I$, p. 95)

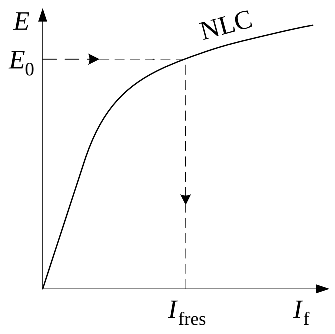
For $R_\mathrm{s} = 0$, we obtain the voltage equation:

$$\underline{U} = \mathrm{j}\omega L_{\sigma\mathrm{s}} \underline{I} + \mathrm{j}\omega (N_\mathrm{s} f_\mathrm{ws}) \frac{\underline{\Phi}_\mathrm{m}}{\sqrt{2}} \text{ and with } \underline{E}_0 = -\mathrm{j}\omega (N_\mathrm{s} f_\mathrm{ws}) \frac{\hat{\Phi}_\mathrm{m}}{\sqrt{2}} \mathrm{e}^{\mathrm{j}\varphi_\mathrm{rez}} .$$

Valid: $\underline{U} = \mathrm{j}X_{\sigma\mathrm{s}} \underline{I} - \underline{E}_0$ (bottom Figure) or: $\underline{U} - \mathrm{j}X_{\sigma\mathrm{s}} \underline{I} = -\underline{E}_0$ (Figure p. 116).

<u>Graphical method for determining the excitation current</u> for a cylindrical machine at a given load $\underline{I}$ and voltage $\underline{U}$



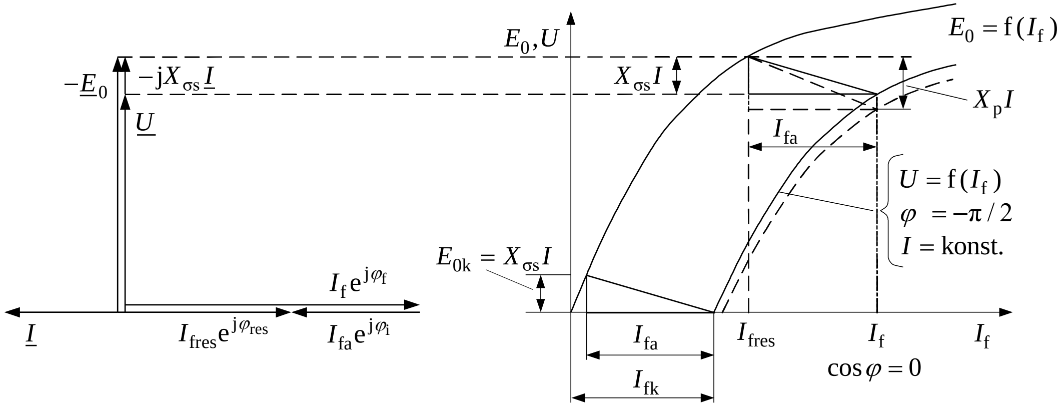


From the known no-load characteristic (NLC), we determine I_{fres} for E_0. We also need to know I_{fa} to obtain I_{f} and the load angle δ or direction U_{Ep}.

b) Internal characteristics

$U = \text{f}(I_{\text{f}})$ for $I = \text{const.}$, $\cos\varphi = 0$ $(\varphi = -\pi/2)$ (overexcited machine)

For $\varphi_{\text{i}} = \varphi = -\pi/2$ and $\varphi_{\text{up}} = \varphi_{\text{u}} + \delta = 0$ valid $I_{\text{fres}}\text{e}^{\text{j}\varphi_{\text{res}}} = (I_{\text{f}} - I_{\text{fa}})\text{e}^{-\text{j}\pi/2}$



A triangle with sides $X_{\sigma\text{s}}I$ and I_{fa} travels along the NLC for $I = \text{const.}$ and we get $U = \text{f}(I_{\text{f}})$. In a short circuit, $U = 0$ and $E_{0\text{k}} = X_{\sigma\text{s}}I$. As the excitation increases from I_{f0} to I_{fN}, the leakage between the poles in the rotor increases and the voltage across the terminals will be lower. We are talking about Potier reactance $X_{\text{p}} > X_{\sigma\text{s}}$, so the actual curve $U(I_{\text{f}})$ will be lower − the dashed line in the Figure.

c) American diagram (ASA)

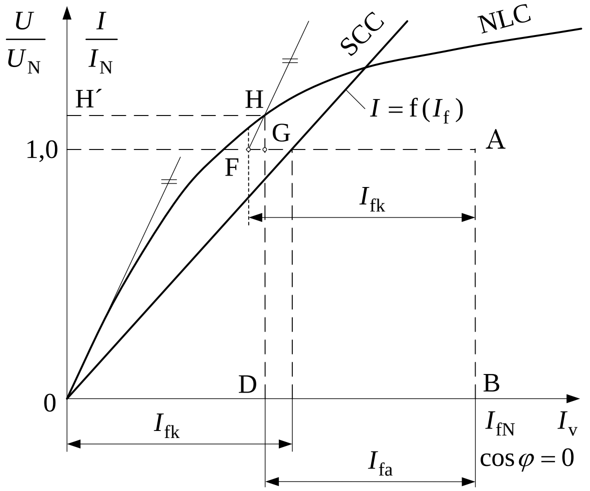
The ASA diagram is used to determine I_{fN} at U_{N}, I_{N} and at any $\cos\varphi$. For the design we need the no-load characteristic (NLC) and the short-circuit characteristic (SCC), I_{fN} at U_{N}, I_{N}, $\cos\varphi = 0$ (ind.) and X_{p} (Potier reactance).

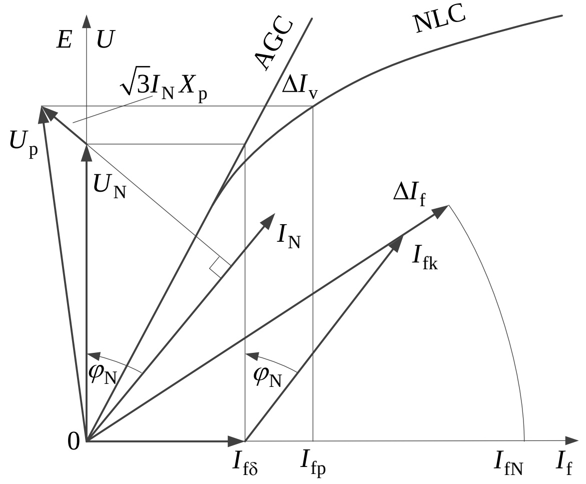
Determination of the Potier reactance:

From SCC $(I = \mathrm{f}(I_\mathrm{f})) \rightarrow I_\mathrm{fk}$ for $I = I_\mathrm{N}$. The distance $\overline{0\mathrm{B}}$ corresponds to I_fN, $\cos\varphi = 0$ (ind.). Subtract from point A $I_\mathrm{fk} \equiv \overline{\mathrm{AF}}$. At point F, draw a parallel to the tangent to the NLC, and obtain the relative Potier reactance $x_\mathrm{p} = \overline{\mathrm{HG}}$. NLC and SCC are drawn in relative units.

ASA diagram

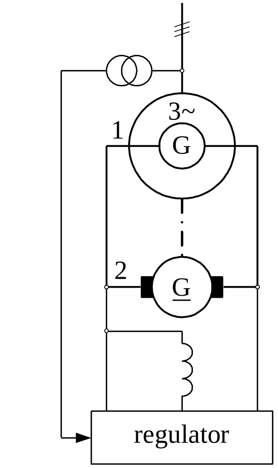
Apply perpendicularly to I_N from the end of U_N the value of the voltage drop $\Delta U_\mathrm{p} = \sqrt{3} I_\mathrm{N} X_\mathrm{p}$ (for the Y connection) and obtain the Potier voltage U_p. The projection of the voltage U_p onto the ordinate gives us, between the AGC and NLC characteristics, the value of the increase in excitation due to saturation ΔI_f. The ASA diagram for the total excitation current is: $\vec{I}_\mathrm{fN} = (\vec{I}_\mathrm{f\delta} + \vec{I}_\mathrm{fk}) + \Delta\vec{I}_\mathrm{f}$.

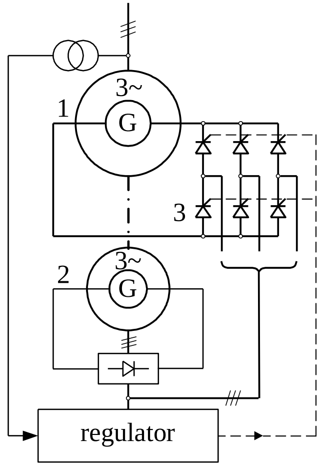




Excitation systems for synchronous generators

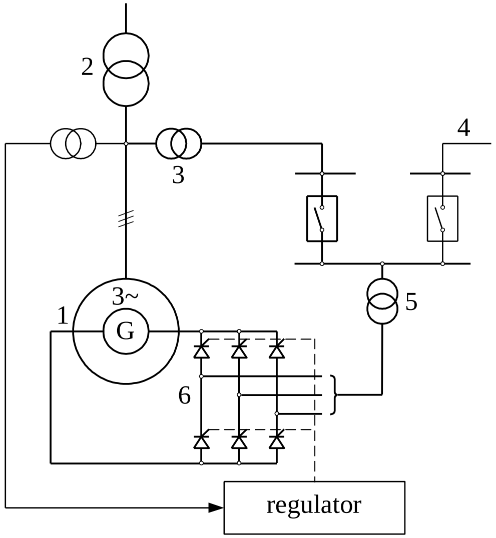
We use DC self-exciting generators to excite small synchronous generators. For larger synchronous machines, we use three-phase synchronous exciters or thyristor rectifiers (static converters). The excitation power is about 1 % of S_N.

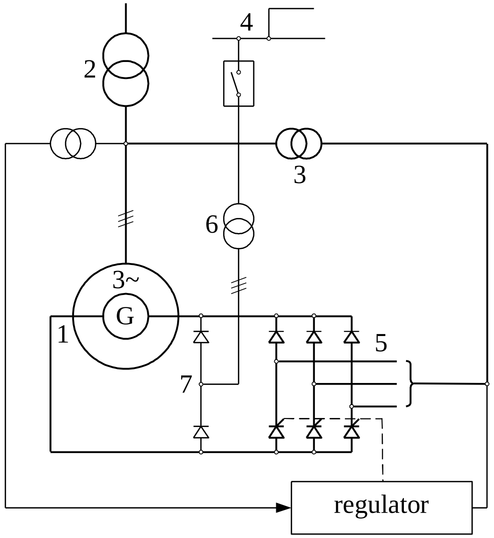




1 three-phase synchronous generator
2 DC excitation generator

2 three-phase synchronous exciter
3 fully controlled thyristor rectifier





2 transformer block
3 own-use transformer
4 foreign grid
5 excitation transformer
6 fully controlled thyristor rectifier

5 half-controlled thyristor rectifier
6 auxiliary transformer
7 auxiliary rectifier

Using permanent magnets for excitation

It is possible to choose between ceramic permanent magnets and permanent magnets made of metal alloys. Ceramic permanent magnets are, e.g., barium or strontium ferrite. Among the metal alloys, the most known are AlNiCo magnets and rare earth-based alloys, e.g., samarium with cobalt, or, more recently, neodymium-iron-boron.

Magnetic hysteresis

In immaterial space, the magnetic flux density will be

$$B = \mu_0 H . \qquad \left(\mu_0 = 4\pi \cdot 10^{-7}\ \mathrm{V \cdot s / (A \cdot m)}\right)$$

B varies in the ferromagnetic material.

$$B = \mu_0 H + B_\mathrm{i} ,$$

where B_i is the magnetic polarization (unit T or mT). The magnetic polarization is:

$$B_\mathrm{i} = \kappa \mu_0 H . \qquad \kappa \text{ is magnetic susceptibility.}$$

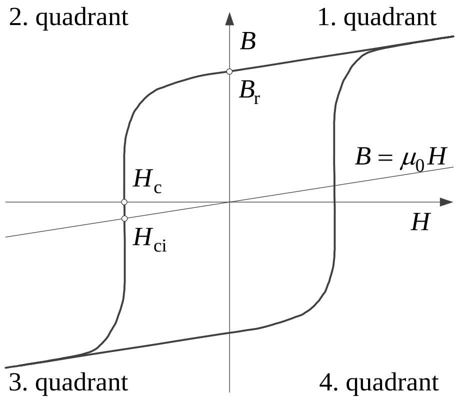
It follows: $B = \mu_0 (1+\kappa) H = \mu_0 \mu_\mathrm{r} H$, where the relative permeability is $\mu_\mathrm{r} = 1+\kappa$.

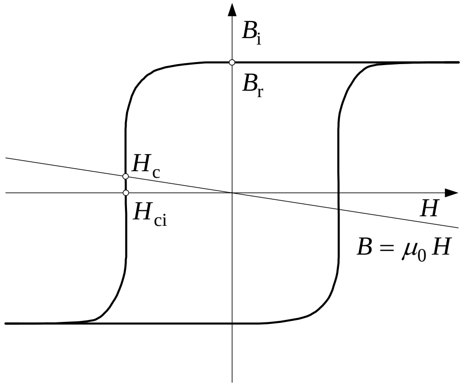
In saturation, $\mu_\mathrm{r} = 1$ will be and the course $B(H)$ will be a straight line. The relative permeability is given for a transformer or dynamo sheet. For permanent magnets the recoil permeability is given $\mu_\mathrm{p} \approx 1 \div 1{,}1$. (Permeabilis is Latin for permeable.)

The magnetic polarization B_i can also be expressed as a function of the magnetic field intensity H :

$$B_\mathrm{i} = B - \mu_0 H .$$

This is the intrinsic magnetic flux density, shown in the right Figure below. For good quality permanent magnets, the point of remanent magnetic flux density is the same in both Figures (B_r) . (Remanere is Latin for remnant.)





The characteristic points for both curves are:

$H = 0 \rightarrow B_r$ (remnant magnetic flux density)

$B = 0 \rightarrow H_c$ (coercive magnetic field intensity) (Coercere is Latin for restraint.)

$B_i = 0 \rightarrow H_{ci}$ (coercive magnetic field intensity polarization)

For excitation with permanent magnets, we use quadrant II (IV). This part of the hysteresis curve is called the demagnetization curve. For excitation with permanent magnets, it is essential that B_r and H_{ci} are as large as possible. What matters is the stored magnetic energy, i.e., the product of (BH).

$(BH) = \text{max.}$ the value at the knee of the demagnetization curve.

Working line

The magnet is embedded in a magnetic circuit (soft iron), which usually has an air gap.
Ignoring leakage, it is:

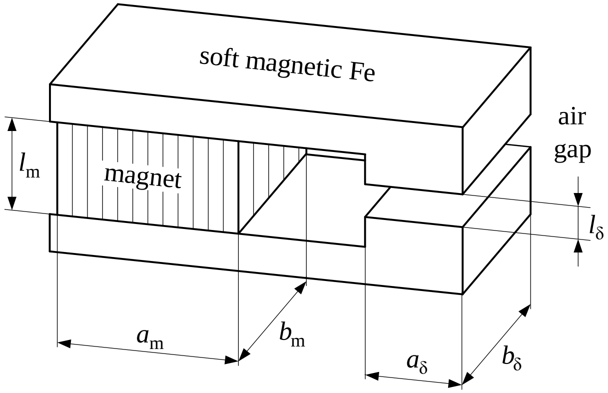
$$\Phi = B_m A_m = B_\delta A_\delta .$$

Ignoring the *MMF* drop in iron, the following is true:

$$\Theta = -H_m l_m = H_\delta l_\delta$$

and follows

$$B_\delta = \mu_0 H_\delta = -\mu_0 H_m \frac{l_m}{l_\delta}.$$



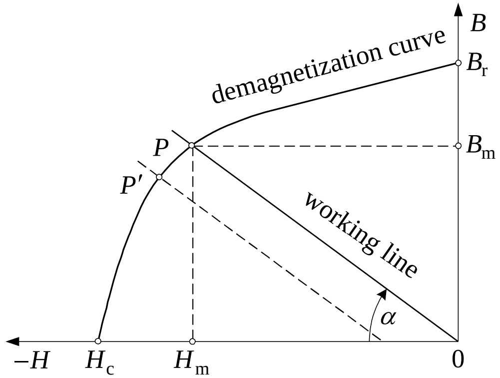
The magnet's operating point will now be:

$$B_m = B_\delta \frac{A_\delta}{A_m} = -\mu_0 H_m \frac{A_\delta}{A_m} \frac{l_m}{l_\delta}$$

and angle of the working line

$$\alpha = \mathrm{arctg}\left(\frac{-B_m}{H_m}\right) = \mathrm{arctg}\left(\mu_0 \frac{A_\delta}{A_m} \frac{l_m}{l_\delta}\right).$$

The reaction of the armature moves the working line from point P to point P′. This must not be below the knee of the curve, otherwise the magnet is demagnetized (weakened) irreversibly.



Permanent magnet synchronous motors

They are characterized by their high economic importance, due to their higher efficiency than induction motors. Synchronous motors of special designs are used in household and technical applications. From an engineering point of view, the synchronous torque is of particular importance.

Stators are built differently:

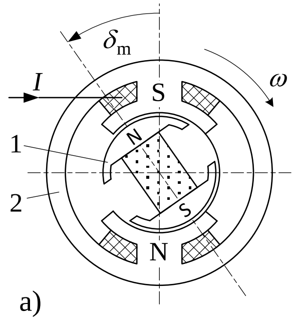
- an annular coil with claw-shaped poles,
- side coils with clawed or toothed poles,

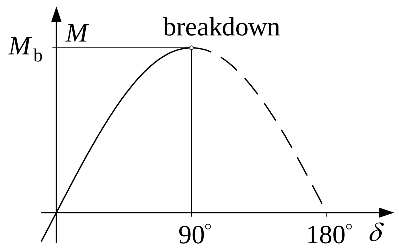
- individual coil poles,
- a normal three-phase stator − identical to the stator of an induction motor (larger units).

Permanent magnet motors are considered to have high synchronous torque, but they do not start on their own.

Synchronous and reluctance torque

We will first establish the relationship between the synchronous torque and the magnet data, assuming that the stator (2) in Figure a) generates excitation (Θ_e with a sinusoidally distributed magnetic field).





When moved by a (mechanical) angle δ_m from the longitudinal direction, the electrical excitation Θ_{el} (sinusoidal in shape) will cause a change in the *MMF* in the permanent magnet on the rotor (1) in Figure a). This results in a change in energy ΔW :

$$\Delta W = \frac{1}{2} \Phi_\delta \, \Theta_{el} \cos(p\delta_m) .$$

In the equation there is: $\Phi_\delta = B_\delta A_\delta$, $\Theta_{el} = I\,N$ and p is the number of pole pairs.

From what is written follows:

$$M = \left| \frac{dW}{d\delta} \right| = p \frac{1}{2} \Phi_\delta \, \Theta_{el} \sin\delta$$

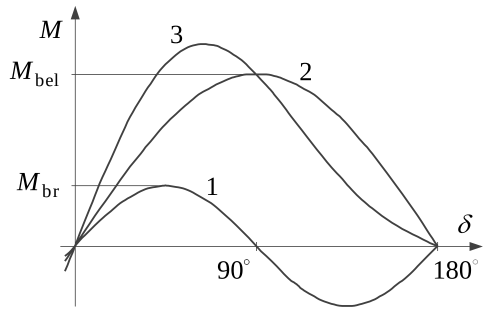
and the maximum, i.e., the breakdown torque at an angle of $\delta = p\,\delta_m = 90^\circ$ (Figure b)

$$M_b = p\frac{1}{2}\Phi_\delta\,\Theta_{el}.$$

For the stator, we use ordinary laminated soft iron. The magnet in the iron poles builds up an opposing magnetic field and so the reluctance (adhesive) torque opposes the rotor twist.

The Figure on the right marks the reluctance torque M_r with 1, the synchronous (electrical) torque M_{el} resulting from the electrical supply to the stator with 2 and the resultant torque with 3.

For the motor to start, it must be $M_{el} > M_r$. It is generally accepted that $M_{el} \approx 3M_r$.



The equation for the breakdown torque can also be written in another way:

$$M_b = p\,\frac{1}{2}\,\Phi_\delta\,\Theta_{el} = p\,\frac{1}{2}\,\Phi_\delta\,\Theta_p\,\frac{\Theta_{el}}{\Theta_p}.$$

In the equation, the flux of the air gap Φ_δ is equal to the flux of the permanent magnet $(\Phi_\delta \approx \Phi_p)$, if we ignore the dissipation in the rotor. The excitation of the permanent magnet is Θ_p. For $\Phi_p = B_m\,A_m$, $\Theta_p = H_m\,l_m$, the specific energy $w_p = B_m\,H_m/2$ and the magnet volume $V_m = A_m\,l_m$, the breakdown torque will be given by the energy:

$$M_b = p\,w_p V_m\left(\Theta_{el}/\Theta_p\right)$$

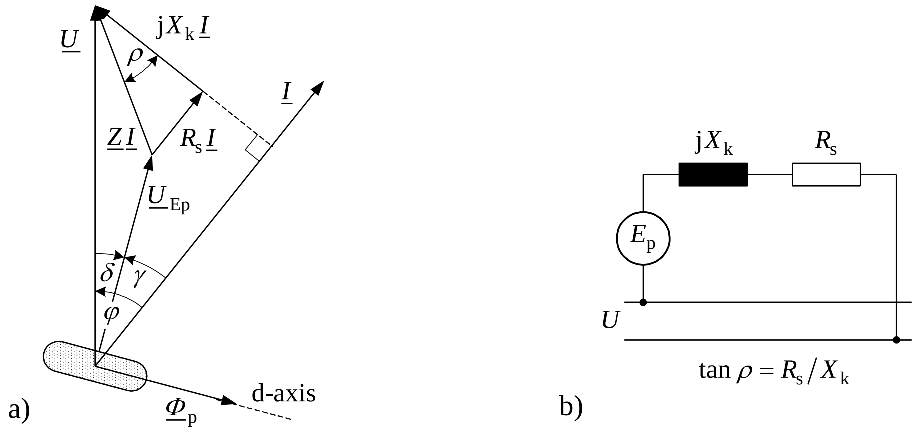
proportional to the magnet volume and dependent on the magnet quality, i.e., the stored energy of the magnet. For this reason, synchronous servomotors today use mainly rare earth magnets, e.g., neodymium-iron-boron $(B_r = 1 \div 1{,}21\ \mathrm{T}$ and $H_{ci} = 690 \div 920\ \mathrm{kA/m})$.

Three-phase synchronous motors

Given a three-phase winding on the stator, the magnet rotates at a mechanical (synchronous) angular velocity $\Omega_{ms} = \omega / p$ and induces a pole wheel voltage

$$E_p = \omega N_s f_{ws} \hat{\Phi}_\delta / \sqrt{2} .$$

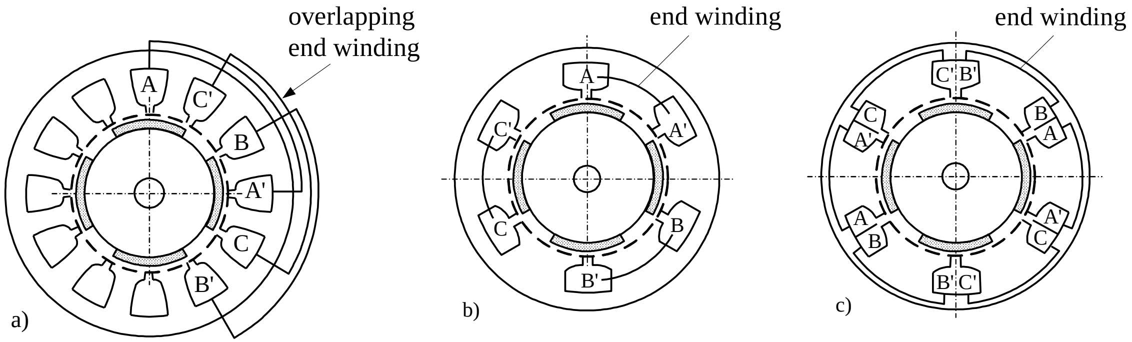
For small motors, the ohmic resistance of the stator winding must not be neglected, which is shown in the phasor diagram (Figure a), i.e., in the Kappa triangle of voltage drops and in the equivalent circuit (Figure b).



For the synchronous (short-circuit) reactance in Figure b), an approximate equation applies: $X_k \approx E_p / I_k$.

Permanent magnet synchronous motors with salient poles on the stator

The permanent magnet synchronous motors are abbreviated in the English literature as PMSMs (permanent magnet synchronous motors). The peculiarity of this type of motors is that the armature winding is not distributed in the stator slots (Figure a) for a 4-pole PMSM with distributed winding for the $Q_s = 12$, $(q_s = 1)$, but concentrically mounted on the teeth of the stator or, according to German literature, wound on the salient poles of the stator. It is considered that the adhesive torque can be reduced at zero flux if such motors have a number of slots on the stator $Q_s \neq 2p$. The number of slots on the stator is: $Q_s = 2p + 2k$ and the factor $k = \pm 0,5, \ \pm 1, \ \pm 2...$ For all motors with stator pole windings, the number of slots per pole and phase shall be $q_s \leq 0,5$. For example, in Figures b) and c): $Q_s = 6$ and $2p = 4$ $(k = 1)$. We have one coil per phase for a single-layer winding, and two coils per phase for a two-layer (two-phase) winding. In general, there are many combinations for Q_s and $2p$: 3/2, 3/4, 6/4, 6/8, 9/8, 9/10, 9/12, 12/10, 12/14, 24/16, 24/22, ... 36/42, etc. The number of periods of adhesive torque at zero flux is equal to the product between Q_s / p and $2p$.

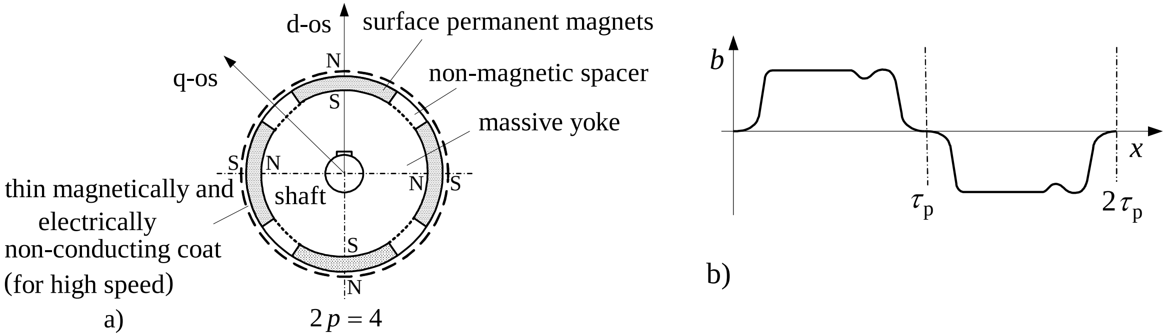


PMSMs motors have $2p$ poles. Thus, the stator winding for $Q_s \neq 2p$ has a relatively large winding factor for the $2p$ period. In the case of $Q_s = 6$ and $2p = 4$, this factor for a concentrated winding with respect to the width of the coil is equal to $\sin(120^\circ / 2) = \sqrt{3}/2 = 0,866$. With such a winding, a virtually sinusoidal induced voltage can be achieved in the stator phases.

These motors do not have a cage in the rotor, and cannot be connected directly to the AC grid. They depend on a frequency-variable power supply with power electronics. Even if they have a cage, stator excitation has very strong harmonic components due to $Q_s \neq 2p$.

The field shape for the example of a motor with permanent magnets on the rotor surface (Figure a) below with the "d" and "q" axes marked) and windings on the salient stator poles for $q_s = 0,5$ is shown in Figure b).

The arc of permanent magnets is usually equal to the arc of the stator slot width $\tau_m = \tau_s = D\pi / Q_s$ to reduce the adhesive torque.



Calculating torque

Assume that the magnetic coupling of the permanent magnets on the stator, with two coils per phase, varies sinusoidally. The maximum magnetic linkage for two coils with N_t turns will be as follows:

$$\hat{\Psi}_p = 2N_t \times B_{\delta e} A_m ,$$

where $B_{\delta e}$ is the magnetic flux density in the equivalent air gap (δ_e) and the area A_m of the magnet in the direction of the air gap. The maximum magnetic linkage will occur in the d-axis, i.e., the symmetry of the permanent magnets, while the q-axis is the symmetry between the magnets (Figure a).

For a sinusoidal arrangement of magnetic linkage: $\Psi_p(\vartheta_r) = \hat{\Psi}_p \sin \vartheta_r$.

ϑ_r is the electrical angle and is *p*-times the mechanical angle ($\vartheta_r = p\vartheta_{rm}$).

$B_{\delta e}$ is given by the equation:

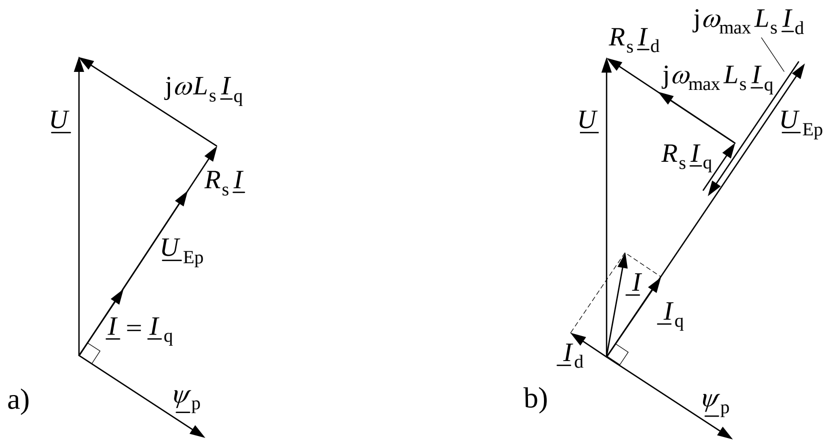
$$B_{\delta e} \approx \frac{B_r}{1+k_r} \frac{l_m}{l_m + \delta_e} .$$

The factor $k_r = 0{,}1 \div 0{,}2$ and takes into account the edge flux. l_m is the thickness of the magnet.

The torque in the case where the armature current is in phase with the induced pole wheel voltage $U_{Ep} = \omega \hat{\Psi}_p / \sqrt{2}$, i.e., in the case for $I_d = 0$, where we have only the "q" component of the current $I_q = I$, is calculated by the well-known equation:

$$M = 3p \frac{\hat{\Psi}_p}{\sqrt{2}} I \; .$$

In the case where the current is in phase with the induced voltage ($I_d = 0$), the phasor diagram in Figure a) is valid, in which we consider the synchronous inductance L_s and the stator resistance R_s . In Figure b) $I_d < 0$, the and reaction reduces the excitation.

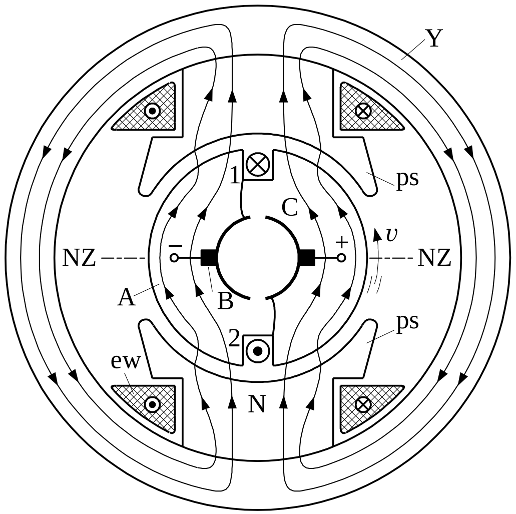


The synchronous inductance in this case is not only equal to the sum of the magnetizing L_m and leakage inductances $L_{\sigma s}$ (i.e., L_d for classical machines), but also to the mutual inductance between the adjacent phases $L_{12} \approx L_m / 3$. In the case where the component of the current I_d is in the opposite direction to the permanent magnet excitation (for control at higher rotor speeds $\omega_{max} > \omega_N$), the phasor diagram in Figure b) applies.

COMMUTATOR MACHINE

Description of construction

A commutating machine is an electrical machine with a commutator in the secondary. The commutator assembly (collector − brushes) can be replaced by electronics. A DC or AC machine can be distinguished depending on the voltage applied. The main components are: a stator and rotor with a commutator.



Sketch of a two-pole DC machine

The stator consists of:

Y – a massive stator yoke,

ps – a pole with a pole shoe,

ew – an excitation winding.

The rotor consists of:

A – armature electrical steel (dynamo sheet metal)

(1, 2 – armature winding),

C – commutator (collector),

B – stationary brushes.

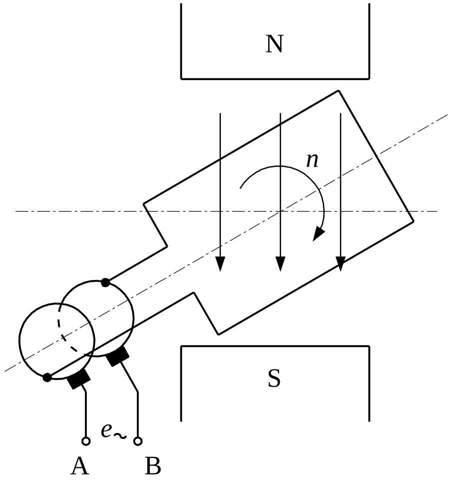
Operation mode

a) Generator

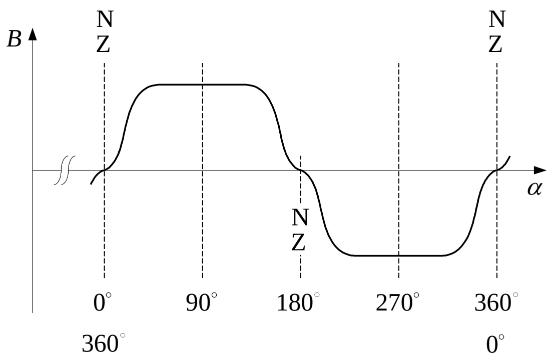
The rotor coil is rotated in a DC magnetic field. A voltage is induced in each side of the coil − the conductor (bar) according to equation:

$$e_{\mathrm{b}} = vBl\,.$$

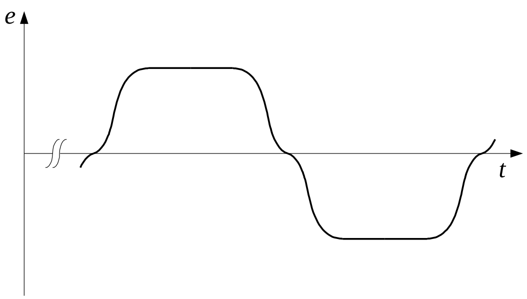
This is identical in shape to the magnetic field.



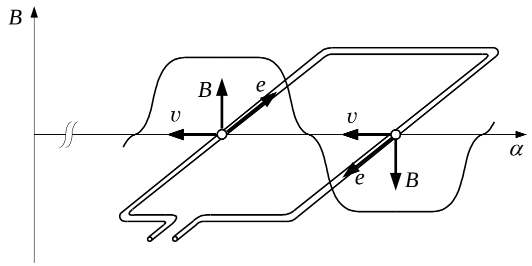
Coil connected to two slip rings



Magnetic field distribution of the salient poles



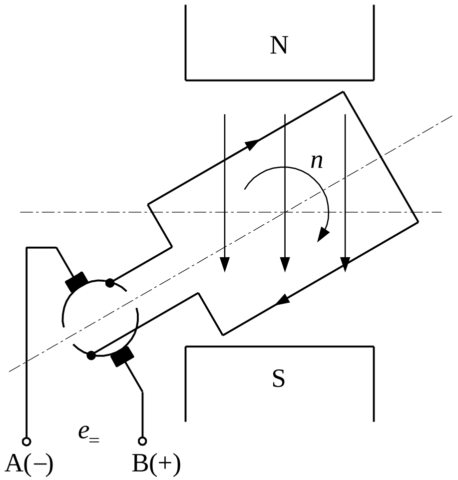
Induced conductor voltage

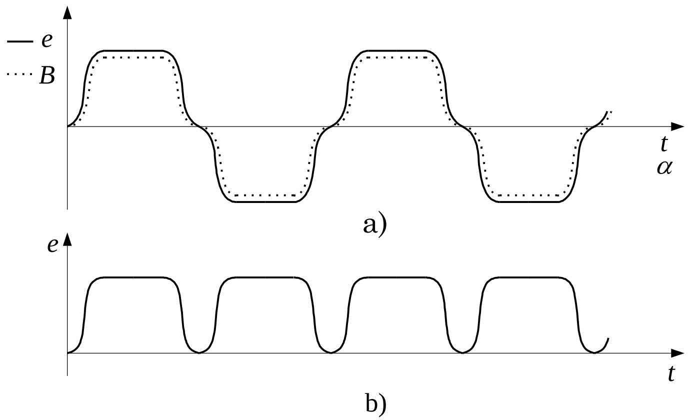


Induced coil voltage

The induced voltages in the two sides of the coil are oriented oppositely. The geometric sum is twice. The voltage is alternating in time. Such a voltage is also obtained on the slip rings.

Instead of two slip rings, we take just one slip ring, which we cut, i.e., two lamellas. The beginning of the coil is connected to one lamella and the end to the other.

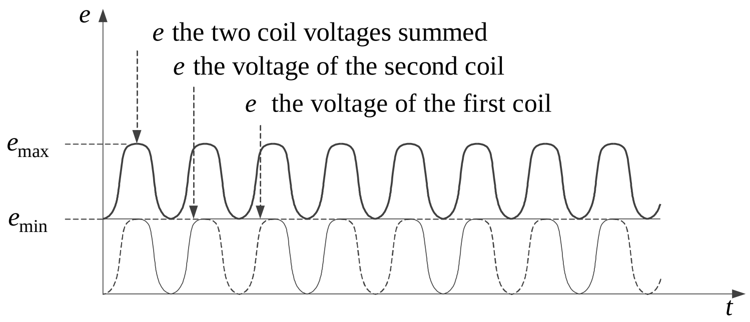




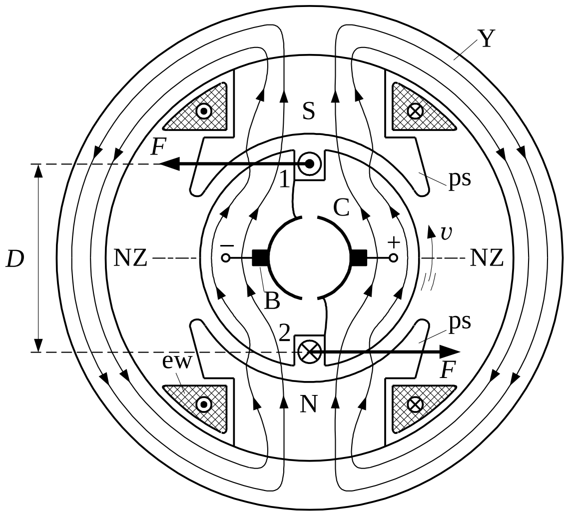
a) Space and temporal Figure of an induced coil voltage

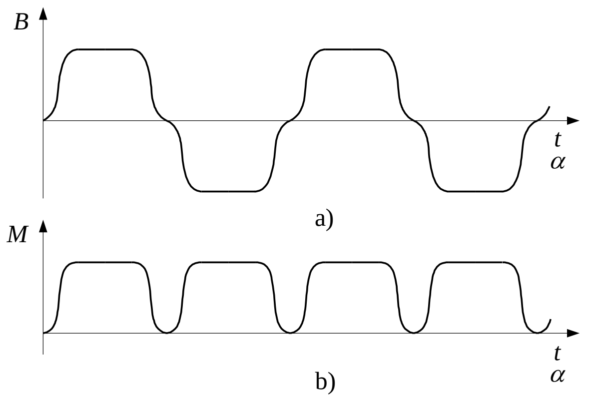
b) Voltage on the brushes

A collector (commutator) is a mechanical rectifier. The induced voltage will always be in the same direction (−) at terminal A, always (+) at terminal B. A single coil has a large voltage ripple. Usually we have at least two (Figure) or more coils.



b) Motor





a) Magnetic flux density

b) Torque

The rotor is connected to a voltage and a current I flow in it. A force is acting on a conductor in a magnetic field of magnetic flux density B: $F = IBl$ and a pair of forces on both sides of the coil, creating a torque:

$$M = F\frac{D}{2} + F\frac{D}{2} = FD\,.$$

Each motor can act as a generator, and vice versa. In a generator, there is also a force opposing the force (torque) of the driving machine.

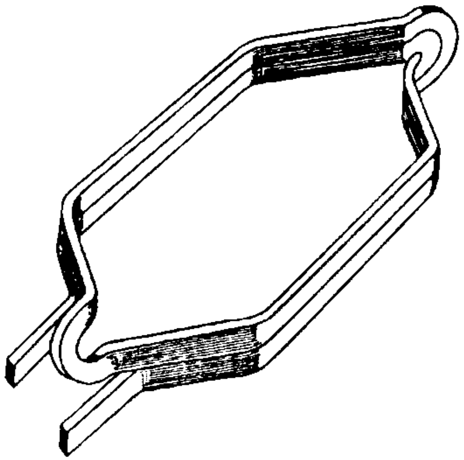
Voltage magnitudes:

generator $U \leq E$

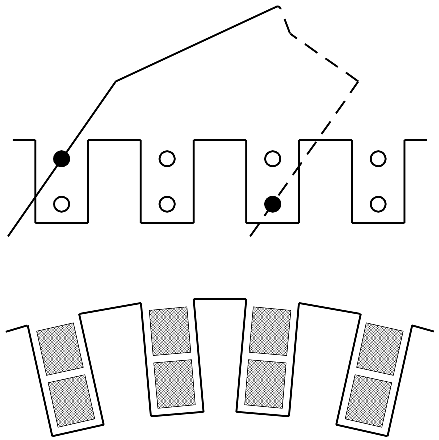
motor $E \leq U$

Commutator machine windings

These are usually two-layer windings. The number of coils is equal to the number of slots.



Coil design



Arrangement of the coil in the slots

The start and end are connected to adjacent lamellas. Therefore, the number of lamellas K is equal to the number of coils or the number of slots Q:

$$K = Q\,.$$

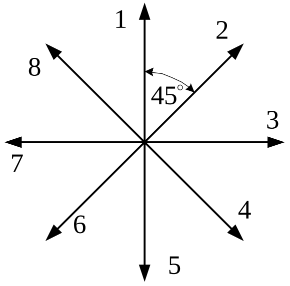
Vector star and induced voltage polygon

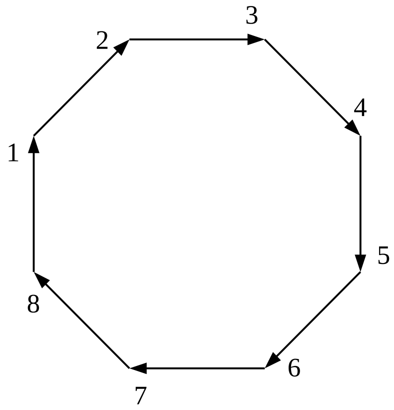
The induced voltage of the individual slots is represented by a vector (phasor). All the conductors in a slot have an induced voltage of the same direction and magnitude. The electrical angle in degrees between the slots will be:

$$\alpha = p\frac{360^\circ}{Q} = p\alpha_\mathrm{Q}\,,$$

where the mechanical angle between the slots is $\alpha_\mathrm{Q} = \dfrac{360^\circ}{Q}$.

Example: $Q = 8$, $2p = 2$, $\alpha = 45^\circ$

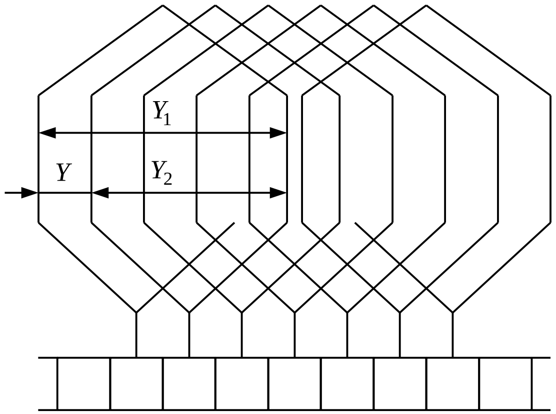




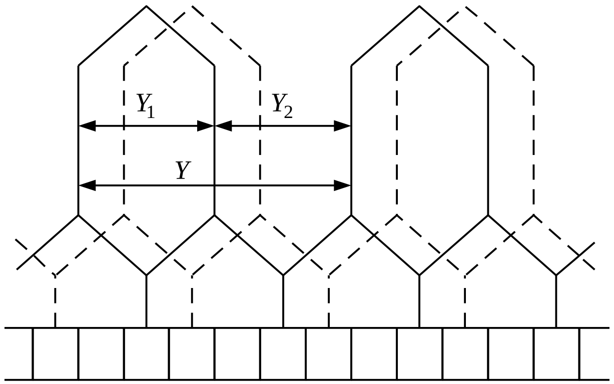
If we connect the end of the first coil with the beginning of the second, etc., we get a polygon. For $Q = \infty$, this is a circle. If we place the brushes at two diametral points of the circle (commutator), the winding breaks into two parallel branches. The voltage across the brushes will be equal to the diameter of the circle.

Winding versions

Depending on the connection of the individual coils, we distinguish between lap and wave windings (or hairpin winding).



Lap winding



Wave winding

In a lap winding, the end of the previous coil is connected to the beginning of the next coil. In a wave winding, we skip some adjacent coils. A winding, lap or wave, is always self-connected or self-linkage.

For a lap winding, the width of the scroll (winding) is: $Y = Y_1 - Y_2$.

For a wave winding, the width of the scroll (winding) is: $Y = Y_1 + Y_2 \neq 2Q_p = \frac{Q}{p} \rightarrow Y = \frac{Q \pm 1}{p}$.

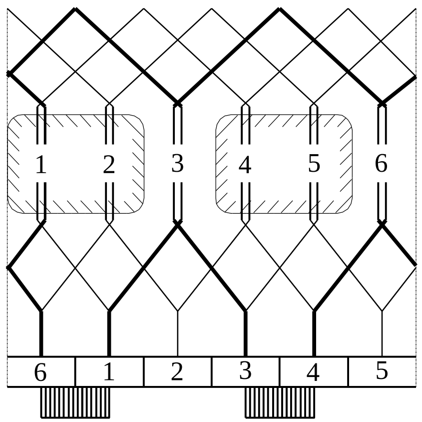
Y_1 is the width of the coils $Y_1 \leq Q_p = \frac{Q}{2p}$. ($Q_p$ is the number of slots per pole.)

Y_2 is the connection width.

Example of a lap winding

$Q = 6, 2p = 2, Y_1 = Q_p = 3, Y = 1$

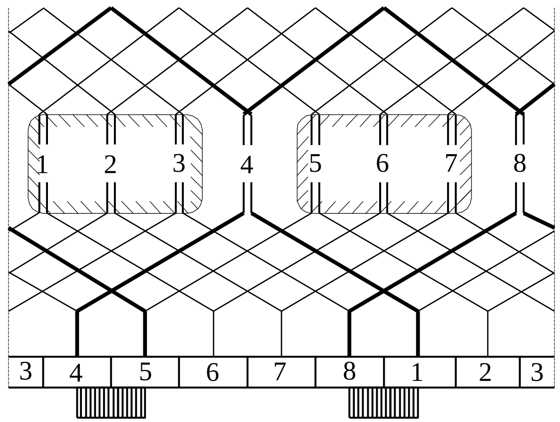
The pitch is $1-(1+Q_p) = 1-4$, $Y_2 = 2$.



Example of a wave winding

$Q = 8, 2p = 2, Y_1 = Q_p = 4, Y = \frac{Q+1}{p} = 9$

The pitch is $1-(1+Q_p) = 1-5$, $Y_2 = Y - Y_1 = 5$.

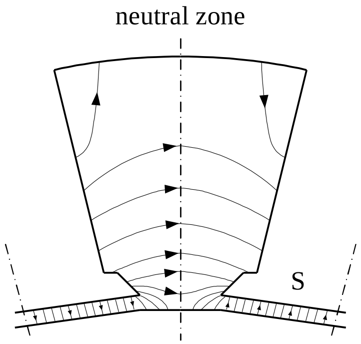


In the pictures we can see that the commutator winding is closed in on itself. The thickly drawn coils are short-circuited via the brushes.

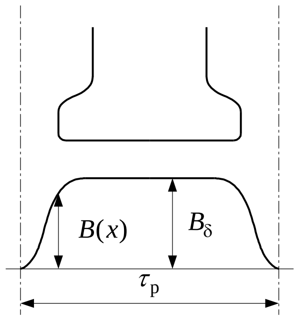
DC machine theory

Induced voltage

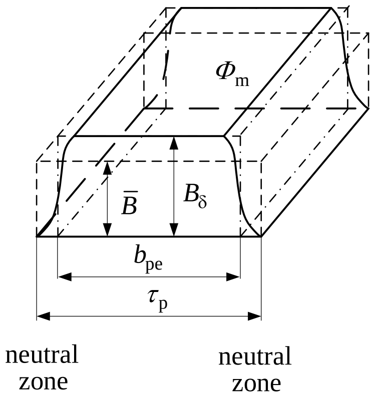
Current in the excitation winding: $I_{\mathrm{f}} \rightarrow \Theta_{\mathrm{f}} = I_{\mathrm{f}} N_{\mathrm{f}} \rightarrow B_{\delta} \rightarrow \Phi_{\mathrm{m}}$



No-load magnetic field image for pole pitch



No-load magnetic field distribution Figure for pole pitch



One pole flux: $\Phi_{\mathrm{m}} = l \int_{0}^{\tau_{\mathrm{p}}} B(x)\,\mathrm{d}x$

$$\Phi_{\mathrm{m}} = b_{\mathrm{pe}}\, l\, B_{\delta} = \tau_{\mathrm{p}}\, l\, \bar{B}$$

b_{pe} is the equivalent peripheral pole width.

Flux is represented as the volume of a geometric body (quad) with sides for the mean value $\bar{B}$, dimensions l and τ_{p}.

The mean value of the induced voltage in a conductor (bar):

$$\bar{E}_{\mathrm{b}} = \upsilon \bar{B} l = 2pn\Phi_{\mathrm{m}}, \text{ if it is } \upsilon = D\pi n = \frac{2pD\pi}{2p} = 2p\tau_{\mathrm{p}} n\,.$$

A winding with "z" conductors $(z = 2N)$ connected in series and with "$2a$" parallel branches, where "a" is the number of parallel branches of half the armature (rotor), will have an induced voltage:

$$E = \frac{z}{2a}\bar{E}_\mathrm{b} = 2p\frac{z}{2a}n\Phi_\mathrm{m} = 4pn\frac{N}{2a}\Phi_\mathrm{m} = 4pnN_\mathrm{a}\Phi_\mathrm{m},$$

where $N_\mathrm{a} = N/(2a)$ is the effective number of turns, i.e., the number of turns of the parallel branch, and the product "pn" is the frequency of the induced voltage in the armature (rotor).

With the introduction of the design voltage constant $c_\mathrm{e} = \dfrac{pz}{a}$:

$$E = c_\mathrm{e}n\Phi_\mathrm{m} = C_\mathrm{E}\Omega_\mathrm{m},$$

and C_E is the magnetic flux coefficient for calculating the induced voltage $\left(C_\mathrm{E} = \dfrac{c_\mathrm{e}}{2\pi}\Phi_\mathrm{m}\right)$. If $\Phi_\mathrm{m} = \mathrm{const.}$, C_E is also constant. The voltage at the generator terminals is:

$$U = E - I_\mathrm{a}R_\mathrm{a} - \Delta U_\mathrm{c}.$$

The voltage is smaller for the voltage drop across the armature resistance $I_\mathrm{a}R_\mathrm{a}$ and for the voltage drop across the commutator brushes ΔU_c. The reverse is true for the motor:

$$U = E + I_\mathrm{a}R_\mathrm{a} + \Delta U_\mathrm{c}.$$

Torque

It is calculated from the mechanical power: $P_\mathrm{m} = M\Omega_\mathrm{m} = EI_a = P_\mathrm{i}$, which is equal to the internal power of the machine (if friction and windage losses can be neglected). This statement is proved by the following derivation:

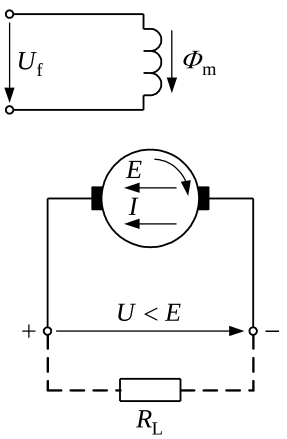
$$P_\mathrm{m} = M\Omega_\mathrm{m} = FD2\pi n = \frac{I_\mathrm{a}}{2a}N\bar{B}l\tau_\mathrm{p}4pn = I_\mathrm{a}\frac{pz}{a}n\Phi_\mathrm{m} = I_\mathrm{a}c_\mathrm{e}n\Phi_\mathrm{m} = EI_\mathrm{a},$$

$$M = \frac{c_\mathrm{e}}{2\pi}\Phi_\mathrm{m}I_\mathrm{a} = c_\mathrm{M}\Phi_\mathrm{m}I_\mathrm{a} = C_\mathrm{M}I_\mathrm{a}$$

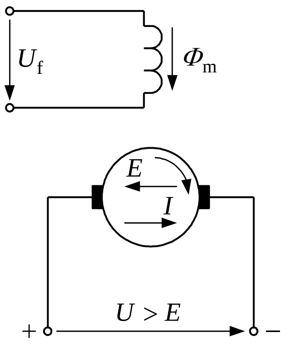
where $c_M = c_e/(2\pi)$ is the design torque constant. The coefficient C_M (or C_T) is the magnetic flux coefficient for calculating the torque. Thus, C_M is the same in value as C_E for the induced voltage if the torque of the rotor friction and ventilation losses can be neglected. The unit of the coefficient is $(N \cdot m/A)$ for torque and $(V \cdot s/rad.)$ for induced voltage. For servomotors is C_E usually given at 1000 rpm. The coefficients for flux and torque are also denoted by the letter K.

DC machines

a) generator



b) motor



$$U = E - IR_a$$

$$U = c_e n \Phi_m - IR_a$$

$$P_{el} = UI = EI - I^2 R_a$$

$$P_m = EI = P_i = P_{el} + P_{Cu}$$

$$P_{el} = P_i - P_{Cu}$$

$$E = U - IR_a = c_e n \Phi_m$$

$$n = \frac{U - IR_a}{c_e \Phi_m}$$

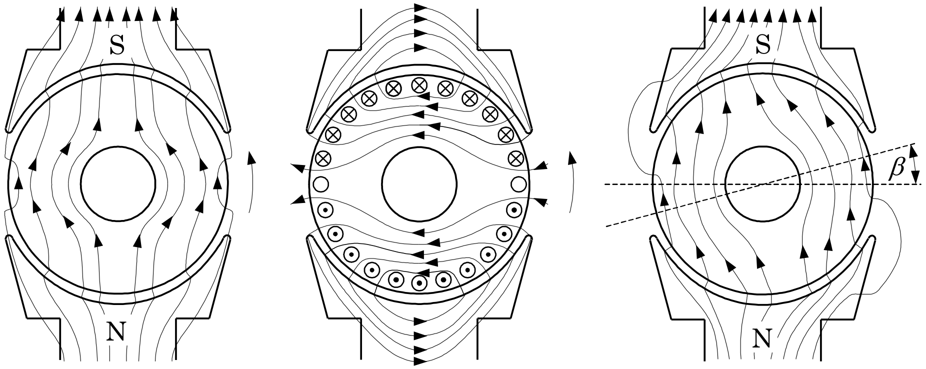
$M_i = c_M \Phi_m I$ (Index "i" is intrinsic.)

$$P_m = M_i \Omega_m = EI = P_i$$

$$P_m = P_{el} - P_{Cu}$$

Armature reaction

In a loaded machine, a load current I_a flows in the armature. $I_a \to \Theta_a \to B_a$ (the magnetic field of the armature reaction). Together with the magnetic field of excitation, we obtain the resulting magnetic field. The physical picture of the action changes. The whole phenomenon and its consequences are called the armature reaction.



a) b) c)

The magnetic field of a DC machine in and near an air gap:

a) no-load − excites the excitation winding,

b) unexcited machine − excites only the armature winding (armature reaction),

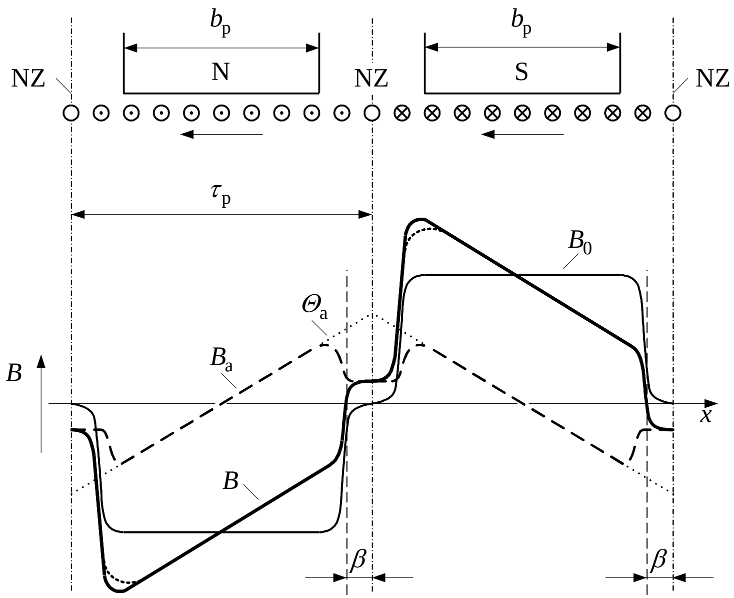
c) sum of a) and b).

The difference between a) and c) is twofold:

1. the magnetic field in the air gap is not distributed homogeneously,

2. the neutral zone is displaced by angle β from the symmetry between the poles.

For generator operation, the spatial distribution of the no-load magnetic field is denoted by B_0, the angular reaction magnetic field by B_a and the resultant magnetic field by *B*. Due to saturation, the increase in the MMF is not equal to the decrease and Φ_m will be smaller.



The consequences of the armature reaction are as follows:

1. reduction of induced voltage E,

2. an increase in iron losses,

3. a shift of the neutral zone,

4. an increase in the inter-lamellar voltage.

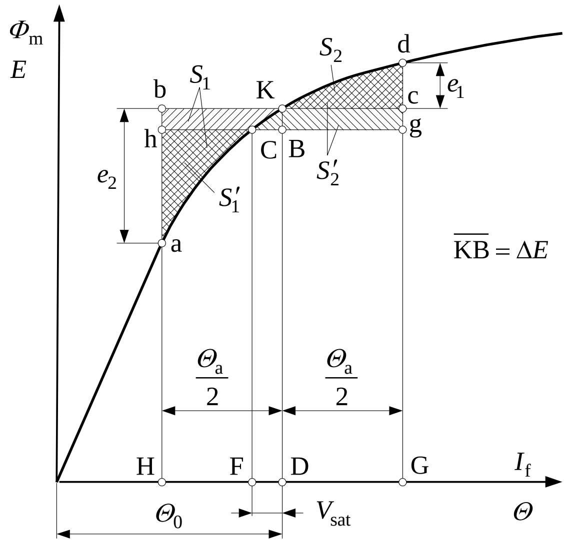
1. Reduction of induced voltage due to armature reaction

a) Generator $E \propto \Phi_m$. The main flux Φ_m falls due to the reaction of the armature, and, hence, E.

b) Motor $U = \text{const.} \rightarrow E = \text{const.}$ (neglecting voltage drop). Φ_m drops due to the armature reaction and the revolution increase, so it remains $E \approx \text{const.}$

Graphical method for determining the effect of armature reaction

We start from the known no-load characteristic (NLC) $E = f(I_f)$.



Distance $\overline{KD} = E$ at excitation Θ_0

$$I_a \to \Theta_a = N_a I_a$$

Resulting excitation:

$$\Theta_{res} = \Theta_0 \pm \Theta_a / 2$$

$$E_2 = \overline{aH} = E - e_2$$

$$E_1 = \overline{dG} = E + e_1$$

$$e_2 = \overline{ab} \text{ and } e_1 = \overline{cd}$$

According to Simpson:

$$\Delta E = E - \frac{E - e_2 + 4E + E + e_1}{6} = \frac{e_2 - e_1}{6}.$$

Graphical procedure to reduce the induced voltage:

The triangle with nodes a, b, K has surface S_1,

and the triangle with nodes K, c, d has surface S_2.

We travel along the NLC from a point K to a point C, which is determined by the fact that the area of the triangles with vertices a, h, C and C, g, d is $(S_1' = S_2')$. $\overline{CF}$ is the reduced induced voltage by a distance $\overline{KB}$ due to the armature reaction. This voltage reduction is due to a drop of the excitation $V_{sat} = \overline{FD}$ (sat − saturation). Therefore, we need to increase the excitation due to the armature reaction when the load is applied by $\Theta_{sat} = V_{sat}$.

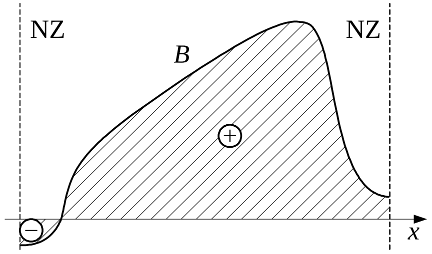
When the load changes:

$$V_{sat} \approx V_{satN} \left(\frac{I_a}{I_{aN}} \right)^2.$$

2. Increase in iron losses due to the armature reaction

Due to the armature reaction $(B_\delta \neq \text{const.})$, the losses in the iron, i.e., mainly in the rotor teeth, increase by approximately $(B_{\text{tmax}} / B_{\text{t0}})^2$.

3. Neutral zone displacement due to the armature reaction



The voltage on the brushes is reduced due to the displacement of the NZ. The commutating coil comes under the influence of the main magnetic field. A voltage is induced in it, which deteriorates the commutation.

4. Increasing the voltage between the lamellas

Mean voltage between the lamellas of a commutator with "K" lamellas:

$$\overline{E}_{\text{K}} = \frac{2pU}{K}.$$

$\overline{E}_{\text{K}} = 16 - 20\ \text{V}$ is permissible.

Due to the deformation of the magnetic field this increases E_{K} and the sparking too. Reducing the effects of an armature reaction can be achieved by:

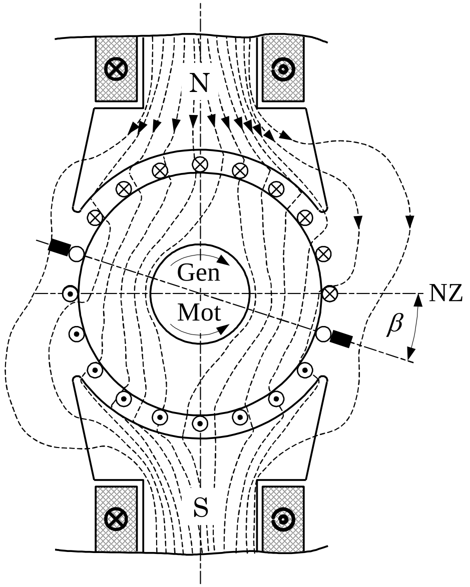
a) moving the brushes,

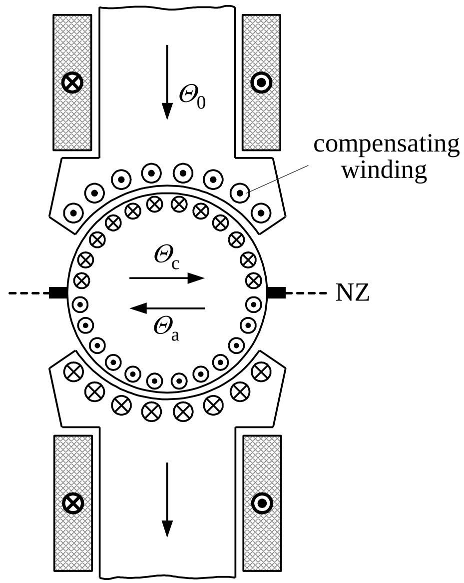
b) a compensating winding.

Depending on the movement of the neutral zone, move the brushes at:

1. the motor in the opposite direction of rotation,

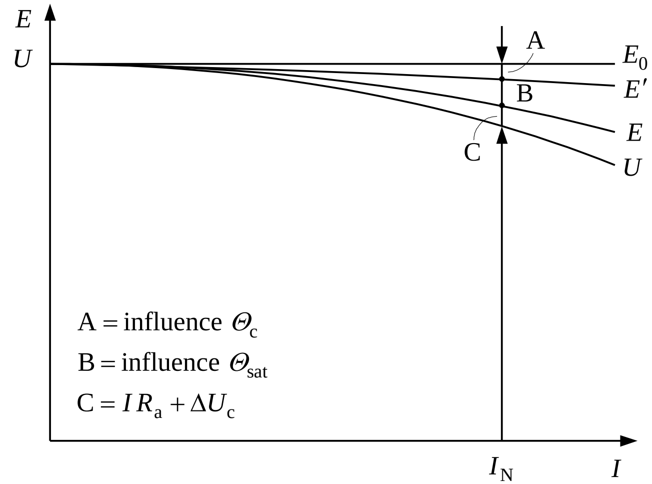
2. the generator in the direction of rotation.





The armature reaction is removed by a compensating winding through which the load current flows. The direction of magnetization shall be opposite to the direction of the armature reaction.

Voltage at the machine terminals



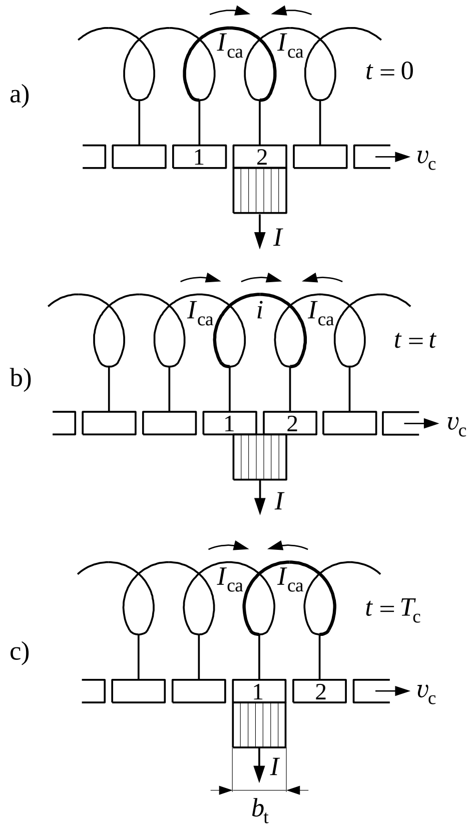
No-load: $E = E_0$

Distance A represents the reduction in voltage due to the movement of the brushes out of the geometric neutral zone.

Distance B illustrates the effect of saturation.

Distance C illustrates the effect of the voltage drops.

Commutation



When a coil passes from one pole region through the NZ to the other pole region, the direction of the induced voltage changes, and so does the direction of the current. This is commutation.

At the time of the current change, the commutating coil is short-circuited across the adjacent lamellas and the brush (Figure b).

If the current does not drop to zero at the moment the number 2 lamella leaves the brush, a spark (electric arc) occurs. This maintains the short circuit until the change in current is complete.

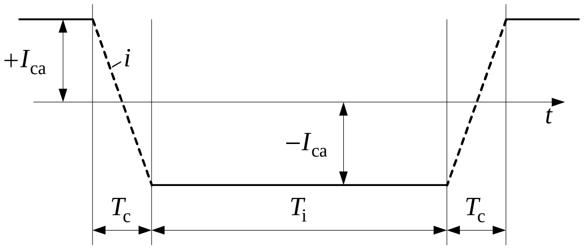
Linear commutation

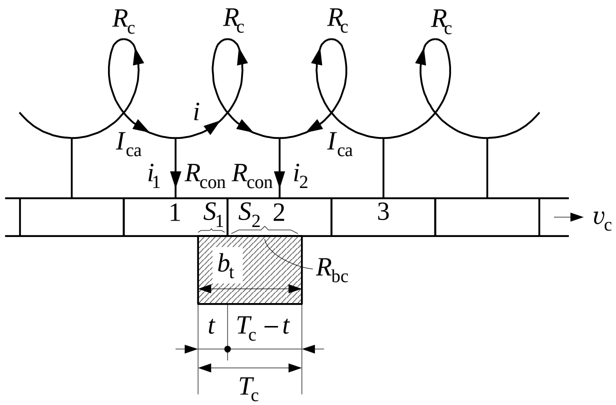
The time T_c waveform of the current during commutation is unknown (dashed line).

The branch current is considered to be $I_{ca} = I/(2a)$, T_c – the duration of the commutation;

and T_i − time of constant current.

The time course of the current is given by Kirchhoff's laws.





Depending on the labels, the equation for the commutation time is:

$$T_c = \frac{b_t}{v_c} = \frac{b_t}{v_a}\frac{D_a}{D_c},$$

where D_a is the rotor diameter, D_c the commutator diameter, b_t the tangential width of the brush, and "v_c" the corresponding speeds.

According to Kirchhoff's first law:

$$i_1 = I_{ca} - i,$$

$$i_2 = I_{ca} + i,$$

where "i" is the instantaneous value of the current, "i_1" and "i_2" are the input currents at lamellas 1 and 2. In the loop, we neglect the resistance of the coil which commutates R_c and the resistance of the connection conductors R_{con}.

The brush-to-commutator contact resistance R_{bc} is taken into account.

According to Kirchhoff's second law:

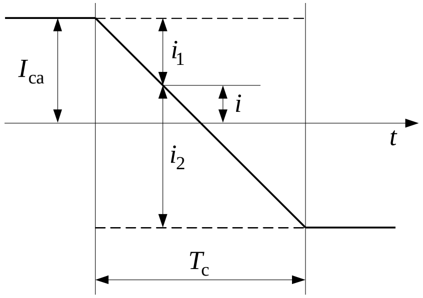
$$i_1 R_1 - i_2 R_2 = 0.$$

According to the Figure, the contact resistance of the individual parts of the brush is proportional to the areas between the brush and the lamella:

$$R_1 = R_{bc}\frac{S_c}{S_1} = R_{bc}\frac{T_c}{t}\,, \qquad R_2 = R_{bc}\frac{S_c}{S_2} = R_{bc}\frac{T_c}{T_c - t}\,,$$

where the surface of the brush $S_c = S_1 + S_2$ or

$$S_1 = S_c\frac{t}{T_c}\,, \qquad S_2 = S_c\frac{T_c - t}{T_c}\,.$$



The shape of the current in the commutating coil is:

$$i = I_{ca}\left(1 - 2\frac{t}{T_c}\right).$$

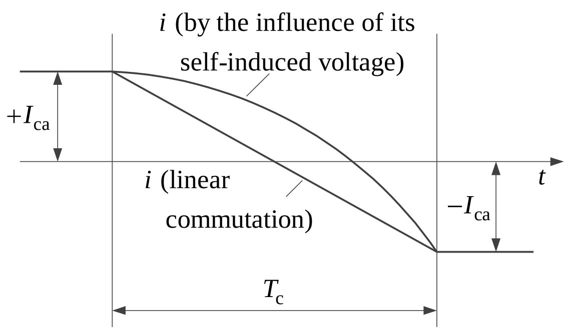
The equation represents a straight line, and we call this type of commutation linear.

Influence of self-induced voltage

According to classical theory, the commutation process is the switching on and off of an inductive circuit, i.e., a commutating coil.

According to Faraday's law:

$$e_c = -N_c\frac{d\Phi_{\sigma c}}{dt} = -L_{\sigma c}\frac{di}{dt}\,,$$



where $\Phi_{\sigma c}$ is the leakage flux of the commutating coil (with the N_c turns) and $L_{\sigma c}$ is its leakage inductance.

Due to the self-induced voltage, the current lags behind the linear commutation. This degrades the commutation (increasing arcing on the brushes, and thus radio interference in the surroundings).

Ways to reduce the impact of self-induced voltage:

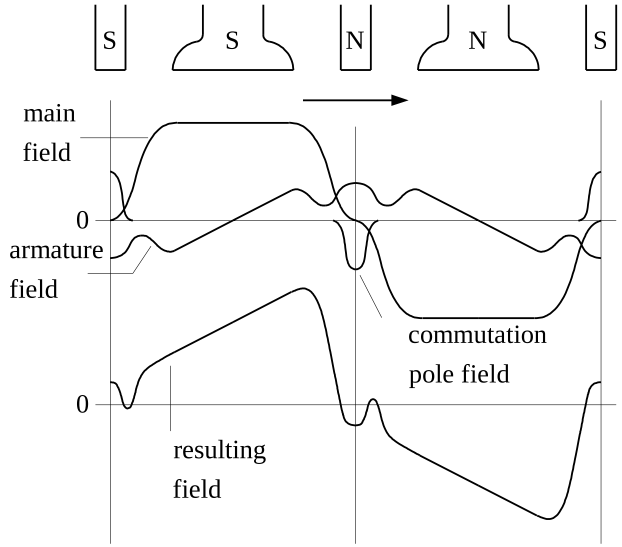
1. a reduction in $\mathrm{d}i/\mathrm{d}t$, i.e., a reduction in current and rotational speed,

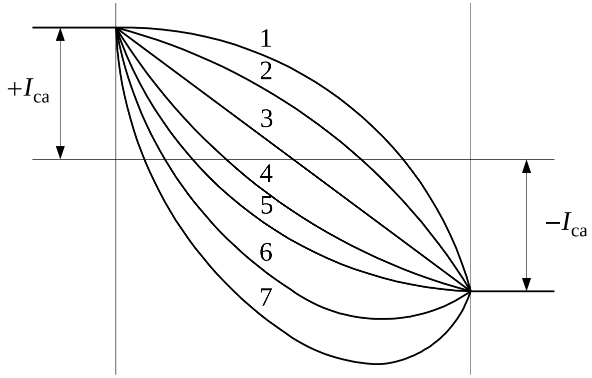
2. reducing inductance $L_{\sigma c}$ by reducing N_c,

3. commutating auxiliary poles that induce a voltage in the opposite direction $(e_{cp} \approx e_c)$.

These are used on larger machines.

Commutation auxiliary poles





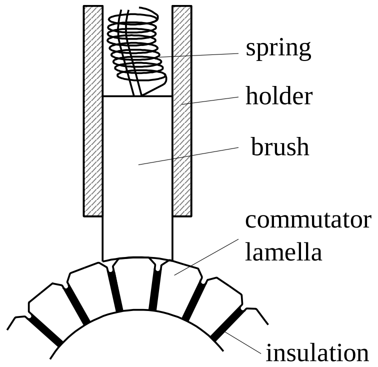
Place them in the neutral zone. The poles shall be narrow and shall correspond to the width of the brush. The Figure is drawn for a generator. For the motor, the opposite direction of rotation or the opposite arrangement of the commutation poles applies.

The load current flows through the windings of the commutator poles and compensates for the effect of the armature reaction. Depending on the dimensioning of these poles, different current curves are obtained:

1 and 2 sub-commutation, 4 and 5 optimal state and 6 and 7 over-commutation.

Commutation assembly

Classic Commutation Assembly

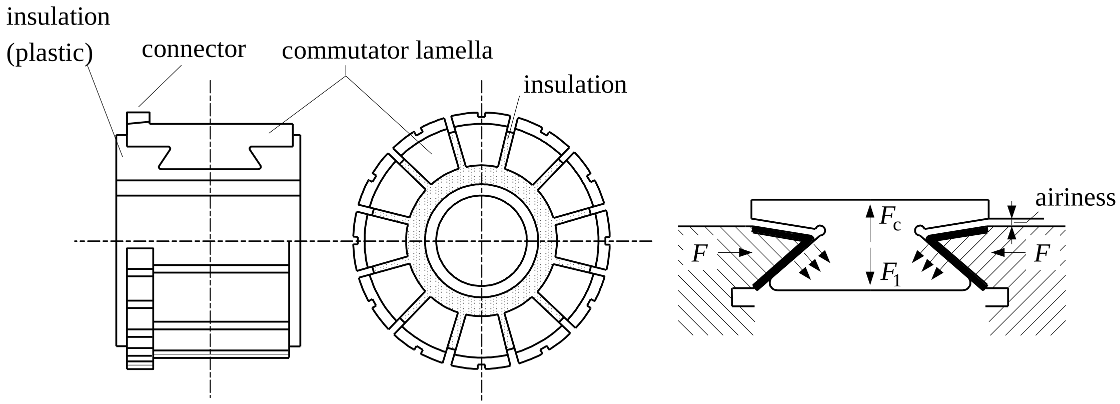


This assembly includes the following parts: commutator, brushes, and brush holders. The picture shows a simple commutation assembly used in small machines. The brushes and brush holders are mounted in holders on the stator and are stationary. The commutator is mounted on the rotor shaft and rotates with it.

There are two different designs of commutators:

1. for small machines where the lamellas are bonded with plastic,

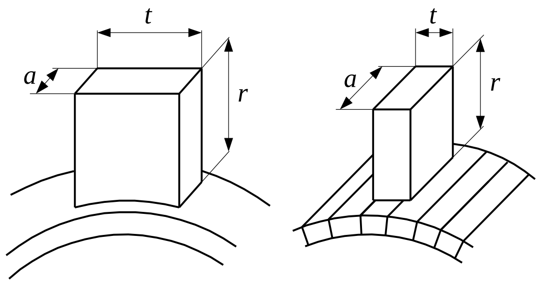
2. for larger machines, we know the swallowtail version.

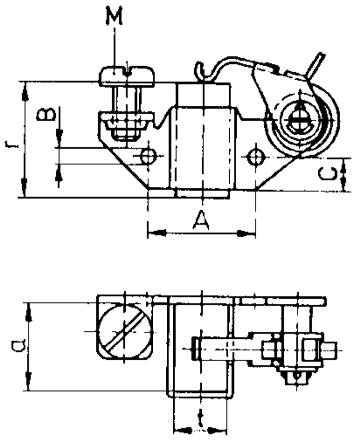


In addition to these designs, there is a disc design for robotic motors, where the lamellas are mounted radially and the brushes in the direction of the machine shaft, and a turbo commutator for high-speed machines.

Depending on the type of material and the manufacturing process, the brushes are divided into: carbon, carbon-graphite, graphite, electro graphite, metal-graphite and resin-bonded graphite. They must have certain properties, e.g., for carbon graphite: specific resistance $30 \div 800\ (\mu\Omega \cdot \mathrm{m})$, current density $5 \div 7\ (\mathrm{A/cm^2})$, peripheral speed up to 20 $(\mathrm{m/s})$, voltage drop across the brush pair $\approx 2{,}8\ \mathrm{V}$, pressure 21 (kPa).

Applications: small DC and universal motors





The dimensions of the brushes are tagged according to the IEC recommendations:

$t \times a \times r$, where it is

t – tangential,

a – longitudinal (axial),

and r – radial dimension.

The connecting conductor to the brush holder is made of (fine) copper braid. The brush holders are of different designs. They are usually radial, but skewed (reaction) brush holders are also possible. The picture shows the holders for universal motors.

Electronic commutation assembly

The commutator-brush assembly replaces a stationary switch. The excitation is on the rotor (permanent magnet) and the armature winding is on the stator. The English abbreviation for these converters is a BLDC motor and stands for brushless (BL) DC motor.

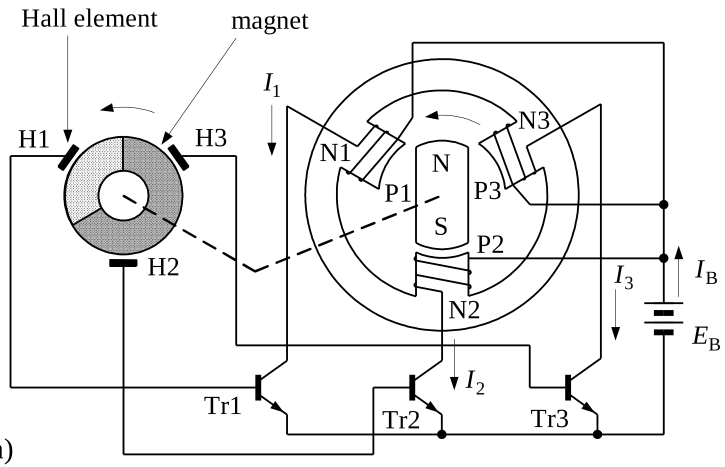
The sensors report the position of the rotor (magnet poles) and provide a signal to control the electronic switches. The types of sensors are:

Hall elements, photo diodes, or photo transistors and inductive encoders.

The armature winding is: a single, two, three and four phase system.

Single and two phase $\rightarrow$ high torque pulsation system

The Figure shows a scheme of a three-phase system with constant polarity (unipolar). The torque is $f(\beta)$.

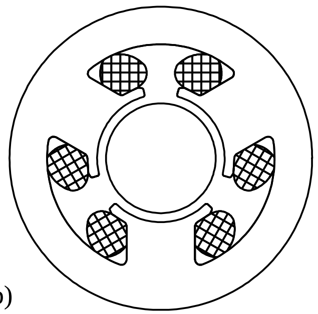


a)

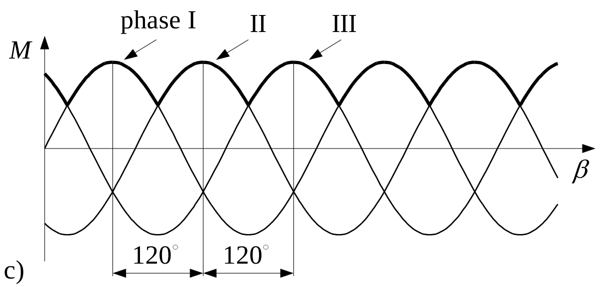
a) operating diagram

b) motor cross-section

c) torque



b)



c)

Types of DC machines

Types of excitations

Classical $\rightarrow$ electromagnet excitation, i.e., one or more excitation windings on the pole shoes. Another option $\rightarrow$ permanent magnets.

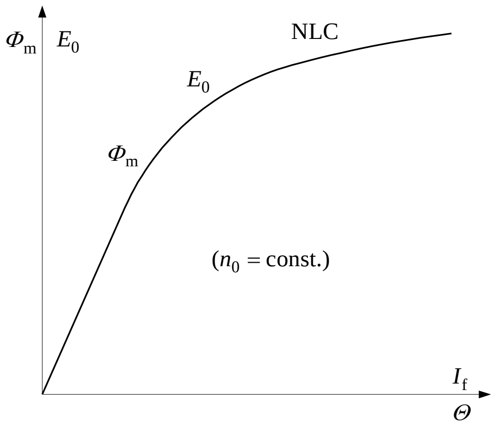
Different machine properties are obtained, depending on the dependence of the excitation on the physical quantities (I, U).

Depending on the connection, we distinguish:

1. foreign excitation,
2. parallel excitation,
3. serial excitation,
4. compound excitation.

Excitation by electromagnets is the first circuit. The second circuit is the winding of the armature and any auxiliary poles and the compensating winding. Different types of machines are distinguished according to the connection of the two circuits.

No-load characteristic (NLC)



This is a basic characteristic.

$E = f(\Phi_m)$ and $\Phi_m = \Theta / R_m \rightarrow E = f(\Theta)$ or

$\Phi_m = f(\Theta)$

The magnetic resistance R_m depends on the saturation.

For NLC we can change the measure for $\Phi_m = f(\Theta)$, because the excitation is

$$\Theta = I_f N_f \rightarrow \Phi_m = f(I_f).$$

Excitation can be a function of voltage, armature current, or a combination of the two. The characteristics of the machines depend on the type of excitation and the connection of the circuits.

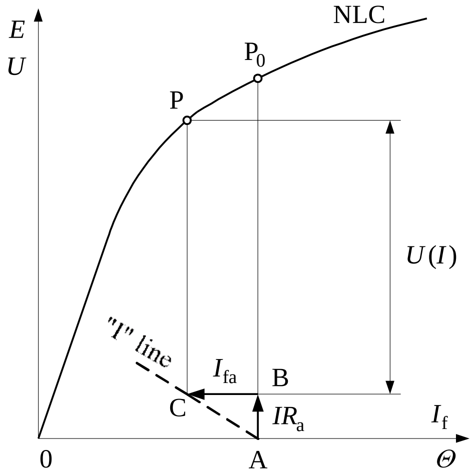
External characteristic by the "I" line method

Generator $\rightarrow U = \mathrm{f}(I)$ $\quad n = \text{const.}$

Motor $\rightarrow n = \mathrm{f}(M)$ $\quad U = \text{const.}$

It is a geometric representation of physical quantities at different drive states. The process is not analytical, such as the circuit diagram of an induction machine or the current characteristics of a synchronous machine, but graphical.

Example for a foreign excitation generator: the starting point is the no-load point $\mathrm{P_0}$ (distance $\overline{\mathrm{P_0A}} = E_0$) of the NLC, i.e., $E = \mathrm{f}(\Theta)$. Distance: $\overline{0\mathrm{A}} = \Theta_0 = I_{\mathrm{f}0} N_{\mathrm{f}}$. Then draw:



$\overline{\mathrm{AB}} = \Delta U = IR_{\mathrm{a}}$ and $\overline{\mathrm{BC}} = \Theta_{\mathrm{sat}} = V_{\mathrm{sat}} = I_{\mathrm{fa}} N_{\mathrm{f}}$.

($I_{\mathrm{fa}} = K_{\mathrm{I}} I$ and K_{I} is the current ratio of the machine.)

$$\Theta_{\mathrm{res}} = \Theta_0 - \Theta_{\mathrm{sat}} = \overline{0\mathrm{A}} - \overline{\mathrm{BC}} \text{ or}$$

$$I_{\mathrm{res}} = I_{\mathrm{f0}} - I_{\mathrm{fa}} .$$

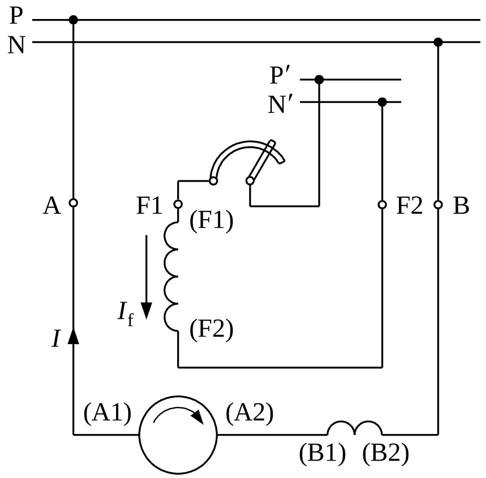
As the load current I changes, ΔU and Θ_{sat} change and the extreme point C travels in the direction of the abscissas and describes the "I" line. The distance $\overline{\mathrm{PC}} = U(I)$ is the voltage of the generator terminals.

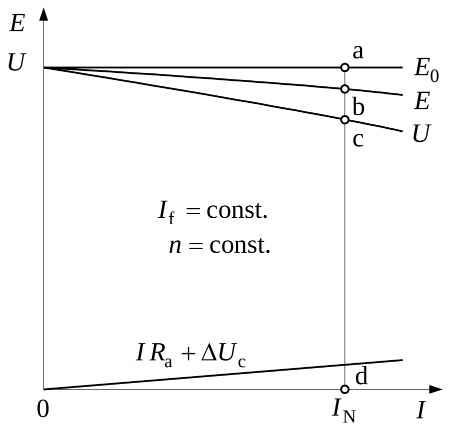
The "I" line thus represents a combination of the geometrical locations of the points of the resultant ampere-turns, and the ohmic voltage drop of a DC machine as a function of the change in load current; hence the name "I" line. Due to the non-linear influence of the armature reaction (saturation), the "I" line is not really a straight line, but a curve.

Generators for direct current

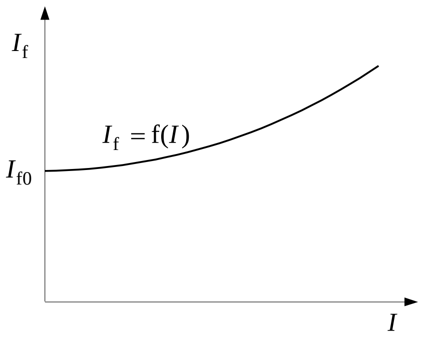
Generator with foreign excitation

We need two separate voltage sources.





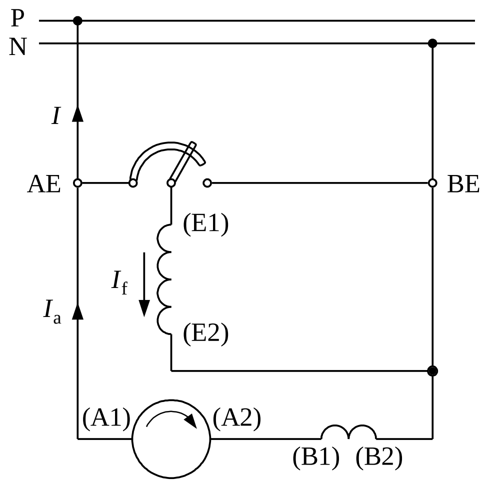
When loaded with I_N is $E < E_0$. $E = \mathrm{f}(I)$ is an internal characteristic. The distance $\overline{\mathrm{ab}}$ is due to the reduction in induced voltage due to the armature reaction. Taking into account $\Delta U = \overline{\mathrm{bc}} = I R_a + \Delta U_c$ we get $U = \mathrm{f}(I)$. The voltage drops and at I_k, there is $U = 0$, $E = \Delta U$.

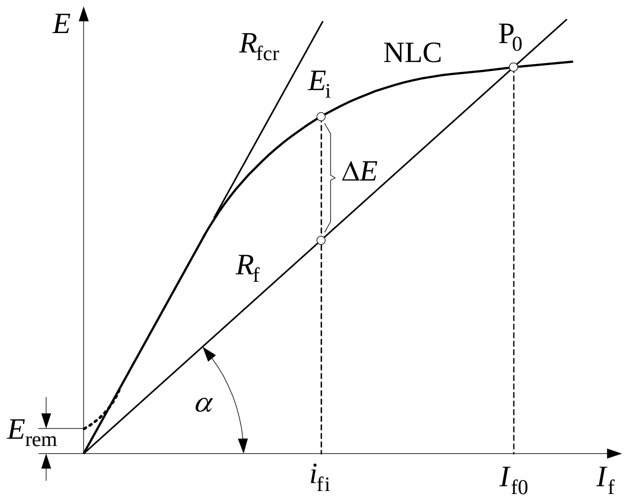


For $U = \mathrm{const.}$ we obtain a control curve $I_f = \mathrm{f}(I)$. This takes into account the influence of the armature reaction and voltage drops.

Parallel excitation generator

The excitation voltage is equal to the rotor voltage $(U_f = U)$ and $I_a = I + I_f$. It is also called self-exciting if there is remanent magnetism. $E_{rem} \to I_f = E_{rem} / R_f$, E is increased gradually up to P_0.





R_f defines a line at an angle α, $\tan\alpha = \dfrac{E}{I_f} = R_f$.

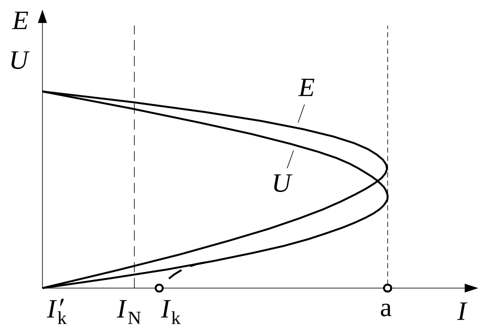
The process is possible if $R_f < R_{fcr}$ (critical).

An instantaneous excitation current raises the voltage difference.

$$\Delta E = E_i - i_{fi} R_f = L_f \frac{di_{fi}}{dt}$$

The external characteristic is softer than for a foreign-excited machine.

$U_f = U \neq \text{const.}$



Generator with series excitation

It is not used as a voltage source. U is highly variable with the current I.

It is usually a series winding combined with a parallel winding in compound generators, where it compensates partly for the effect of the armature reaction.

DC motors

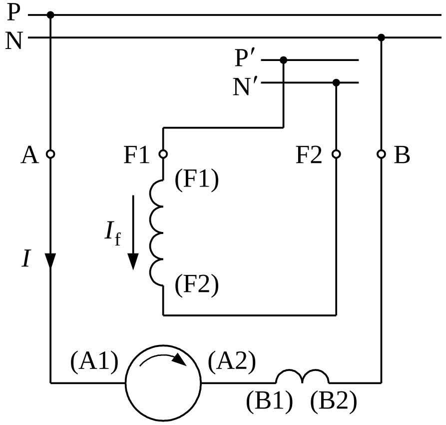
Each generator can work like a motor.

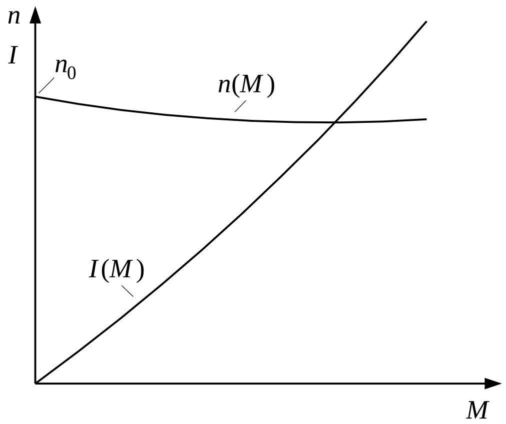
Only the direction of current at the rotor terminals is changed for the same direction of rotation.

Motor with foreign excitation

We are interested in the external characteristic $n = \mathrm{f}(M)$ and the current characteristic $I = \mathrm{f}(M)$.

$$\left.\begin{array}{l} I = \mathrm{f}(M) \rightarrow M = \mathrm{f}(I) \\ n = \mathrm{f}(M) \rightarrow n = \mathrm{f}(I) \end{array}\right\} \text{load characteristics}$$





In stationary operation $n = \dfrac{1}{c_e}\dfrac{E}{\Phi_m}$.

For $I = \text{const.}$, E will be such that $E = U - IR_a - \Delta U_c$ holds, and will be:

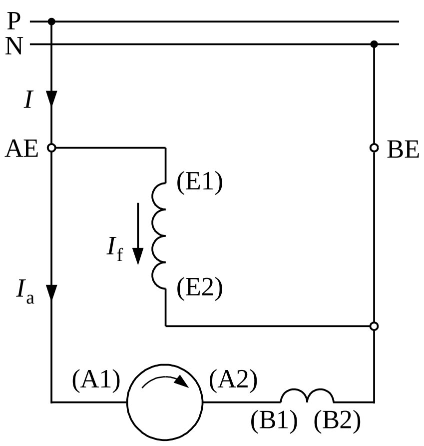
$$n = \frac{1}{c_e} \frac{U - I R_a - \Delta U_c}{\Phi_m} .$$

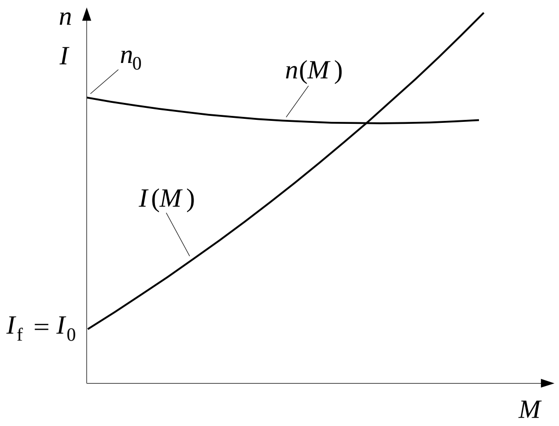
For $\Phi_m = \text{const.}$ is $M = c_M \Phi_m I$ and $M \propto I$.

Due to the armature reaction Φ_m decreases and the speed increases from a certain load, i.e., an unstable operating range. The unstable operating region occurs when the influence of the voltage drops becomes less than the influence of the armature reaction (which varies with the square of the armature current at higher load).

Parallel excitation motor

If the grid is sufficiently rigid, a parallel excitation motor has the same characteristics as a foreign excitation motor.





Difference: grid current $I = I_a + I_f \Rightarrow I_a = I - I_f$

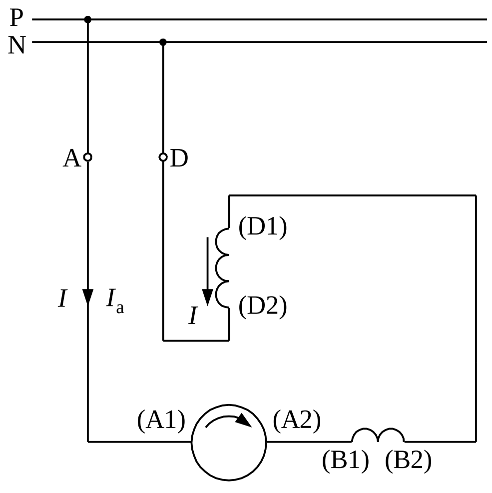
The difference can be seen in the current characteristic $I = f(M)$, because the current has an initial value $I_f = I_0$, whereas in a foreign excitation motor it is zero.

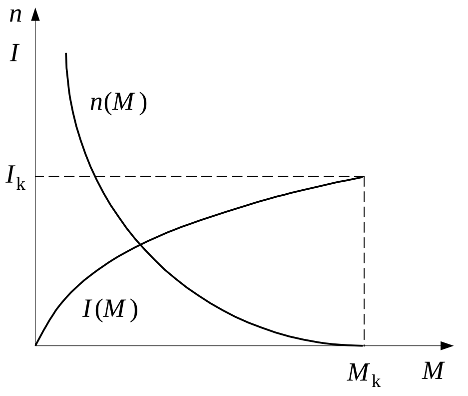
Series excitation motor

The series motor is considered to be $\Phi_m \propto I$, and thus:

$$M = c_M \Phi_m I = c_1 I^2 \rightarrow I = \frac{1}{c_1}\sqrt{M},$$

$$n = \frac{1}{c_2}\frac{E}{I} = c_3 \frac{E}{\sqrt{M}}.$$



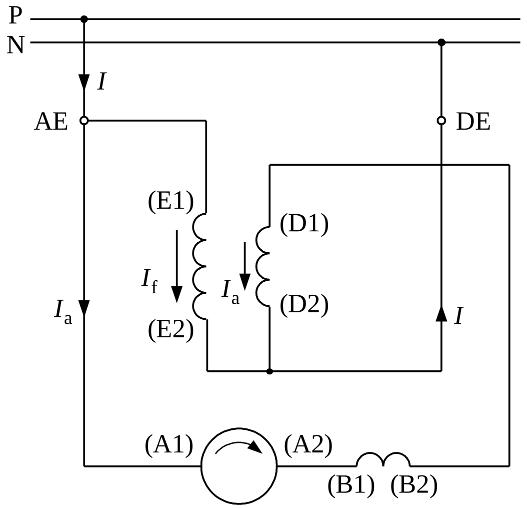


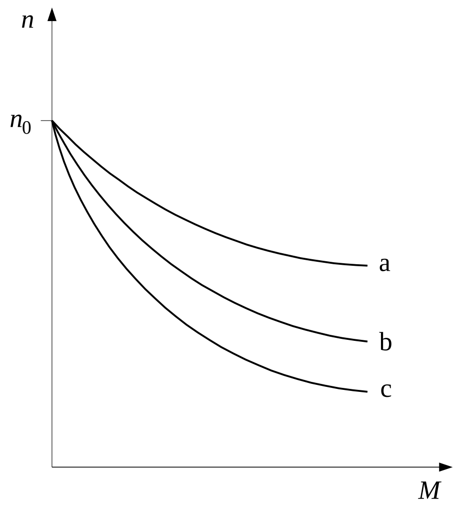
In start-up (short-circuit) $I = I_s = I_k$, $M = M_k$ and $n = 0$.

For low loads (no-load): $M \rightarrow 0$, $n \rightarrow \infty$

Compound excitation motor

In most cases the series winding supports a parallel winding. The winding that has the greater influence determines the shape of the characteristic. The motor has a harder characteristic (curve a) if the parallel winding dominates, and a softer characteristic if the series winding dominates (curve c).



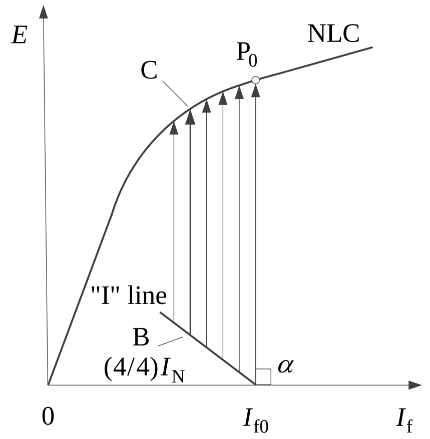


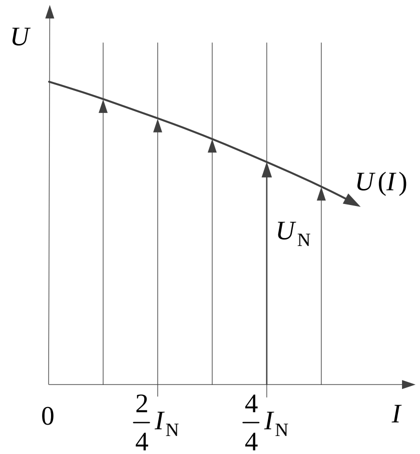
Constructing characteristics

Constructing the external characteristics of generators *U*(*I*)

Assume: the "I" line is a straight line, choose, e.g., $I = (1/4 \div 5/4) I_N$.

a) Generator with foreign excitation



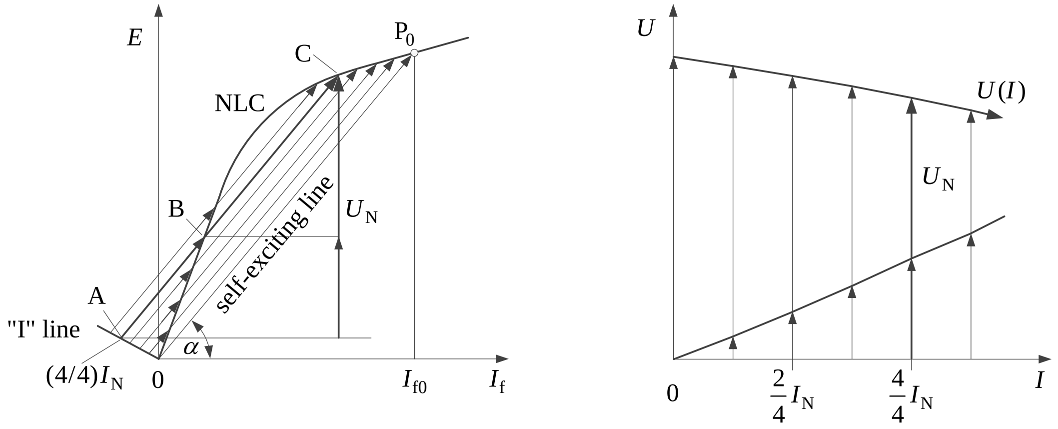


Point B on the 'I' line is defined for the rated current, i.e., the armature current $I = (4/4)I_N$.

The magnitude of the voltage for a given load, e.g., $I = I_N$, is obtained from the relation:

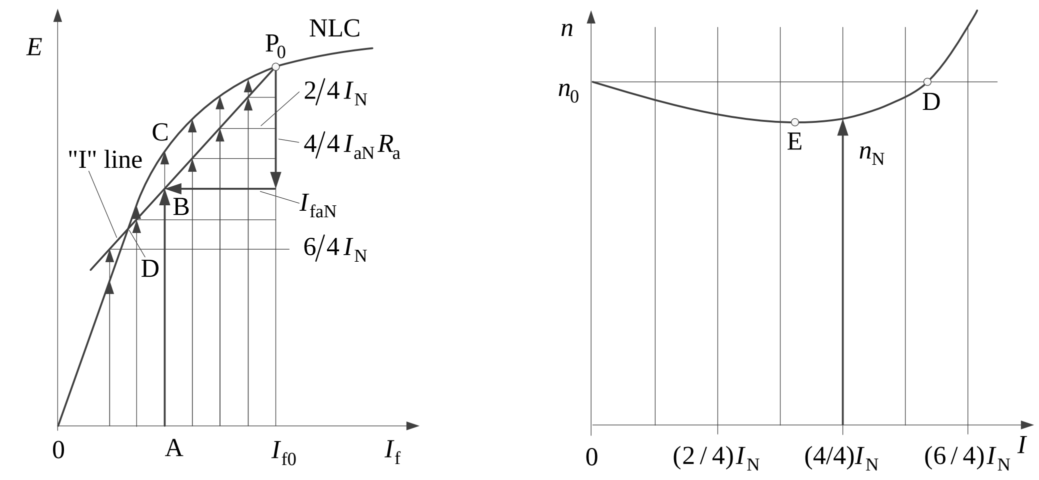
$U = E_0 \times \overline{BC} / \overline{P_0 I_{f0}}$.

b) Parallel generator



Direction of self-excitation (angle) α: $\sin\alpha = \overline{P_0 I_{f0}} / \overline{P_0 0} = E_0 / \overline{P_0 0}$ and $U = E_0 \times \overline{AC} / \overline{P_0 0}$.

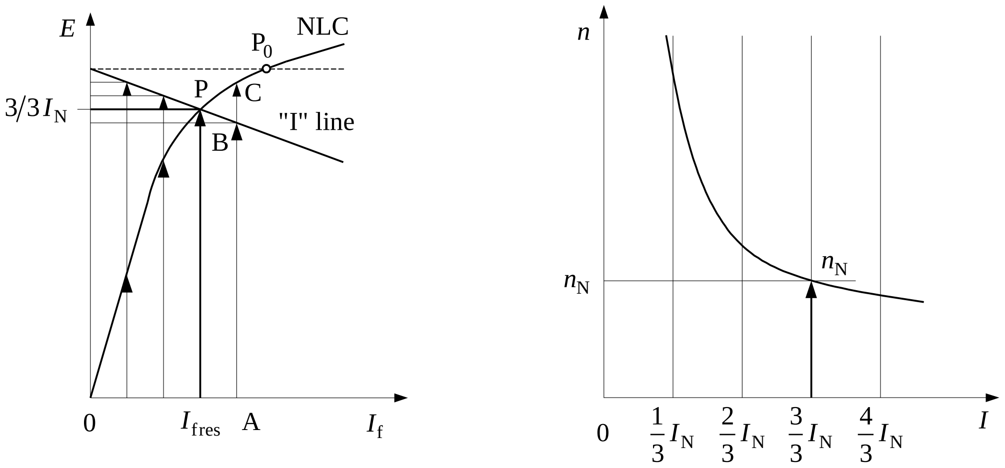
Constructing motor rotational speed characteristics *n*(*I*)



a) Foreign-excited and parallel motor

The characteristic designs $n(I)$ for the foreign-excited and parallel motors are identical. The revolutions for a given load, e.g., $I = I_{aN}$, are given by: $n = n_0 \overline{AB} / \overline{AC}$. From point E on the curve $n(I)$ is the unstable operating range.

b) Serial motor



The rotations for an arbitrary load are obtained from the relation: $n = n_N \dfrac{\overline{AB}}{\overline{AC}}$, because NLC is valid for n_N.

Starting and changing the speed of the motors

Starting: $n = 0 \;\; \rightarrow \;\; E = 0$ and $I_k = \dfrac{U}{R_a}$, $M_k = C_M I_k$.

The current is reduced by adding an additional (starting) resistor, or by reducing *U*.

Changing the rotational speed of foreign-excited motors:

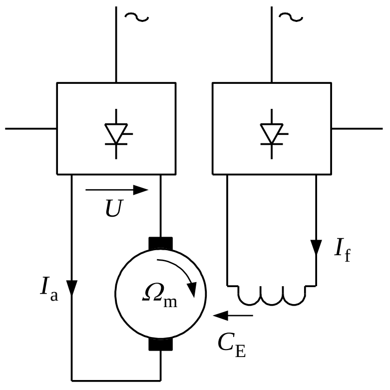
$$\Omega_m = 2\pi n = \frac{U - I_a R_a}{C_E} = \frac{U}{C_E} - \frac{R_a M}{C_E C_M} = \Omega_{m0} - \frac{R_a M}{C_E C_M}\frac{U}{U} = \Omega_{m0}\left(1 - \frac{M}{M_k}\right).$$

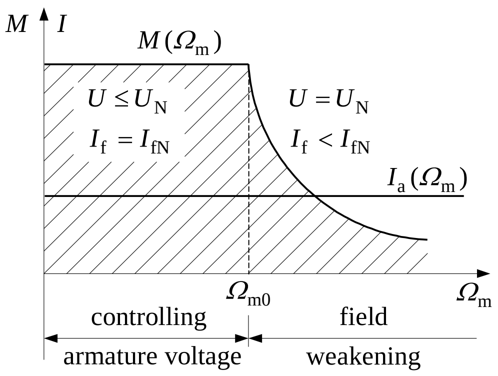
Options for varying the speed of foreign-excited engines:

1. armature voltage control U ,

2. magnetic field control $\Phi_m = f(U_f)$,

3. with additional resistance R_{add} .





From 0 to Ω_{m0}, vary U (I = const., Φ = const., M = const.). We weaken the magnetic field from Ω_{m0} $(U = U_N)$ onwards. For $\Omega_m < 0$ $(M_L > M)$, we switch to generator braking.

The speed control of the series engine is a particular problem. A change in U causes a change in I_f . Therefore, we vary the speed by applying a shunt to the excitation winding.

Non-stationary operation

Changes are possible after a transient phenomenon determined by the excitation time constant $T_f = L_f / R_f$ and armature $T_a = L_a / R_a$ $\left(T_f \approx (5 \div 20) T_a\right)$. This is electrical inertia. Mechanical inertia is determined by the masses of the rotor and the load, or the total moment of inertia J .

Basic equations for non-stationary operation (drive)

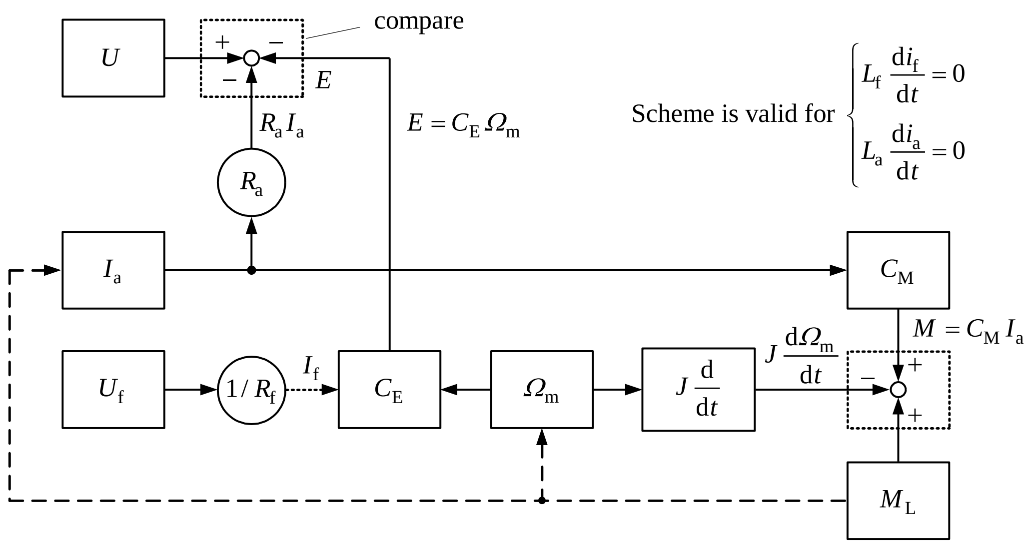
$$u_f = R_f i_f + L_f \frac{di_f}{dt}$$

$$u = R_a i_a + e + L_a \frac{di_a}{dt} \quad \text{(induced voltage } e = C_E \omega_m\text{)}$$

$$J \frac{d\omega_m}{dt} = m + m_L \qquad \text{(torque } m = C_M i_a\text{) For the motor } m_L < 0 .$$

The familiar equations in stationary operation apply for $i_f = I_f$ and $i_a = I_a$ is $d/dt = 0$.

The following block scheme shows the physical picture of the effects of quantities in a foreign excited machine.

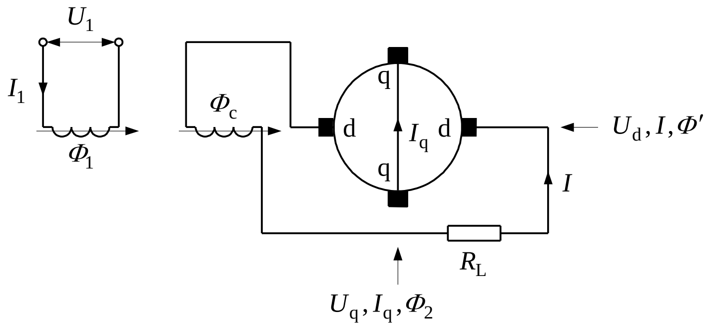


Special DC machines

Most of them have only historical significance. One of the remaining is amplidyne, which served as an exciter in synchronous generators.

Amplidyne

It's a power amplifier. It has two pairs of brushes. The first – normal brushes (q-q) are short circuited, and a rotor excitation greater than the fundamental excitation is obtained. The consumer is connected to the second pair of brushes (d-d).



Amplification is: $U_d I \approx 100 U_q I_q \approx 10000 U_1 I_1$.

Because of the rotor (armature) reaction, it has a compensating winding (Φ_c compensates the flux of the armature reaction Φ').

AC commutator machines

Implementation $\rightarrow$ motors only

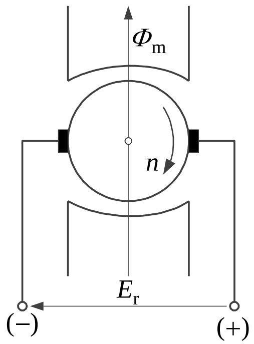
Only a single-phase commutator motor of small power remains.

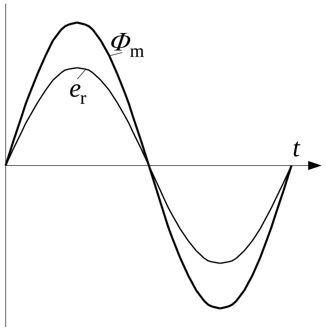
Use $\rightarrow$ for household appliances and power hand tools. Also known as a universal motor.

Induced voltages of a single-phase commutator motor

U_f (alternating) $\rightarrow$ Φ_m (alternating) $\rightarrow$ E_r (alternating)

E_r is in phase with Φ_m.





If $N_a = N/(2a) = z/(4a)$ is the number of effective turns of the armature, then the rotational or motion voltage will be:

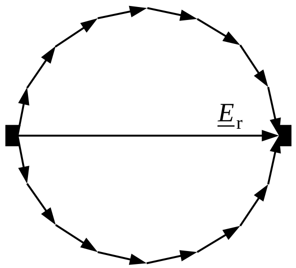
$$E_r = 2\sqrt{2}\,p\,n N_a \hat{\Phi}_m .$$

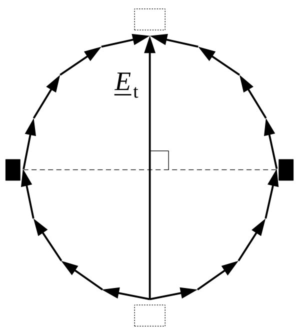
The alternating magnetic field also results in a transformer voltage:

$$E_t = \frac{2\pi}{\sqrt{2}}\, f\, N_a f_w \hat{\Phi}_m .$$

The winding factor is given by $f_w = \dfrac{2r}{\pi r} = \dfrac{2}{\pi}$ (where r is the radius of the potential circle)

and therefore the transformer voltage will also be $E_t = 2\sqrt{2}\, f\, N_a \hat{\Phi}_m$. The space position of the two can be seen in the Figures below.





For $pn = f$, will $E_r = E_t$.

The voltage across the brushes in the neutral zone is equal to $E_t = 0$.

The corresponding transformer voltage is also induced in the excitation winding.

Calculation of torque

For alternating $i(t)$ and $\Phi(t)$ valid:

$$M(t) = c_{\mathrm{M}}\, \Phi_{\mathrm{m}}(t)\, i(t)\,.$$

If it is $i(t) = I\sqrt{2}\sin(\omega t)$, it will be $\Phi_{\mathrm{m}}(t) = \hat{\Phi}_{\mathrm{m}} \sin(\omega t - \varphi')$,

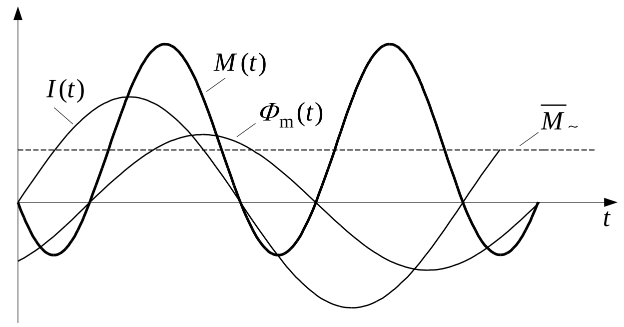
where φ' is the phase shift between the current and magnetic flux. Taking the trigonometric transformation into account, we obtain:

$$M(t) = \frac{c_{\mathrm{M}}}{\sqrt{2}} \hat{\Phi}_{\mathrm{m}}\, I \left(\cos\varphi' - \cos(2\omega t - \varphi')\right).$$

The first component is the mean torque:

$$\overline{M}_{\sim} = \frac{c_{\mathrm{M}}}{\sqrt{2}} \hat{\Phi}_{\mathrm{m}} I \cos\varphi'\,,$$

around which the torque oscillates at double the frequency, the mean value of which is zero over one period.



$\hat{\Phi}_{\mathrm{m}\sim}$ at AC supplied voltage shall be equal to the flux at the DC supplied voltage

$$(\hat{B}_{\mathrm{m}\sim} = B_{\mathrm{m}=})\,.$$

That's why:

$$M_{\sim} = M_{=} \frac{\cos\varphi'}{\sqrt{2}}\,.$$

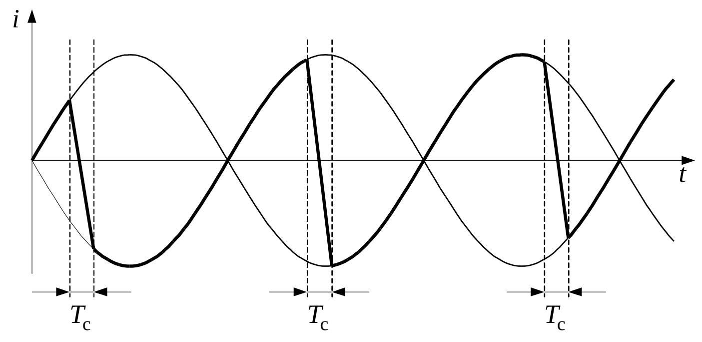
From the equation for rotational voltage E_{r}, we express Φ_{m}, and note that $N_{\mathrm{a}} = N/(2a) = z/(4a)$ and the constant $c_{\mathrm{M}} = pz/(2\pi a)$. The torque value is then:

$$M = \frac{E_r}{\Omega_m} I \cos\varphi' \text{ and } P = M\Omega_m = E_r I \cos\varphi' .$$

Commutation of a single-phase commutator machine

Physically, the events during commutation are the same as in a DC machine. For the commutation time: $T_c << T = 1/f$.

The current remains alternating after commutation. The problem is the transformation voltage E_t . This is greatest in the coil that commutates. It therefore worsens the commutation process.



Single-phase commutator motor in series connection

The torque is maximum for $\cos\varphi' = 1$ $(\varphi' = 0)$, if I and Φ_m are in phase, i.e., in series connection.

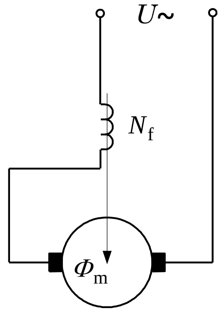
Application example: small universal motors in household appliances or hand tools with power $5 \div 1000$ W and up to 20000 rpm. The advantage is the high starting torque. It has no compensating winding and no commutating poles. The name universal is used because it can be connected to DC or AC voltages of the same peak value.

Current $I = I_a = I_f \rightarrow \Theta_f$

The excitation winding with N_f turns excite the main flux:

$$\hat{\Phi}_\mathrm{m} = \frac{\hat{\Psi}_\mathrm{f}}{N_\mathrm{f}} = \frac{L_\mathrm{f}\, I\sqrt{2}}{N_\mathrm{f}} = \frac{\omega L_\mathrm{f}\, I\sqrt{2}}{\omega N_\mathrm{f}} = \frac{X_\mathrm{f}\, I\sqrt{2}}{\omega\, N_\mathrm{f}}\ , \qquad \hat{\Phi}_\mathrm{m} \to E_\mathrm{r} \ \text{ and } \ E_\mathrm{t}\,.$$

Equivalent scheme



Taking the equation for $\hat{\Phi}_\mathrm{m}$, it will be:

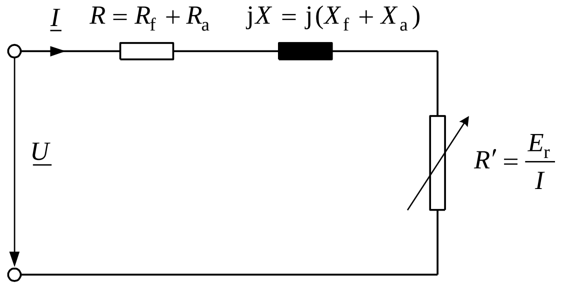
$$E_\mathrm{r} = \frac{2}{\pi}\frac{pn}{f}\frac{N_\mathrm{a}}{N_\mathrm{f}} X_\mathrm{f} I = c\frac{n}{n_\mathrm{s}} I\ ,$$

where c is the constant of the unsaturated machine

$$c = \frac{2}{\pi}\frac{N_\mathrm{a}}{N_\mathrm{f}} X_\mathrm{f}\,.$$

E_r is in phase with I, and is, therefore, $E_\mathrm{r} = R'I$. $R' = cn/n_\mathrm{s}$ is the equivalent (fictitious) resistance.

The result is a similar replacement circuit as for the rotor of an induction machine.



The equivalent resistance R' also represents the internal mechanical power:

$$P_\mathrm{m} = I\, E_\mathrm{r} = I^2 R'\,.$$

The current is through an equivalent circuit:

$$\underline{I} = \frac{\underline{U}}{R + c\dfrac{n}{n_\mathrm{s}} + \mathrm{j}X}\,.$$

For $n = 0$, i.e., short-circuited, the following applies:

$$I_\mathrm{k} = \frac{U}{\sqrt{R^2 + X^2}}\,.$$

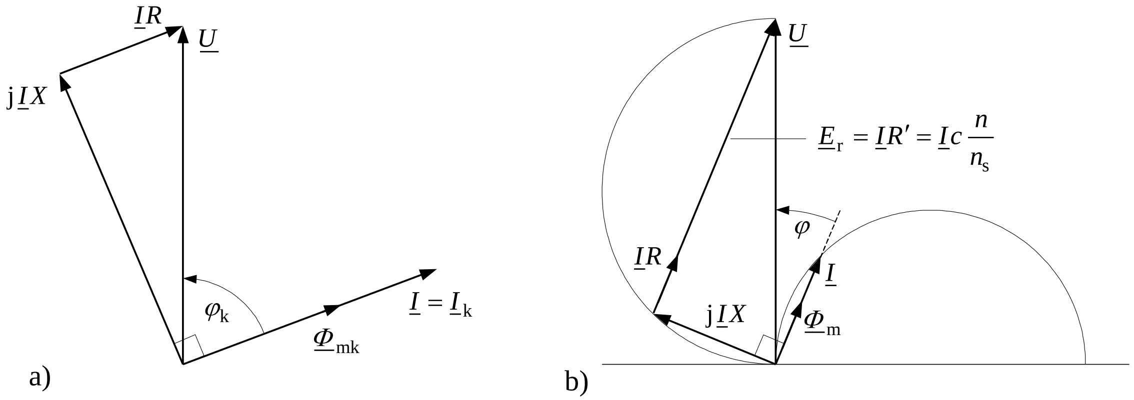
Such a machine, like an induction machine, has a maximum current value at $n = 0$, i.e., at a standstill.

Phasor diagram

We draw it for two examples:

a) $n = 0$ (short-circuited motor),

b) $n \neq 0$ (any number of revolutions).



For $n = 0$ it is also $E_\mathrm{r} = 0$, and so $I = I_\mathrm{k} = \dfrac{U}{Z_\mathrm{k}}$.

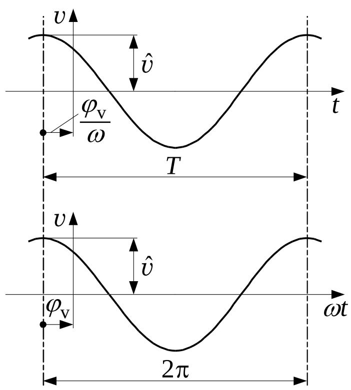
For $n \neq 0$, E_r will be in phase with $I(\Phi_\mathrm{m})$ for $I < I_\mathrm{k}$ and $\cos\varphi > \cos\varphi_\mathrm{k}$. The power factor $\cos\varphi_\mathrm{N} \approx 0{,}95$.

COMPLEX CALCULATION

The time-sine quantity of magnitude "υ", whose waveform is shown in the Figure, can be expressed as a function of time t or as a function of the argument ωt as follows:

$$\upsilon = \hat{\upsilon}\cos(\omega t + \varphi_{\mathrm{v}}) .$$

The amplitude $\hat{\upsilon}$ represents the maximum value of a sinusoidal quantity. The proportionality factor before time in the argument of the cosine function is the electric angular frequency ω. During the period T, the argument increases by the value of the angle 2π. From the condition $\omega t = 2\pi$ we get:



$$\omega = \frac{2\pi}{T} = 2\pi f .$$

Frequency is defined in the equation as: $f = \dfrac{1}{T}$.

The phase angle φ_{v} gives the negative displacement of the maximum of the cosine quantity from the origin of the time coordinate. The cosine function is taken to use complex calculus.

Using the Euler notation $\mathrm{e}^{\mathrm{j}x} = \cos x + \mathrm{j}\sin x$, the complex equation can be expressed as:

$$\upsilon = \mathrm{Re}\left(\hat{\upsilon}\,\mathrm{e}^{\mathrm{j}(\omega t + \varphi_{\mathrm{v}})}\right) = \mathrm{Re}\left(\hat{\upsilon}\,\mathrm{e}^{\mathrm{j}\varphi_{\mathrm{v}}}\mathrm{e}^{\mathrm{j}\omega t}\right).$$

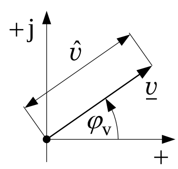
There are now three equal quantities in the equation: the amplitude $\hat{\upsilon}$, the phase factor $\mathrm{e}^{\mathrm{j}\varphi_{\mathrm{v}}}$ and the frequency factor $\mathrm{e}^{\mathrm{j}\omega t}$. Of interest are the amplitude and the phase position, which, together, represent a complex quantity:

$$\underline{\upsilon} = \hat{\upsilon}\,\mathrm{e}^{\mathrm{j}\varphi_{\mathrm{v}}} .$$

The corresponding instantaneous value of the complex quantity is given by the following basic equation using the previous equation:

$$\upsilon = \mathrm{Re}\left(\underline{\upsilon}\,\mathrm{e}^{\mathrm{j}\omega t}\right).$$

The Figure on the right shows a complex quantity $\underline{\upsilon}$ as a phasor in the complex plane.



Mathematical operations of complex quantities

Multiplying a sine quantity by a constant

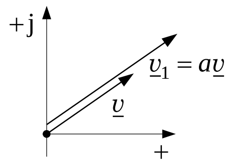
For multiplication of a complex quantity, the equation:

$$\upsilon_1 = \mathrm{Re}\left(\underline{\upsilon}_1\,\mathrm{e}^{\mathrm{j}\omega t}\right) = a\,\upsilon = \mathrm{Re}\left(a\,\underline{\upsilon}\,\mathrm{e}^{\mathrm{j}\omega t}\right).$$

From it we get $\underline{v}_1 = a\underline{v}$, respectively

$$\hat{v}_1 \mathrm{e}^{\mathrm{j}\varphi_{\mathrm{v}1}} = a\hat{v}\mathrm{e}^{\mathrm{j}\varphi_{\mathrm{v}}} .$$

We can see that multiplying a complex quantity by a constant, changes its amplitude and preserves its phase position. Multiplication by a constant is illustrated in the Figure on the right.



Adding two sinusoidal quantities

This is an example of the law of nodes or the law of the loop. Using the basic equation, we derive

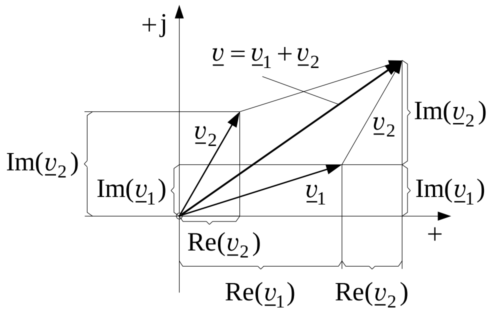
$$v = \mathrm{Re}\left(\underline{v}\,\mathrm{e}^{\mathrm{j}\omega t}\right) = v_1 + v_2 = \mathrm{Re}\left(\underline{v}_1\,\mathrm{e}^{\mathrm{j}\omega t} + \underline{v}_2\,\mathrm{e}^{\mathrm{j}\omega t}\right) = \mathrm{Re}\left((\underline{v}_1 + \underline{v}_2)\mathrm{e}^{\mathrm{j}\omega t}\right)$$, it is:

$$\underline{v} = \underline{v}_1 + \underline{v}_2 .$$

With the introduction of real and imaginary parts, it applies:

$$\mathrm{Re}(\underline{v}) + \mathrm{j}\,\mathrm{Im}(\underline{v} = \mathrm{Re}(\underline{v}_1) + \mathrm{Re}(\underline{v}_2) + \\ + \mathrm{j}\left(\mathrm{Im}(\underline{v}_1) + \mathrm{Im}(\underline{v}_2)\right) .$$

The Figure shows the phasors in the complex plane, which are summed geometrically (vectorially).



Differentiating a sinusoidal quantity by time

This is the case for the law of induction. Using the basic equation, we derive:

$$v' = \mathrm{Re}\left(\underline{v}'\,\mathrm{e}^{\mathrm{j}\omega t}\right) = \frac{\mathrm{d}v}{\mathrm{d}t} = \mathrm{Re}\left(\frac{\mathrm{d}}{\mathrm{d}t}\left(\underline{v}\,\mathrm{e}^{\mathrm{j}\omega t}\right)\right) = \mathrm{Re}\left(\mathrm{j}\omega\underline{v}\,\mathrm{e}^{\mathrm{j}\omega t}\right)$$

and it is

$$\underline{v}' = \mathrm{j}\omega\underline{v} .$$

Differentiating by time in the domain of instantaneous values means, in the complex domain, multiplying by $\mathrm{j}\omega$. Taking into account $\mathrm{j} = \mathrm{e}^{\mathrm{j}\pi/2}$ and

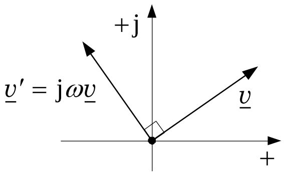
$$v'\mathrm{e}^{\mathrm{j}\varphi_{\mathrm{v}}} = \omega\hat{v}\,\mathrm{e}^{\mathrm{j}(\varphi_{\mathrm{v}} + \pi/2)}$$

we get the ratio between the amplitudes $\Rightarrow\ \hat{v}' = \omega\hat{v}$

and phase angles $\Rightarrow\ \varphi'_{\mathrm{v}} = \varphi_{\mathrm{v}} + \pi/2$.

The differentiated quantity overtakes the original quantity by 90°.

Corresponding to the derivative equation, draw a phasor diagram of the derivative $\underline{\upsilon}'$ and the original quantity $\underline{\upsilon}$.



Time integration of a sine quantity

Such a use case occurs in a voltage − current coupling on a capacitor such as:

$$u = (1/C)\int i\,\mathrm{d}t\,.$$

The basic equation gives:

$$\upsilon_1 = \mathrm{Re}\left(\underline{\upsilon}_1\,\mathrm{e}^{\mathrm{j}\omega t}\right) = \int \upsilon\,\mathrm{d}t = \mathrm{Re}\left(\int \underline{\upsilon}\,\mathrm{e}^{\mathrm{j}\omega t}\mathrm{d}t\right) = \mathrm{Re}\left(\frac{1}{\mathrm{j}\omega}\underline{\upsilon}\,\mathrm{e}^{\mathrm{j}\omega t}\right) \text{ and it is}$$

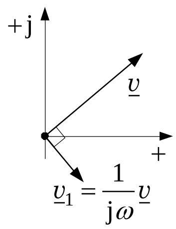
$$\underline{\upsilon}_1 = \frac{1}{\mathrm{j}\omega}\underline{\upsilon}\,.$$

Integrating over time in the domain of instantaneous values means, in a complex domain, dividing by $\mathrm{j}\omega$. For $1/\mathrm{j} = \mathrm{e}^{-\mathrm{j}\pi/2}$, the equation can also be represented as:

$$\hat{\upsilon}_1\mathrm{e}^{\mathrm{j}\varphi_{\mathrm{v}1}} = \frac{1}{\omega}\hat{\upsilon}\,\mathrm{e}^{\mathrm{j}(\varphi_{\mathrm{v}}-\pi/2)}\,.$$

The following is the relationship between the amplitudes $\Rightarrow$ $\hat{\upsilon}_1 = \frac{1}{\omega}\hat{\upsilon}$ and phase angles $\Rightarrow$ $\varphi_{\mathrm{v}1} = \varphi_{\mathrm{v}} - \pi/2$.

The integrated quantity lags the original quantity by 90°. Plot the corresponding equations on the phasor diagram of the integrated quantity $\underline{\upsilon}_1$ and the original quantity $\underline{\upsilon}$.



The feasibility of a computational operation in the complex domain fails in the case of multiplication of two sinusoidal quantities corresponding to the expression, as required in the case of determining the instantaneous power value.

The cause of the failure is conditional on the term:

$$\mathrm{Re}\left(\underline{\upsilon}_1\,\mathrm{e}^{\mathrm{j}\omega t}\right)\mathrm{Re}\left(\underline{\upsilon}_2\,\mathrm{e}^{\mathrm{j}\omega t}\right) \neq \mathrm{Re}\left(\underline{\upsilon}_1\,\mathrm{e}^{\mathrm{j}\omega t}\,\underline{\upsilon}_2\,\mathrm{e}^{\mathrm{j}\omega t}\right).$$

To determine the instantaneous value, we need to consider the individual instantaneous values.

Calculation of power

Calculating single-phase power in complex calculus

The power flowing into the terminals of the converter at the applied voltage $u = \sqrt{2}U\sin(\omega t + \varphi_{\mathrm{u}})$ and current $i = \sqrt{2}I\sin(\omega t + \varphi_{\mathrm{i}})$ is calculated using the equation:

$$p = u\,i = U\,I\cos(\varphi_{\mathrm{u}} - \varphi_{\mathrm{i}}) - U\,I\cos(2\omega t + \varphi_{\mathrm{u}} + \varphi_{\mathrm{i}})$$

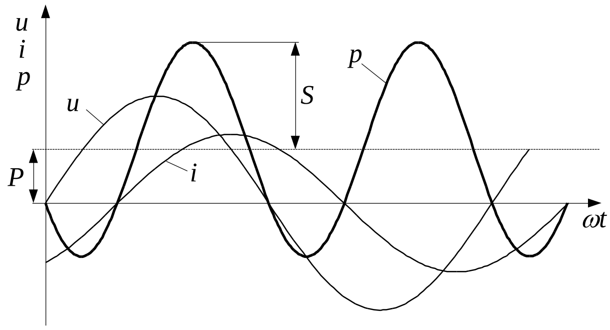
The power oscillates at twice the frequency of the voltage or current around the mean value

$$P = U\,I\cos\varphi$$

and is known as the working (watt) power. The phase angle between voltage and current is

$$\varphi = \varphi_{\mathrm{u}} - \varphi_{\mathrm{i}}\,.$$

The time waveform of power when the current and voltage are sinusoidal quantities of the same frequency is shown in the Figure for the case where the current lags voltage.



Without taking into account the phase shift of the current with respect to the voltage, we obtain an expression for the total or apparent power:

$$S = U\,I\,.$$

In addition to the total or apparent power, there is, quite formally, the reactive power, which is calculated as:

$$Q = U\,I\sin\varphi\,.$$

It is not possible to determine the instantaneous power value from the complex voltage and current, but the power components can be determined. Complex power $\underline{S}$ is introduced by multiplying the complex voltage by the conjugate complex current. This is the case:

$$\underline{S} = \underline{U}\,\underline{I}^{*} = U\,I\,\mathrm{e}^{\mathrm{j}(\omega t+\varphi_{\mathrm{u}})}\mathrm{e}^{-\mathrm{j}(\omega t+\varphi_{\mathrm{i}})} = U\,I\,\mathrm{e}^{\mathrm{j}(\varphi_{\mathrm{u}}-\varphi_{\mathrm{i}})} = U\,I\cos\varphi + \mathrm{j}U\,I\sin\varphi = P + \mathrm{j}Q\,.$$

It is necessary to emphasize that this complex power is a complex quantity of a different kind, and behind it is not subject to the basic equation.

Calculation of three-phase working power for different winding connections

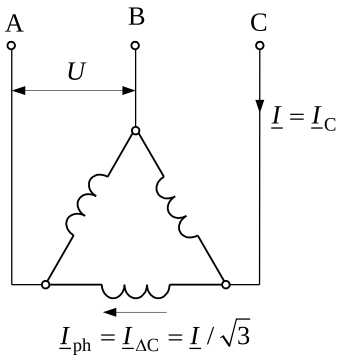
The three-phase (symmetrical) winding is connected in a delta or star connection. A zigzag or broken star is also used in transformers. In all three cases, the equation for the working power applies:

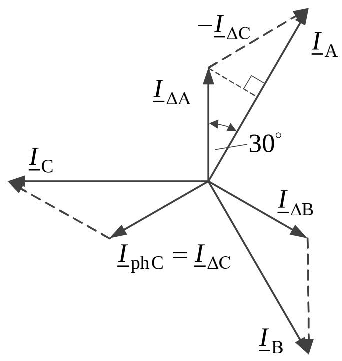
$$P = 3U_{\mathrm{ph}}\,I_{\mathrm{ph}}\cos\varphi\,,$$

where U_{ph} and I_{ph} are the phase quantities, i.e., the RMS values of the voltages and currents in the winding.

According to the Figure for a three-phase winding connected in a delta circuit, the currents in the terminals (lines) are considered to be the difference of two adjacent phase currents, depending on the sequence of the phases. Therefore, the equations for the currents in the supply lines apply:

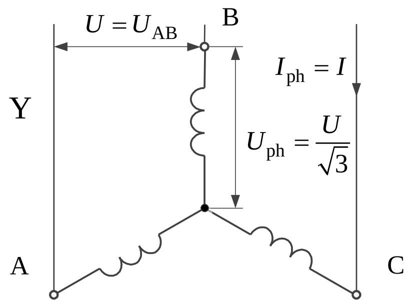
$\underline{I}_{\mathrm{A}} = \underline{I}_{\Delta\mathrm{A}} - \underline{I}_{\Delta\mathrm{C}} = 2\underline{I}_{\Delta\mathrm{A}}\cos 30^{\circ} = \underline{I}_{\Delta\mathrm{A}}\sqrt{3}$, $\underline{I}_{\mathrm{B}} = \underline{I}_{\Delta\mathrm{B}} - \underline{I}_{\Delta\mathrm{A}} = \underline{I}_{\Delta\mathrm{B}}\sqrt{3}$ and $\underline{I}_{\mathrm{C}} = \underline{I}_{\Delta\mathrm{C}} - \underline{I}_{\Delta\mathrm{B}} = \underline{I}_{\Delta\mathrm{C}}\sqrt{3}$.

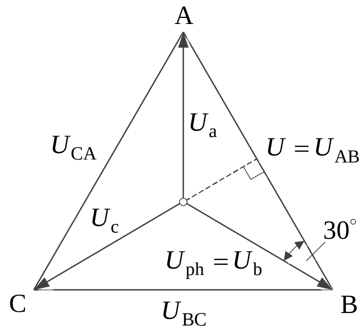




The equations apply to a symmetrical system where the currents in the individual phases are equal to each other: $I_{\mathrm{ph}} = |\underline{I}_{\Delta\mathrm{A}}| = |\underline{I}_{\Delta\mathrm{B}}| = |\underline{I}_{\Delta\mathrm{C}}|$ respectively $I_{\mathrm{ph}} = I/\sqrt{3}$.

In a delta connection, $U_{\mathrm{ph}} = U$ gives the equation for the working power in a three-phase system, expressed in terms of the effective magnitudes at the terminals: $P = \sqrt{3}\,U\,I\cos\varphi$





In a symmetrical star or zigzag connection, the relationship for the voltages is:

$$U = U_{\mathrm{AB}} = U_{\mathrm{BC}} = U_{\mathrm{CA}} = 2U_{\mathrm{ph}}\cos 30^\circ = U_{\mathrm{ph}}\sqrt{3} \text{ or } U_{\mathrm{ph}} = U/\sqrt{3}.$$

The currents in a star or zigzag connection are the same in the windings as in the supply $(I_{\mathrm{ph}} = I)$. Taking the values for the phase quantities gives the equation for the working power:

$$P = \sqrt{3}\,U\,I\cos\varphi.$$

We find that the working power in a three-phase system can also be calculated from the input (line) quantities using the same equation, i.e., independently of the type of winding connection.

Calculation of losses for different winding connections

Assume a symmetrical three-phase winding, connected in a delta, star or zigzag connection. The resistances are given between the supply terminals $R_{\mathrm{t}} = R_{\mathrm{AB}} = R_{\mathrm{BC}} = R_{\mathrm{CA}}$ and the current in the supply terminals $I = I_{\mathrm{A}} = I_{\mathrm{B}} = I_{\mathrm{C}}$. The winding losses are calculated using the equation $P_{\mathrm{Cu}} = 3I_{\mathrm{ph}}^2 R_{\mathrm{ph}}$, i.e., from the phase quantities. For a delta circuit, the phase currents are assumed to be given by the preceding derivations: $I_{\mathrm{ph}} = I/\sqrt{3}$.

The phase resistance is obtained from the resistance between the terminals with respect to the parallel connection of one phase and the two series connected resistances of the adjacent phases according to the equation:

$$\frac{1}{R_{AB}} = \frac{1}{R_{ph}} + \frac{1}{2R_{ph}} = \frac{3}{2R_{ph}} \rightarrow R_{ph} = \frac{3}{2} R_{AB}$$ and the losses for a delta connection are

$$P_{Cu} = 3I_{ph}^2 R_{ph} = 3\left(\frac{I}{\sqrt{3}}\right)^2 \frac{3}{2} R_{AB} = 1{,}5\, I^2 R_{AB} = 1{,}5\, I^2 R_t \, .$$

For a star or zigzag, $I_{ph} = I$ and $R_{ph} = R_{AB} / 2 = R_t / 2$ apply, and we calculate the losses using equation:

$$P_{Cu} = 3I_{ph}^2 R_{ph} = 3I^2 \frac{R_t}{2} = 1{,}5\, I^2 R_t \, .$$

The same equation (independent of the connection) is used to calculate the winding losses as a function of the input quantities.

The power losses in a three-phase winding in the complex region are calculated by multiplying the complex current (in the terminals) by the conjugate complex current value. That is:

$$P_{Cu} = 1{,}5\, \mathrm{Re}\left(\underline{I}\, \underline{I}^*\right) R_t \, .$$

The law of induction in complex calculus

The induced voltage in the turn is governed by Faraday's law of electromagnetic induction:

$$\underline{e} = -\underline{\phi}' = -\frac{\mathrm{d}\underline{\phi}}{\mathrm{d}t} \, .$$

As a rule of thumb, the magnetic field (or flux) is usually given as the "maximum", i.e., the peak value of the time sine function:

$$\underline{\phi} = \hat{\Phi} \mathrm{e}^{\mathrm{j}\omega t} \, .$$

Considering the differentiation − derivative equation, we derive:

$$\underline{e} = -\mathrm{j}\omega\, \underline{\phi} \, .$$

Flux multiplied by $-\mathrm{j}$, means that the induced voltage lags the flux by an angle of 90°. The effective value of the induced voltage of one turn (the turn voltage) will thus be:

$$E = \frac{\hat{E}}{\sqrt{2}} = \frac{\omega}{\sqrt{2}} \hat{\Phi} = \frac{2\pi}{\sqrt{2}} f \hat{\Phi} = 4 \frac{\pi / 2}{\sqrt{2}} f \hat{\Phi} = 4{,}44 f \hat{\Phi} = 1{,}11 \frac{2\hat{\Phi}}{T/2} = 1{,}11 \frac{\Delta \hat{\Phi}}{\Delta t} \, .$$

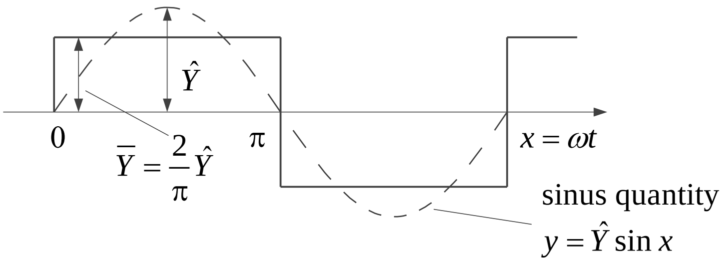
The induced voltage is equal to twice the change in flux over the half-period ($T / 2$).

In the equation $1{,}11 = \left(\pi / 2\right) / \sqrt{2} = \left(1 / (2 / \pi)\right) \cdot (1 / \sqrt{2})$ is the form factor for alternating quantities of the sinusoidal shape. The ratio of the mean value to the peak value is $2 / \pi$, $\sqrt{2}$ the ratio of the peak value to the RMS value. (Note: $\hat{\Phi}$ is the peak mean flux.)

Calculating the mean and RMS value of an alternating quantity

To calculate the mean value of an alternating quantity of sinusoidal shape (amplitude) for half the period (Figure), the equation is used:

$$\overline{Y} = \frac{1}{\pi}\int_0^{\pi} Y \sin x \,\mathrm{d}x = \frac{1}{\pi} Y(-\cos x)\Big|_0^{\pi} = \frac{2}{\pi} Y \; .$$



To calculate the RMS value of a sinusoidal quantity from the peak value, use the following derivation:

$$Y = \sqrt{\frac{1}{\pi}\int_0^{\pi} (\hat{Y}\sin x)^2 \mathrm{d}x} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} \hat{Y}^2 \left(1 - \cos(2x)\right) \mathrm{d}x} = \hat{Y}\sqrt{\frac{1}{2\pi}\left(x - \frac{1}{2}\sin(2x)\right)\Big|_0^{\pi}} = \frac{\hat{Y}}{\sqrt{2}} .$$

To add the RMS values of the sine magnitudes of the fundamental and higher order frequencies, we follow the rule:

$$Y = \sqrt{Y_1^2 + Y_3^2 + Y_5^2 + \ ......+ Y_\nu^2} \; .$$

In electromechanical converters, the order ν (usually) contains only odd (unpaired) higher harmonic components ($\nu = 1,\ 3,\ 5,\ ....\infty$), because the quantities are (usually) symmetrical with respect to the y-axis. Higher harmonic components occur, e.g., in the magnetizing current of an electromechanical converter as a result of saturation.

Note: To calculate the RMS value of the induced voltage, you would have to write the mean value of the flux (which is in fact the peak mean value) in the equation for the induced voltage in the form $\overline{\hat{\Phi}} = (2/\pi)\hat{\Phi}$. We do not write it this way, but omit the sign for the mean, and write only the sign for the amplitude or peak value of the flux amplitude. It is essential to emphasize the peak flux value, which is important for calculating the correct value of the magnetic flux density, and, hence, the excitation required.

It is also a rule that alternating electrical quantities (voltage, current, etc.) are given as RMS values, and magnetic quantities (flux, magnetic flux density, etc.) are given as peak or peak-mean values.

FOURIER ANALYSIS

Examples for excitation curves

In rotating electromechanical converters, a given periodic excitation curve $\theta = \mathrm{f}(x)$ must be replaced by a period $2\tau_\mathrm{p} \equiv 2\pi$ of exactly or approximately the trigonometric sum:

$$s_\mathrm{v}(x) = \frac{1}{2}a_0 + a_1\cos x + a_2\cos(2x) + \ldots + a_\mathrm{v}\cos(\nu x)$$
$$+ b_1\sin x + b_2\sin(2x) + \ldots + b_\mathrm{v}\sin(\nu x)\ .$$

The best approximation of $s_\mathrm{v}(x)$ to the curve $\mathrm{f}(x)$ is to choose for the coefficients a_v and b_v $(\nu = 0,\ 1,\ 2,\ \ldots)$ the Fourier coefficients of the given function:

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi}\mathrm{f}(x)\,\mathrm{d}x\,,$$

$$a_\mathrm{v} = \frac{1}{\pi}\int_0^{2\pi}\mathrm{f}(x)\cos(\nu x)\,\mathrm{d}x\,,$$

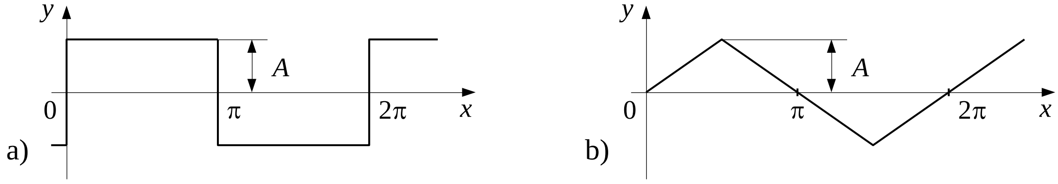
$$b_\mathrm{v} = \frac{1}{\pi}\int_0^{2\pi}\mathrm{f}(x)\sin(\nu x)\,\mathrm{d}x\,.$$

In the case of rotating electromechanical converters, the function $\mathrm{f}(x)$ is odd, i.e. $\mathrm{f}(-x) = -\mathrm{f}(x)$ (type II symmetry), and, in addition, symmetrical with respect to the x-axis $\mathrm{f}(x+\pi) = -\mathrm{f}(x)$ (type III symmetry), so this is a type IV symmetry. In this case, $a_\mathrm{k} = b_{2\mathrm{k}} = 0$ and

$$b_\mathrm{v} = \frac{4}{\pi}\int_0^{\pi/2}\mathrm{f}(x)\sin(\nu x)\,\mathrm{d}x\,.$$

$(\nu = 2k+1)$ and it is $k = 0,\ 1,\ 2,\ \ldots$. We see that only sinusoidal unpaired higher harmonic components are obtained.

In the following examples we will learn about some of the most typical excitation curve functions and their mathematical solutions. Figure a) shows the most typical excitation curve of a single coil with excitation amplitude $y = A$ for $0 \le x \le \pi$.



The evolution of the function into a trigonometric series for the first seven higher harmonic components is as follows:

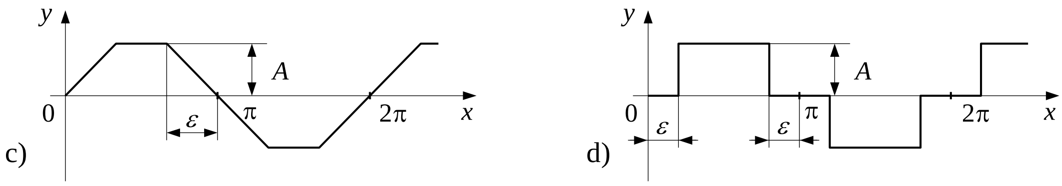
$$y = \frac{4}{\pi} A \left(\sin x + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x) + \frac{1}{7}\sin(7x) + \dots \right).$$

For an infinite number of coils, uniformly distributed around the circumference of the machine, with the excitation amplitude of $y = A$ for $-\pi/2 \le x \le \pi/2$ in Figure b), we obtain the solution:

$$y = \frac{8}{\pi^2} A \left(\sin x - \frac{1}{3^2}\sin(3x) + \frac{1}{5^2}\sin(5x) - \frac{1}{7^2}\sin(7x) \pm \dots \right).$$

In the case that a winding with an infinite number of coils is distributed over only part of the circumference of the converter and $y = A$ for $\varepsilon \le x \le \pi - \varepsilon$ (Figure c), the solution is:

$$y = \frac{4}{\pi}\frac{A}{\varepsilon} \left(\sin\varepsilon \sin x + \frac{1}{3^2}\sin(3\varepsilon)\sin(3x) + \frac{1}{5^2}\sin(5\varepsilon)\ \sin(5x) + \dots \right).$$



Normally, the winding is only distributed over $2/3$ of the circumference of the machine $(\varepsilon = \pi/3)$, giving the equation the form:

$$y = \frac{6\sqrt{3}}{\pi^2} A \left(\sin x - \frac{1}{5^2}\sin(5x) + \frac{1}{7^2}\sin(7x) \mp \dots \right).$$

All unpaired harmonic components divisible by three are dropped.

The last example in Figure d) shows the excitation of a coil for salient poles, where the pole width is narrower than the pole pitch and $y = A$ is by $\varepsilon \le x \le \pi - \varepsilon$. The solution is given by the equation:

$$y = \frac{4}{\pi} A \left(\cos\varepsilon \sin x + \frac{1}{3}\cos(3\varepsilon)\sin(3x) + \frac{1}{5}\cos(5\varepsilon)\ \sin(5x) + \dots \right).$$

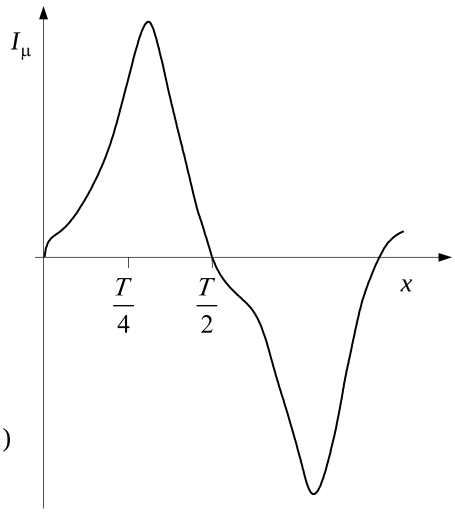
If the pole width is $2\pi/3$ and $\varepsilon = \pi/6$ (usually the pole width is slightly larger), the equation takes the form:

$$y = \frac{2\sqrt{3}}{\pi} A \left(\sin x - \frac{1}{5}\sin(5x) - \frac{1}{7}\sin(7x) \mp \dots \right).$$

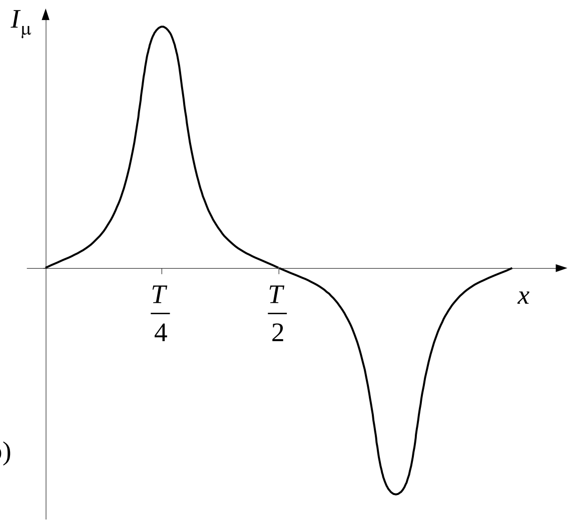
Example for transformer no-load current

The shape of the no-load current of a real transformer is shown in Figure a) (next page). Since the current is a periodic function of time with period T, the value on the abscissa axis is $x = t$. The magnetizing current is symmetrical in shape with respect to the x-axis and asymmetrical with respect to the y-axis. The function is odd because it is $f(-x) = -f(x)$, i.e. symmetry of the second kind and at the same time it is $f(x+T/2) = -f(x)$, i.e., symmetry of the third kind.

For this form of current, only unpaired higher harmonic components ($\nu = 2k+1$) are obtained for $k = 0, 1, 2, \ldots$. For respectively $k = 0$ or $\nu = 1$, i.e. for the harmonic component of the fundamental frequency, the transformer has the sine and cosine components of the current. One of the two components, the larger one, is the magnetizing current (the reactive component) and the other, the smaller one, is the losses in the iron (the working component). For all other higher harmonic components, according to the Figure, only the sinusoidal harmonic components are obtained, i.e. the magnetizing current of the higher harmonic components. For the case of a shift of the x coordinate system, only the cosine higher harmonic components are obtained, while the fundamental harmonic component of the current always has a sine and a cosine term.



a)



b)

If the effect of hysteresis loop losses and eddy current in the iron core can be neglected, the no-load current picture of a transformer normally operating in saturation, i.e. at the knee of the magnetic curve, is perfectly symmetrical. Such a simplified form of the current is shown in Figure b). The current is symmetrical with respect to the x and y axes, i.e. a type IV symmetry. For the magnetizing current in Figure b), we obtain only the cosine or only the sine of the fundamental component and the unpaired higher harmonic components of the transformer magnetizing current.

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LIST OF VARIABLES

a number of parallel branches p. 136; a (m) longitudinal dimension of brushes p. 148

A ($\mathrm{m^2}$) area or cross-section p. 2

b $\left(\mathrm{T = V \cdot s / m^2 = N / (A \cdot m)}\right)$ instantaneous value of the magnetic flux density p. 8

b (m) [11] width code p. 7 or pole arc (width) p. 135

B $\left(\mathrm{T = V \cdot s / m^2 = N / (A \cdot m)}\right)$ magnetic flux density p. 3

c $\left(\mathrm{J / (kg \cdot K}\right)$ specific heat p. 18; c design constant p. 99; c (Ω) motor constant p. 166

C $\left(\mathrm{W / (m^2 \cdot K^4)}\right)$ radiant constant p. 17; C $\left(\mathrm{N \cdot m/A \text{ or } V \cdot s/rad}\right)$ coefficient p. 136;

C $\left(\mathrm{F = A \cdot s / V}\right)$ capacitance p. 170

d mathematical operator of the derivative of a function (differential) p. 3; d (m) thickness p. 16

D (m) diameter (stator bores) p. 8; delta winding connection p. 54

e (V) induced voltage p. 11; $\mathrm{e} \approx 2.71828$ number of p. 18; e (%) ratio error p. 53

E (V / m) electric field intensity p. 11; E (V) induced voltage p. 15

f ($\mathrm{Hz = s^{-1}}$) frequency p. 1; f ratio − winding factor p. 8 and p. 65

F (A) magnetomotive force p. 3; F (N) force (in magnetic field) p. 12

H (A / m) magnetic field intensity p. 3

i (A) instantaneous value (of current) p. 6

I (A) RMS current p. 1; DC value p. 93

$\mathrm{j = e^{j\pi/2}}$ phase shift in the complex plane p. 25

J $\left(\mathrm{A / m^2}\right)$ or ($\mathrm{A / mm^2}$) current density p. 4; J $\left(\mathrm{kg \cdot m^2}\right)$ Moment of inertia p. 161

k factor p. 127; constant p. 136

K ratio (transformation ratio) p. 12; constant p. 99;

K number of commutator lamellas p. 132; K ($\mathrm{N \cdot m / A = V \cdot s}$)) coefficient p. 137

l (m) length p. 3, longitudinal (axial) length (machine package) p. 67

L ($\mathrm{H = \Omega \cdot s}$) inductance p. 3

m number of phases of AC machine winding p. 1; m (kg) mass p. 16

M ($\mathrm{N \cdot m}$) torque (force torque) p. 2

n ($\mathrm{s^{-1}}$) or ($\mathrm{min^{-1}}$) speed p. 2;

N number of turns p. 4

p number of pole pairs p. 8

P (W) working or loss power p. 1

q $(\mathrm{A \cdot s / m})$ line charge p. 12; q relative reactive power p 46;

q number of slots per pole and phase p. 62

Q $(\mathrm{A \cdot s})$ charge p. 12; Q number of slots p. 62; Q $(\mathrm{V \cdot A})$ reactive power p. 46

r relative resistance p. 46; r (m) radius p. 14 or radial dimension of brushes p. 148

R $\left(H^{-1} = (\Omega \cdot \mathrm{s})^{-1}\right)$ magnetic resistance p. 3; R (Ω) ohmic resistance p. 16

s slip (induction machine) p. 68

S $(\mathrm{V \cdot A})$ apparent power p. 50; S (m^2) area or cross-section p. 145

t (s) time p. 8; t (m) tangential dimension of brushes p. 148

T (s) repeat period p. 9, time constant p. 18

u (V) voltage p. 14; u relative voltage p. 46

U (V) RMS voltage value p. 1

v $(\mathrm{m / s})$ speed p. 9

V (m^3) volume p. 127; V (A) drop of excitation p. 140

w $(\mathrm{J/m}^3)$ specific magnetic energy p. 127

W (J) energy p. 13

x (m) coordinate (abscissa) p. 7; x relative value of reactance p. 46

X (Ω) inductive resistance − reactance p. 32

y actual to nominal value ratio factor p. 50

Y three-phase star winding connection p.54; Y width of winding coils p. 63

z number of conductors (coils) p. 136

Z (Ω) impedance (complex resistance) p. 40; Z three-phase zigzag winding connection p.54;

α (rad.) electrical or mechanical angle p. 8

α $\left(\mathrm{W / (m^2 \cdot K)}\right)$ coefficient (heat) p. 16, α transformation constant p. 85

β pole form factor p. 105, β (rad) electrical or mechanical angle p. 138

γ $(\mathrm{S \cdot m / mm^2})$ specific electrical conductivity p 30; γ (rad.) phase angle p. 87

δ (rad.) wheel or pole angle p. 99

δ (m) air gap p. 6; δ_e (m) equivalent air gap p. 7

Δ mathematical sign of the difference p. 17; delta winding connection p. 81

ε absorption ratio (heat) p. 17

η efficiency p. 2

ϑ (rad.) electric space angle p. 8; ϑ $(^\circ\mathrm{C})$ temperature p. 16

θ (A) instantaneous value of the magnetic voltage (excitation) p.8

Θ (A) magnetic voltage (excitation) p. 4; Θ (K) absolute temperature p. 17

κ magnetic susceptibility p. 4

λ $(\mathrm{W/(m \cdot K)})$ specific thermal conductivity p. 16

Λ $(\mathrm{H} = \Omega \cdot \mathrm{s})$ magnetic conductivity p. 3; Λ (W / K) thermal conductivity p. 16

μ relative permeability p. 4; $\mu_0 = 4 \cdot \pi \cdot 10^{-7}$ $(\mathrm{V \cdot s/(A \cdot m)})$ vacuum permeability

ν spatial order of higher harmonic components p. 38

ξ (degrees or minutes) angle error p. 53

$\pi = 3.1415926$ Ludolf's number p. 7

σ field leakage factor p. 86

Σ mathematical sum operator p. 4

τ (m) arc on the perimeter of the machine p. 7; τ (s) time constant p. 88

φ (rad.) phase angle or angle of displacement p. 25

ϕ $(\mathrm{Wb} = \mathrm{V \cdot s})$ magnetic flux (instantaneous value) − flux p. 11

Φ $(\mathrm{Wb} = \mathrm{V \cdot s})$ magnetic flux − flux p. 2; Φ_t (W) heat flow p. 16

ψ $(\mathrm{V \cdot s})$ magnetic linkage (instantaneous value) p. 12

Ψ $(\mathrm{V \cdot s})$ magnetic linkage (RMS value) p. 3

ω (rad./ s) or (s^{-1}) electrical angular frequency (circular speed) p. 8

Ω (rad./ s) ali (s^{-1}) mechanical angular velocity (circular velocity) p. 2

Greek alphabet

α, A	alpha	ι, I	iota	ρ, P	rho
β, B	beta	κ, K	kappa	σ, ς, Σ	sigma
γ, Γ	gamma	λ, Λ	lambda	τ, T	tau
δ, Δ	delta	μ, M	mu	υ, Υ	upsilon
ε, E	epsilon	ν, N	nu	φ, ϕ, Φ	phi
ζ, Z	zeta	ξ, Ξ	xi	χ, X	chi
η, H	eta	o, O	omicron	ψ, Ψ	psi
ϑ, θ, Θ	theta	π, Π	pi	ω, Ω	omega

Abbreviations

AC alternating current, DC direct current, IM induction machine, *MMF* magnetomotive force, MMF magnetomotive field, *EMF* electromotive force, NZ neutral zone, RMS root mean square

Electrical and Electromechanical Converters: Lecture Notations

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DOI
https://doi.org/10.18690/um.feri.1.2025

ISBN
978-961-286-945-8

This publication presents the basics of electromechanical conversion and the four basic electrical machines: the transformer, the induction machine, the synchronous machine and the commutator machine. The publication is divided into five chapters. The content of each chapter is presented below. Introduction: magnetic field, excitation of windings, induction of voltage, forces and torque, conversion of electrical power into electrical or mechanical power, losses, efficiency, heating and cooling. Transformer: construction elements, ideal and real single-phase transformer, three-phase transformer, special transformer designs. Induction machine: description of construction with windings and mode of operation, starting motors and varying speed and torque, induction generator, single-phase induction motors. Synchronous machine: description of construction and operation, operation on a rigid grid, approximate treatment of a saturated machine, excitation systems and the use of permanent magnets for excitation, and permanent magnet synchronous motors. Commutator machine: description of the construction with windings and mode of operation, armature reaction, commutation problems, characteristics of different stator-rotor winding connections, variation of rotational speed, AC (universal) commutator machines.

Keywords:
fundamentals of electromagnetics,
transformer,
induction machine,
synchronous machine,
commutator machine

University of Maribor Press

University of Maribor

Faculty of Electrical Engineering
and Computer Science