PRIMARY PRESERVICE TEACHERS' DRAWINGS OF NUMBER-BASED TWO-DIGIT ADDITION AND SUBTRACTION

DOI
https://doi.org/
10.18690/um.pef.2.2024.17

ISBN
978-961-286-899-4

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Whole number addition and subtraction are required topics in early-grade mathematics curricula. This study, with a sample size of N=117, introduces a unique method of using self-designed drawings to explore visual representations of mathematical concepts. The collected drawings show how preservice teachers communicate mathematical ideas, and reveal insights into their grasp of the place value concept and potential addition/subtraction teaching visual representations. Results showed that even though a significant portion of the participants demonstrated a clear comprehension of how to teach key mathematical concepts, a prominent trend emerged: about one-third of preservice teachers did not employ base ten grouping in their drawings of two-digit numbers. Similarly, one-third illustrated arithmetic operations symbolically, merely converting numbers into iconic forms, and additionally often misrepresented subtraction. These findings point to specific areas where preservice teachers' meta-representational abilities could be improved. By emphasizing and strengthening these areas in teacher education programs, the pedagogical skills of future educators can be enhanced.

Keywords:
representations,
metavisualisation,
addition,
subtraction,
two-digit numbers

University of Maribor Press

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Ključne besede;
ponazoritev,
metavizualizacijska,
seštevanje,
odštevanje,
dvomestno število

Univerzitetna založba
Univerze v Mariboru

RISBE ŠTUDENTOV RAZREDNEGA POUKA ZA DVOMESTNO SEŠTEVANJE IN ODŠTEVANJE

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Seštevanje in odštevanje števil do 100 je obvezna vsebina v učnih načrtih za matematiko v prvem triletju. Naša študija z vzorcem N=117 uvaja edinstveno metodo uporabe lastnih risb za raziskovanje vizualnih predstavitev matematičnih konceptov. Risbe pokažejo, kako bodoči učitelji razrednega pouka sporočajo matematične ideje, in razkrijejo vpogled v njihovo razumevanje koncepta mestne vrednosti in morebitnih vizualnih predstavitev za poučevanje seštevanja/odštevanja. Rezultati kažejo, da čeprav je velik del udeležencev pokazal jasno razumevanje, kako poučevati ključne matematične koncepte, se je pokazal pomemben trend: približno tretjina udeležencev na svojih risbah dvomestnih števil ni uporabljala grupiranja po deset. Podobno je ena tretjina aritmetične operacije ponazorila simbolno, kjer so števila zgolj pretvorili v slikovne oblike, pogosto pa so tudi napačno prikazali odštevanje. Te ugotovitve kažejo na specifična področja, kjer bi lahko izboljšali metavizualizacijske zmožnosti bodočih učiteljev. S poudarjanjem in krepitvijo teh področij v programih za izobraževanje učiteljev lahko izboljšamo pedagoške spretnosti bodočih učiteljev.

1 Introduction

Methods of addition and subtraction that emphasize flexible decomposition and combination of numbers, referred to as number-based algorithms or strategies, bolster students' comprehensive grasp of numbers. In everyday scenarios, these computational approaches can often be done mentally. Research on children's usage of strategies in the additive realm indicates that children efficiently and adaptively employ a range of number-based strategies before being introduced to digit-based algorithms (Hickendorff et al., 2019). On the other hand, teacher knowledge is undoubtedly crucial in the challenging and complex process of teaching mathematics. Teacher training programs therefore increasingly promote teachers' mathematical knowledge for teaching that can help preservice teachers (PTs) better prepare for future teaching in multi-number addition and subtraction (Fuson, 1992). To effectively guide PTs in developing this knowledge, educators should understand and expand upon the PTs' foundational conceptions, which can occasionally be elusive (Thanheiser, 2009). Primary PTs come to their mathematics courses fluent in using digit-based algorithms for adding and subtracting multidigit whole numbers, but many are unaware of the essential features inherent in understanding the base-ten place-value system (i.e., grouping, place value, base) that are used for number-based algorithms. Understanding these features is crucial to understanding and teaching multi-digit addition and subtraction. To help PTs develop such knowledge, educators need to know and build upon the PTs' initial conceptions (Thanheiser, 2009). Research on children's conceptions of multidigit whole numbers and computations can serve as a starting point for investigating PTs' conceptions. Two main conceptual aspects of numbers have been identified in literature: understanding of the underlying structure of powers and understanding of how a number can be grouped and regrouped (Valeras & Becker, 1997). This study focuses on the first one.

Employing the drawing-based research method, like other approaches, for example children development tests that include drawings of human figures, has proven valuable in various domains. We posit that drawings created by teachers can serve as indicators of the visual aids they might utilize in instruction. This approach therefore particularly aims to examine teachers' meta-representational (metavisualisation) competence in these mathematical areas. By exploring these aspects, this study seeks to bridge the existing gap in the literature, particularly enhancing the understanding of visualisation theory in the context of mathematics education, with a focus on two-digit numbers and basic arithmetic operations. By addressing these research inquiries, this study aims to bridge the existing gap in literature concerning visualization theory within mathematics education.

2 Theoretical Background

Strategies for multi-digit arithmetic differ from those for single-digit arithmetic. In single-digit arithmetic, an important distinction is between computational strategies and retrieval. By contrast, in multi-digit arithmetic retrieval of the outcome as an arithmetic fact is not feasible: the outcome needs to be computed. Hence, in multi-digit arithmetic the question is how the numbers are manipulated to find the answer. This is called a (solution) strategy. School children's strategy use in multi-digit arithmetic is a well-researched domain with two subdomains – number-based strategies and digit-based strategies (Torbeyns et al., 2017). In the realm of multi-digit arithmetic strategies, a fundamental distinction lies in the approach taken towards the manipulation of numbers, particularly in relation to the preservation or disregard of their place values. This critical differentiation culminates in two primary categories of strategies: number-based strategies and digit-based strategies (Verschaffel et al., 2007). The most prevalent digit-based strategies encompass written algorithms for operations such as long addition, subtraction, multiplication, and division, which systematically engage with individual digits, typically proceeding from right to left. In contrast, number-based strategies encompass calculation methods that operate on the numeric values of integers within the problem, drawing upon a profound understanding of the fundamental characteristics of the number system and arithmetic operations, a refined numerical intuition, and a solid grasp of elementary number facts. The distinction between the digit-based algorithm, often referred to as traditional, vertical, or operational algorithms, and the number-based approach, sometimes denoted as mental computations, horizontal, or conceptual algorithms, revolves around the treatment of place values within the numbers under consideration. In digit-based algorithms, place values are disregarded (e.g., in traditional or vertical algorithms, 57 - 34 is simplified to 5 - 3 and 7 - 4). Conversely, the number-based approach meticulously respects the place value, subtracting 34 from 57 while employing distinct strategies. The introduction of a digit-based algorithm marks a significant juncture in the realm of multi-digit arithmetic. Prior to its introduction, children typically rely consistently on number-based strategies.

However, following the introduction of digit-based strategies, children tend to heavily favour the utilization of these digit-based algorithms (Torbeyns & Verschaffel, 2016). Nevertheless, the outcomes concerning the efficiency of number-based versus digit-based algorithms exhibit a degree of inconsistency in the existing literature (Torbeyns et al., 2017; Hickendorff et al., 2019). In Slovenian mathematics textbooks, the number-based approach is instructed from second grade onward, and it is not before the end of the third grade that the digit-based is instructed.

Research indicates that problems centred around addition and subtraction provide an effective context for learning place value concepts (Carpenter et al., 1998). Place value is therefore not only fundamental for computation but also aids learners in understanding the concept as they explore their computing methods (Ebby et al., 2020). Although results differ depending on what and how manipulatives are used in learning situations, learning with manipulatives is correlated positively with later development of mental mathematics achievement, and understanding (Sowell, 1989). For example, Dienes base-10 blocks (a popular mathematics manipulative that contains small units for ones, thin rods for tens, ten-by-ten flats for hundreds, and a large ten-by-ten-by-ten block for thousands place values) improves students' conceptual understanding of arithmetic operations. Manipulatives reinforce understanding of the concept of place value. They help learners grasp the notion of "a ten" as both a single entity and a collection of ten individual units. The models used must be proportional, meaning the representation for ten should be physically ten times larger than that of a single unit (Trimurtini et al., 2019). Non-proportional models, such as those where the representation of ten is not physically larger (like money or chips assigned different values based on colour), are not ideal for introducing the concept of place value. One of the often-used non-proportional manipulatives is a place-value abacus consisting of beads of the same size on vertical wires, where the number of beads on most right wire represents ones, the number of the beads on the next left wire represents the number of tens and so on. Trimurtini et al. (2019) found that 2nd graders who used Dienes cubes (a proportional manipulative) performed statistically better than those who utilised place-value abacus.

In 1986, Shulman emphasized that effective teaching requires more than just subject knowledge. He introduced the concept of pedagogical content knowledge (PCK), which involves understanding how to present content to students appropriately. Later, Ball et al. (2008) expanded on this idea in the context of mathematics, delineating mathematical knowledge for teaching (MKT) into two main components: subject matter knowledge (SMK) and PCK. PCK is further divided into three subcategories: knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC). KCS involves understanding how students interact with content, including their typical errors and misconceptions. KCT focuses on selecting appropriate teaching methods, tools, and tasks that align with the content. KCC relates to understanding how the content fits within the broader curriculum, including its connections with other subjects.

SMK encompasses common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge (HCK). CCK is the mathematical knowledge shared by most educated people, used for performing standard mathematical operations. SCK, however, is unique to teaching; it includes an understanding of the mathematical concepts' background and how to effectively teach them. For example, in teaching place value, a mathematics teacher should not only know how to use manipulatives to represent place value but also understand the conceptual underpinnings of regrouping and how to evaluate students' understanding. The boundaries between these knowledge categories are fluid and change with a teacher's professional development. A teacher's ability to adapt their teaching based on students' responses or new educational contexts reflects their proficiency in SCK and KCS. Additionally, understanding how different subjects interconnect within the curriculum is crucial, as highlighted by HCK and KCC.

Sun et al. (2019) emphasised the importance of teachers' PCK in whole number addition and subtraction for improving student understanding. Several studies have examined the context of PTs' PCK about multi-digit computations. There is a consistent pattern showing the relationship between teacher knowledge and teaching strategies. Specifically, Özel et al. (2022) reported that PTs justified student solutions based on operational knowledge (traditional algorithms) rather than the conceptual understanding (e. g. number-based algorithms) of multiplication. Verzosa (2020) found that over 90% of the reasoning by Philippine PTs was based mainly on rules and procedures, without linking to the quantities in the problem. Kalinec-Craig et al. (2019) report that an overwhelming majority of the PTs used a digits strategy for two-digit subtraction, similar results were found by McClain (2009). Thanheiser (2009), who examined PTs' explanations of standard algorithms for multi-digit computations, reported that two-thirds of the PTs' in her study showed a lack of understanding about digit-based algorithms. In the USA, Son (2016) reported that one fourth of PTs did initially not recognise students number-based strategy as a legitimate method for subtracting. In summary, a prevalent trend among PTs is emphasising operational over conceptual knowledge, which has implications for teaching and student understanding in multi-digit computations. Different methodologies have been used to reach PTs' mathematical knowledge for teaching multidigit computations. Özel et al. (2022) selected problem posing and justification of students' solutions, Verzosa (2020) used interviews in which PTs were asked to describe how they would introduce the concept of multidigit subtraction to their students. McClain (2009) engaged PTs in activities from an instructional sequence designed to support conceptual understanding of both place value and multidigit addition and subtraction and analysed PTs' learning trajectory. Son (2016) analysed PTs' reasoning and responses to students' informal and formal strategies.

This study posits that drawings can serve as a strategy for solving problems and provide insight into the learners' understanding of the mathematical concept (Verschaffel et al., 2020). Selecting efficient drawings for teaching can be challenging for teachers and researchers, as highlighted by diSessa (2004, pp. 293). DiSessa introduced the concept of meta-representational competence, which involves creating and evaluating new representations, understanding their purposes and effectiveness, and explaining their use. This skill, initially studied in learners, is particularly vital for STEM (Science, Technology, Engineering and Mathematics) teachers who must constantly choose effective representations for teaching. These competencies, also known as metavisualisation competences (Chang et al., 2023), form a part of the knowledge of teaching (KCT), emphasizing the selection of adequate and effective presentations in teaching contexts.

Presmeg (2014) stated several questions that need to be addressed in this research domain and pointed out that an overarching visualisation theory in mathematics education has not yet been established. Drawings can be seen as representations or as processes of meaning-making. In this study, the first approach was adopted. It was assumed that there is a matching representation for the underlying concept; accordingly, there is a corresponding (matching, adequate, appropriate) drawing for a given concept. The line of research by Jitendra and colleagues (e.g., Jitendra & Hoff, 1996) demonstrated the effectiveness of the approach where children were identifying, drawing, and completing a teacher-imposed schematic drawing in a wide variety of types of word problems, age levels, and target groups. Since drawings are usually chosen by teachers it is especially important to know which drawings PTs produce.

This study aimed to fill the gap in literature regarding PTs' mathematical knowledge for teaching number-based algorithms. However, an innovative strategy based on drawing as a research method that was not used so far, was used in this study. Additionally, this study posits that teacher-generated drawings are a good indicator of visual representations that teachers will use in teaching. Models (including visual representations) used in teaching mathematics are key predictors of students' mathematics achievements (Presmeg, 2014). Answering research questions arising from this research problem therefore addresses the gap in the literature regarding visualisation theory in mathematics education.

The research questions of the study were to find out:

How do primary preservice teachers represent two-digit numbers?

How do primary preservice teachers represent addition and subtraction with two-digit numbers?

3 Methodology

The study utilized a drawing-based approach to probe the mathematical understanding of PTs at the University of Maribor. The participants in the study were PTs in the 3rd and 4th-year of the Primary School Teaching program at the Faculty of Education, University of Maribor (N=117). The sample was chosen through convenience sampling. Data were collected through an anonymous questionnaire, the results of which did not affect PCTs' grades in any way. The data collection occurred in the year 2022. Participants were provided with very scant instruction, namely to: Draw pictures representing: (a) $25 + 37$, (b) $45 - 17$.

Arithmetic expressions were written horizontally since writing arithmetic computation problems horizontally instead of vertically prompts learners to rely less on standard algorithms (Humphreys & Parker, 2015). The data analysis was conducted in three distinct phases. In the initial stage, inappropriate drawings were filtered out, setting the groundwork for a systematic categorization. This categorization hinged on the portrayal of two-digit numbers and the representation of mathematical operations. Following the qualitative analysis, quantitative analysis methods were applied. In the continuation of this section, a more detailed exploration of each of these phases is provided.

In the study's initial phase, drawings were analysed for their appropriateness in representing mathematical concepts. Only drawings that meaningfully depicted both numbers in a mathematical expression, rather than just the result, were selected for further analysis. Drawings that either failed to present the given numbers, showed only the computation or result, or included figures unrelated to the mathematical expression, were deemed inappropriate and excluded. Figure 1 in the study illustrates examples of such inappropriate drawings.

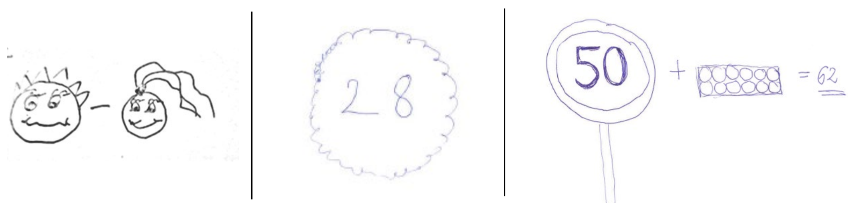

Figure 1: Three examples of inappropriate drawings: the first one contains only a figure unrelated to mathematical concepts, the second one presents the result of a given mathematical expression, while the third one depicts a non-initial one step in the computation of a given addition expression.

For each appropriate drawing the following aspects were examined: the presentation of two-digit numbers; the presentation of mathematical operation.

Hence, in the second phase of this research we looked for codes through a systematic coding process; then these codes were combined into categories (types of drawings). Precise criteria were established for categorising participants' drawings into a particular code through coding. The presentation of findings is illustrated with concrete examples of participants' drawings to strengthen the validity and reliability of the results.

From the perspective of representing two-digit numbers, the first step involved determining the symbolic nature of each representation. A presentation of a number is designated as a symbolic representation when a given number is written with a symbol, and there are no additional reasonable presentations representing numbers (not symbols). In Figure 2, three examples of symbolic and non-symbolic representations of numbers are shown.

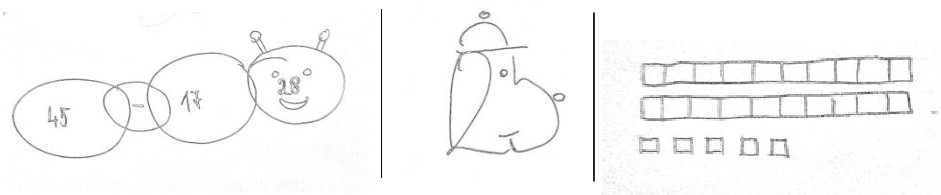

Figure 2: A symbolic representation of numbers 45 and 17 (left figure), a symbolic representation of a number 25 integrated into a depiction of a dog (second figure), and non-symbolic representation of number 25 (right figure).

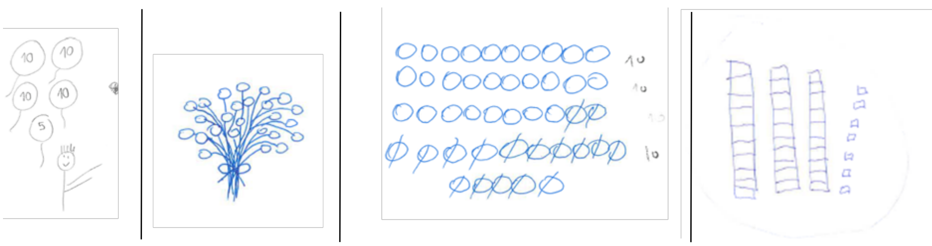

Figure 3: A non-proportional model for presenting the number 45, a proportional non-grouped model for presenting the number 25, a proportional model for presenting the number 45 (grouped model with implicit grouping), and a proportional model for 37 (grouped model with explicit grouping).

Further, for each non-symbolic representation it was ascertained whether the model for each two-digit number was proportional or non-proportional. If a model was proportional, we checked whether the elements representing numbers were grouped or not. As a grouped proportional model, we considered representations of numbers using elements that were either explicitly (the group is illustrated) or implicitly (the group is not illustrated, but it is evident from the image) grouped together into the groups with the same number of elements. Examples of each mentioned category are shown in Figure 3. In cases where a grouped proportional model was employed, the number of elements within these groups was evaluated.

In the realm of representing mathematical operations, the focus of this study extended to determining whether the drawings presented a symbolic or non-symbolic representation of a given operation. More precisely, an investigation was conducted to ascertain whether the drawings included symbols for addition (i.e., the symbol "+") and subtraction (i.e., the symbol "-"). Furthermore, our emphasis lay in determining whether the generated drawings correspond to the underlying concept of addition and subtraction of two-digit numbers. Note that the (separate, distinct) presentation of both given numbers from an expression without the inclusion of a symbol for the given operation between them is consistent with the concept of addition, but not with the concept of subtraction. Consequently, each such representation is considered as appropriate in the case of addition, but inappropriate for subtraction. Furthermore, each representation of an operation that includes a symbol for the operation is deemed appropriate from the perspective of operations. Figures 4 and 5 illustrate instances of both of symbolic and non-symbolic representations of mathematical operations.

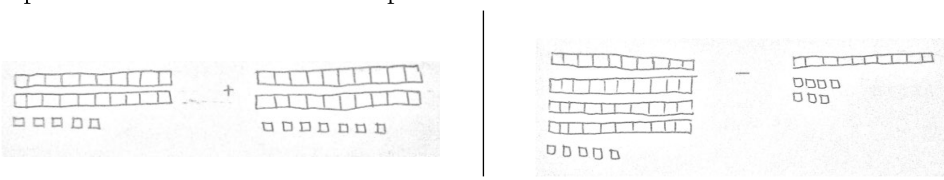

Figure 4: Symbolical representations of addition and subtraction (including symbol "+" or "-") corresponding to the underlying concept.

The left and right drawings in Figure 4 show the interlacing of the addition (+) and subtraction (-) symbols and the iconic representations of the numbers over which the two operations are performed. The symbolic notation of the numbers is only translated into iconic form (squares), the context of the mathematical operation (addition or subtraction) is still encoded in the symbol.

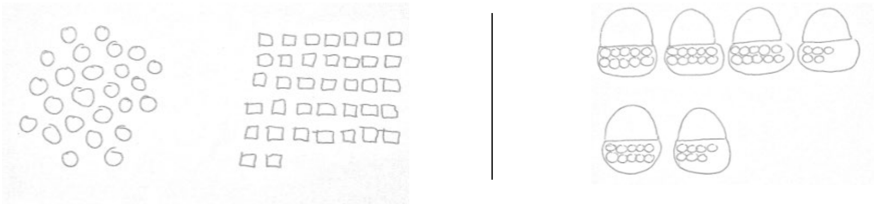

Figure 5: Representations of addition and subtraction without usage of symbols "+" or "-": the first one represents the expression 25+37 and is appropriate, while the second one depicts the expression 45-17 (35-17) and is not appropriate.

The left and right drawings in Figure 5 show addition and subtraction without the use of the + and - symbols. The addition results in a correct representation of the part-part-whole structure. In the case of subtraction, however, the drawing obtained by translating only the minuend and subtrahend into iconic form (circles) is not correct, as it is usually interpreted as the addition of the minuend and subtrahend.

To ensure the accuracy of the analysis in both phases of the study, three researchers conducted the coding, resolving any discrepancies through discussion and re-evaluation. The study acknowledges potential limitations in using drawings as data, as highlighted in existing literature (Pehkonen et al., 2016). A key limitation was that participants were not asked to explain their drawings, possibly leading to different interpretations of their meaning compared to the authors' inferences. This contrasts with Badillo et al.'s (2015) study where children verbally explained their drawings.

4 Results

Among all the drawings, there were eight inappropriate drawings representing addition expressions and eleven drawings representing subtraction expressions. Several of them solely depicted the result of the expressions or their computations, while others included only graphical representations devoid of numerical values. In the continuation of this section, only appropriate drawings are considered. The presented findings are related to the presentation of two-digit numbers and the presentation of mathematical operations.

From the perspective of the presentation of two-digit numbers, the initial inquiry of this study focused on whether the research participants employed symbolical or non-symbolical representations of numbers. Four drawings from this study portray addition expressions and six drawings depict subtraction expressions, which contain only a symbolical representation of numbers without the iconic presentation. Further, in instances of non-symbolic representations of numbers, the utilization of proportional or non-proportional models was investigated. As indicated in Table 1, more than 94% of the students used a proportional model for representing numbers in addition expression. In the case of subtraction, the proportion of students employing a proportional model was 91% or more.

Table 1: Usage of a proportional model.

Usage of a proportional model	25 (+) f (f %)	37 (+) f (f %)	45 (-) f (f %)	17 (-) f (f %)
No	6 (5,7)	5 (4,8)	9 (9)	8 (8)
Yes	99 (94,3)	100 (95,2)	91 (91)	92 (92)
Total	105 (100)	105 (100)	100 (100)	100 (100)

The next step was to determine whether the elements representing numbers, when the proportional model was utilized, were grouped together. Results are presented in Table 2.

Table 2: Grouping in the representations, using the proportional model.

Grouped model	25 (+) f (f %)	37 (+) f (f %)	45 (-) f (f %)	17 (-) f (f %)
Yes	65 (65,7)	68 (68)	63 (69,2)	63 (68,5)
No	34 (34,3)	32 (32)	28 (30,8)	29 (31,5)
Total	99 (100)	100 (100)	91 (100)	92 (100)

In instances where groupable proportional models were employed, the quantity of elements within these groups was analysed. As shown in Table 3, the majority of students who used a groupable proportional model formed groups of ten elements. Note that in this table only the results for the cases of groups are presented, when at least three students represent them.

Table 3: The number of elements in group when some groupable proportional model was used.

The number of elements in a group	25 (+) f (f %)	37 (+) f (f %)	45 (-) f (f %)	17 (-) f (f %)
5	17 (26,2)	7 (10,3)	9 (14,3)	9 (14,3)
6	0 (0)	4 (5,9)	0 (0)	1 (1,6)
7	1 (1,5)	2 (2,9)	0 (0)	1 (1,6)
9	0 (0)	0 (0)	3 (4,8)	3 (4,8)
10	46 (70,7)	49 (72,1)	43 (68,3)	42 (66,7)
11	0 (0)	0 (0)	2 (3,2)	2 (3,2)
12	0 (0)	0 (0)	2 (3,2)	1 (1,6)
20	0 (0)	0 (0)	3 (4,8)	3 (4,8)
Total	65	68	63	63

In Figure 6 some examples of grouping into groups of order five, seven and ten are shown.

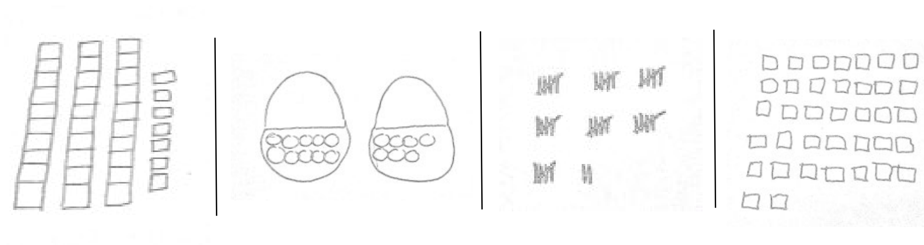

Figure 6: Grouping of elements into groups of order ten (first and second figure), five (third figure) and seven (fourth figure).

Next, the results pertaining to the presentation of mathematical operations are presented. Results showed that 40 (38,1%) representations for addition expressions are symbolic (include a symbol for addition, i.e., the symbol "+"). Similarly, 34 (34%) of representations for subtraction expressions that contained a symbol for subtraction (i.e., the symbol "-"). Recall that each representation of an operation that includes a symbol "+" or "-" is appropriate.

As mentioned, the separate presentation of both given numbers from an expression without the inclusion of a symbol for the given operation between them aligns with the concept of addition, but not with the concept of subtraction. There are 65 (61,9%) representations for addition that do not include the symbol "+" and are thus considered appropriate. In contrast, there is 13 (13%) representations for subtraction that present both numbers from the expression but omit the symbol "-" and are therefore deemed inappropriate. The described results are summarized in Table 4.

Table 4: The usage of symbols "+" and "-".

	Addition: usage of symbol »+« f (f %)	Subtraction: usage of symbol »-« f (f %)
No	65 (61,9)	66 (66)
Appropriate	65 (61,9)	53 (53,0)
Inappropriate	0 (0)	13 (13,0)
Yes	40 (38,1)	34 (34,0)
Appropriate	40 (38,1)	34 (34,0)
Inappropriate	0 (0)	0 (0)
Total	105 (100)	100 (100)
Appropriate	105 (100)	87 (87,0)
Inappropriate	0 (0)	13 (13,0)

5 Discussion

The first aim of this study was to identify how PTs' drawings depicted multidigit numbers. In the initial analysis, an evaluation was made of whether PTs opted for a proportional model in their representations. As highlighted in Table 1, an overwhelming majority, over 90% of PTs, chose to use a proportional model for illustrating numbers in addition and subtraction tasks. This suggests that the PTs possess a solid grasp of effective representations suitable for young learners. Several studies, including those by Trimurtini et al. (2019) and Rojo et al. (2021), have reported that proportional models are superior in efficacy compared to non-proportional models. Specifically, Rojo et al. (2021) argued that models which adhere to a physical base-10 proportion, such as proportional models, are optimal for teaching the inherent multiplicative patterns found in place value, contrasting non-proportional models like place value chips which don't maintain this proportion. Slovenian future primary teachers seem well-aware of this distinction, suggesting that, from the standpoint of choosing between proportional and non-proportional models, their SCK regarding manipulatives appears to be robust.

The second important characteristics of models is whether they are pregrouped. Approximately one third of students (see Table 2) did not use groups in their pictures. Base ten models can be broadly classified into two categories: groupable (like bundles of tens and ones, or cups of tens and ones) and pregrouped (for instance, Dienes blocks). The pregrouped models are frequently depicted in textbooks and often employed in instructional activities (Rojo et al., 2021), however it seems that using groupable manipulatives maintains germane cognitive load where new information is processed into long-term memory (Sweller et al., 1998). It could be that one third of PTs' who did not use groups thought that this type of manipulative would benefit children more, however this is highly unlikely. A more plausible explanation is that these PTs do not have well-developed SCK regarding place value.

The third characteristic examined was the use of groupings by tens. Surprisingly, approximately one third (see Table 3) of the students who presented groupable proportional models did not employ groupings of ten in their drawings. Previous research, such as that by McClain (2009), highlighted that PTs often struggle with the place value of multi-digit numbers, especially when considering addition or subtraction in number bases other than ten. In Slovenia, recommendations by McClain (2009) regarding activities in number bases different from ten have been integrated into mathematical courses, emphasizing the efficiency of the 10-base system. Consequently, it was anticipated that a larger portion of PTs would instinctively use groups of ten. Dishearteningly, the findings of this study resonate with several studies suggesting PTs' challenges with comprehensively understanding the place value of whole numbers. Thanheiser (2009) noted that PTs frequently miss the relationship between different unit types (such as ones, tens, hundreds). Consistently, in our data, among students who presented groupable proportional models there are just slightly over 60% of them effectively represented numbers using groups of ten. This result further indicates that their SCK for teaching place-value may not be sufficiently robust.

The following section of the discussion focuses on the results concerning arithmetic operations, namely addition and subtraction. This combination of representing numbers visually as objects while also interspersing them with arithmetic symbols (compare right drawing on Figure 1) has parallels in earlier studies. For example, both children's conceptual renderings (e.g., Lipovec, 2023; Lipovec & Podgoršek Mesarec, 2017) and those of preservice primary educators (e.g., Lipovec & Antolin Drešar, 2015; Lipovec & Podgoršek Mesarec, 2016) have shown similar patterns. The data collected in this study showed that around a third of PTs' drawings (as referenced in Table 4) incorporated symbols for addition or subtraction. Fuson (1992) also observed a similar trend in children's drawings (as depicted in Figure 5 of Fuson's study) but did not delve deeper into the implications of this observation. What stands out about this type of representation is that it does not neatly fit into the categorization set forth by Hegarty and Kozhevnikov (1999). According to their classification, drawings are typically either "pictorial" (i.e., realistic visualizations of problem objects), or "schematic" (i.e., illustrations emphasizing the essential spatial relationships of a problem). Of these, Hegarty and Kozhevnikov (1999) noted that employing schematic representations tends to bolster success in mathematical problem-solving. The observed "interplay" (combining symbolical signs "+" and "-" with pictorial drawings of objects representing numbers in arithmetic expressions 25+37 and 45-17) in this study hints at a nuanced middle ground, underscoring the evolving complexities of visual mathematical representation. There were around 48% of participants using interplay in addition and 34% of PTs for subtraction (see Table 4). Those participants exhibited weaker SCK regarding the representations of addition and subtraction for younger students. It is likely that such PTs might not opt for schematic drawings when teaching, which, as noted by Hegarty and Kozhevnikov (1999), can be more beneficial for mathematical problem-solving. This deficiency in SCK could have implications for their effectiveness in conveying mathematical concepts to students.

It is noteworthy that almost a half of the primary PTs, 47%, chose to depict the subtraction of 45-17 by illustrating 45 objects followed by an additional 17 objects. Note that if such depiction contains a sign for subtraction, then it is appropriate (34% of depictions), otherwise it is inappropriate (13% of depictions). Traditionally, one would represent subtraction by drawing 45 objects and then crossing out 17 of them. This often-used unconventional method suggests a potential gap or misunderstanding in their foundational comprehension of subtraction. These observations further underscore concerns about some PTs having a weaker SCK when it comes to basic arithmetic operations.

6 Conclusion

Teachers' ability to choose effective visual methods for teaching mathematical concepts (teachers' meta-representational skills) was highlighted in the study through PTs' self-designed drawings. These drawings particularly shed light on their grasp of the place value system and how they visually represent addition and subtraction concepts. Although many PTs demonstrated a clear understanding of foundational concepts, a notable trend was observed among those who struggled with grouping numbers in sets of ten, indicating areas for improvement in their mathematical knowledge for teaching. The study contributes to the debate on digit-based versus number-based algorithms in mathematics education, highlighting the need for conceptual understanding and strategy flexibility. It suggests that primary PTs, influenced by their experience with digit-based algorithms, might not be as adept as young children in using number-based strategies. The study underscores the potential benefits of "number talks" (Humphreys & Parker, 2015), a method of engaging students in discussing diverse computational strategies, though it notes that this approach is underutilized in Slovenian classrooms.

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