Development of an Innovative Lattice Boltzmann Hydraulic Model for Shallow Water Flows

Student: Jessica Padrone

Industrial and Civil Engineering, PhD level, Niccolò Cusano University,
Via Don C. Gnocchi 3, 00166, Rome, Italy
jessica.padrone@unicusano.it

Mentors: Silvia Di Francesco [1], Sara Venturi [2]

[1] Niccolò Cusano University, Via Don C. Gnocchi 3, 00166, Rome, Italy
[2] Department of Civil and Environmental Engineering, University of Perugia,
Via G. Duranti 93, 06125, Perugia, Italy
silvia.difrancesco@unicusano.it, sara.venturi@unipg.it

Abstract*. The development of an innovative Lattice Boltzmann hydraulic model for shallow water flows is presented. The model (CaLB) uses a multi relaxation time (MRT) approach and a cascaded collision operator based on central moments, different from the standard linear collision operator generally used. CaLB is considered for different purposes: reproducing the dynamics of multilayer shallow water flows and modeling large-scale urban floods. Although a variety of models allowing numerical investigation of single and multi-layer shallow water flows, based on continuum and particle approaches, have been widely discussed, there are still some computational aspects that need further investigation. The use a multilayer model (CaLB-N) allows obtaining a description of the vertical variation of hydrodynamic quantities of large-scale geophysical flows and deepening the computational aspects of the density layered shallow water flows. The simulation of flood events in urban areas risk is also investigated, introducing a new porosity approach, aimed at developing technical solutions for the assessment and mitigation of flood risk. The model is validated through comparisons with experimental and numerical results from test cases available in literature, yielding very promising results.*

Keywords. Lattice-Boltzmann cascaded model, multi-layer shallow water flow, gravity currents, idealized dam break flows, large-scale urban floods, porosity

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1 Introduction

A multi relaxation time (MRT) cascaded (CaLB) Lattice-Boltzmann model (LBM) [1] is developed and enhanced to reproduce the dynamics of a multilayered liquid, made of immiscible shallow-layers of different density, bounded by a free surface and to model large-scale urban floodings.

The LBM, based on a first-order linear partial differential equation, it is used to solve the shallow water equations (SWE), derived from depth averaging Navier–Stokes equations, which offer nonlocal second order terms; hence the method is advantageous from a computational point of view and allows to obtain stratified horizontal flow velocities at various depths.

The inclusion of a collision operator based on central moments, which is distinct from the typical BGK (Bhatnagar Gross Krook) [2] of recent models reported in literature, is an essential novelty of the present study.

In the cascaded model, the use of central moments as a basis allows to overcome the Galilean invariance defects of the original MRT method to improve stability and accuracy and the use of a MRT collision operator allows to increase the number of adjustable parameters.

The multi-layer Shallow Water Equations method (MSWE) leads to have a description of the planar velocity along the vertical profile, neglected in the case of a single layer. As the number of layers increases, greater accuracy is achieved tending towards an almost three-dimensional model.

At first, to verify the accuracy and robustness of the model, only two layers are taken in account (CaLB-2). Then, the two-layer model was extended to represent a n-layer flow (CaLB-N) and simulate free surface currents where the three-dimensional aspects of the flow are not negligible, introducing a new formulation of the interchange forces between layers.

The validation phase is conducted considering various experimental and numerical results available in literature [3], [4].

On the other side, the LBM is employed to simulate large-scale urban inundation modeling, solving the traditional SWE on grids. Although CFD methods have already been employed to simulate flow in estuaries and coastal lagoons, capturing localized flow features in large scale is still a challenge, especially when dealing with very shallow flow, temporary submergence and time-dependent flow domains, and complex morphology. The time required for the resolution of the equations is still very long and gets longer with the increase of the spatial resolution and, therefore, of the required accuracy.

The introduction of new a porosity-based Lattice Boltzmann model could allow to capture the effects imposed by structures and small-scale obstructions, providing an accurate representation of the source term to simulate realistic shallow water flows, while at the same time exhibiting a notable reduction in computational times.

2 Mathematical model

The mathematical model proposed solves the SWE, where the fluid motion description is based on the evolution of the particles distribution functions (PDFs), through the discrete Boltzmann equation for individual layers, adopting the two-dimensional nine-speed reticle (D2Q9).

The PDFs $f_i(x,t)$ represent the probability of finding the fluid particle in the neighbourhood of a given position with a given velocity [5]:

$$f_i(\mathrm{t}+\Delta\mathrm{t},\mathbf{x}+\boldsymbol{e}_i\Delta\mathrm{t})-f_i(\mathrm{t},\mathbf{x})=\boldsymbol{\Omega}+\boldsymbol{F}+\boldsymbol{S_p} \qquad i=1,\ldots,n \tag{1}$$

Where $\mathbf{x}$ represents the particle position in discrete space at the time t, $\mathbf{e}_i$ the set of speeds allowed along the n directions of the lattice, $\boldsymbol{\Omega}$ the collision operator, based on the use of central moments [6] and $\mathbf{F}$ the external force acting on the individual layers, depending on gravity and the bottom friction [3]. The source term $\boldsymbol{S_p}$ includes the interchange forces $\boldsymbol{\Phi}$ between layers, the formulation of which was obtained experimentally, taking into account the pressure exerted by the upper layers on the lower ones and the influence of the friction exerted by the lower layers on the upper ones.

Considering n layers, with the same density ρ, the force $\boldsymbol{\phi}^s$ exerted on each single layer *s*, with thickness $h_s$becomes:

$$\boldsymbol{\phi}^s=-gh_s\frac{\partial}{\partial_x}(h_{tot}-h_s)\mathbf{i}-gh_s\frac{\partial}{\partial_y}(h_{tot}-h_s)\mathbf{j} \tag{2}$$

Where $h_{tot}=\sum_{s=1}^{n}h_s$.

Regarding large scale urban inundation modelling, the introduction of a new porosity-based Lattice Boltzmann model is made considering the storage porosity ϕ (Dewals), in the SWE (Guinot) and introducing it in the source term $\boldsymbol{S_p}$ of the eq. 1:

$$\boldsymbol{S_p}=-\frac{h}{\phi}\left(u\frac{\partial\phi}{\partial_x}+u^2\frac{\partial\phi}{\partial_x}+vu\frac{\partial\phi}{\partial_x}\right)\mathbf{i}-\frac{h}{\phi}\left(v\frac{\partial\phi}{\partial_y}+uv\frac{\partial\phi}{\partial_y}+v^2\frac{\partial\phi}{\partial_y}\right)\mathbf{j} \tag{3}$$

Where *h, u, v* are respectively the water surface elevation and the velocity components along the x and y axes.

3 Results, conclusions and ongoing developments

Some of the results obtained to validate the model are reported. The correctness of the model has been first verified with the classical Riemann problem (Fig. 1), where the fluid domain is constituted of a tank filled with the same fluid at different levels.

The results, compared with Stoker's analytical solution, show a correct implementation and that the water depths and velocity magnitude recovered are satisfactory. We also consider the case of a gravity current originating from two liquid layers, separated by a rigid bulkhead in a tank half-filled with the fluid with density ρ_1 and half-filled with the fluid with density ρ_2, ($\rho_1 > \rho_2$), both at the same height h_0. The results obtained from the model is compared with those got by La Rocca et al. [3], for different values of density ratio ρ_1/ρ_2, highlighting a good correspondence of results.

The model was also tested with the case of an asymmetrical ideal dam break, consisting of simulating a wave generated by the partial collapse of a dam, and the results have been compared with those deriving by a continuous 2-D shallow water approach [7].

The results obtained in reproducing the dynamics of shallow water flow_are satisfactory and very promising for the new CaLB-N, demonstrating that the model is well established and ready for the most varied engineering applications.

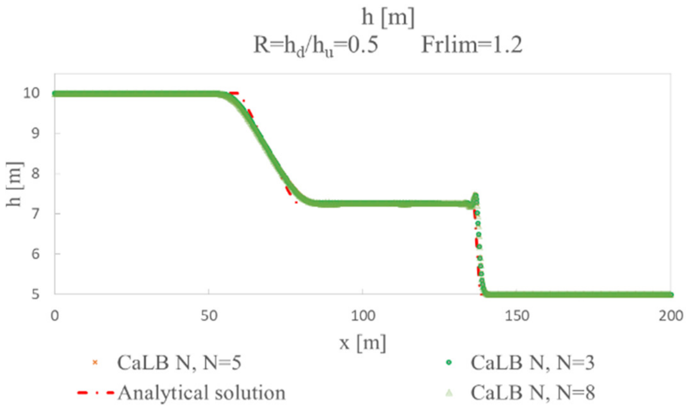

Figure 1. Comparisons between analytical solution and results obtained from the CaLB-N, for the one-dimensional Riemann problem, in terms of levels h [m] for h_d/h_u=0.5 ($h_{u=}$ upstream level; h_d=downstream level).

In future works author will make the model suitable in simulating multilayer flows also in high resolution domains and effective for the most varied engineering applications (e.g. modeling dispersion of pollutants, sediment transport and salinity distribution, wastewater effluents and thermal discharges in riverine and coastal waters).

At least the introduction of the porosity approach could give a more accurate representation of the dynamic of flood events in areas with complex geometries and topography, for a better management of hydraulic risk.

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