WAVE PROPAGATION IN HYDRAULIC TRANSMISSION LINES - STATE OF THE ART IN EFFICIENT SIMULATION MODELS

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Transmission lines are the most simple elements in a fluid power system from the point of view of traditional fluid power system design, simply connecting two individual ports of different components without introducing any additional dynamic effects. In reality, this wishful thinking works only when the system operates quasi-statically with respect to the time constants of wave propagation across the lines. An increasing number of industrial applications faces problems by pushing fluid power systems to operating frequencies high enough to excite significant dynamic effects in transmission lines. In order to mitigate these problems in a model-based systems engineering framework, efficient computational methods for the modelling and simulation of hydraulic transmission lines are crucial. This paper gives an overview of the state of the art in computationally efficient simulation models with a special emphasis on very compact low order models.

Keywords:
hydraulic system design,
transmission lines,
dynamics,
wave propagation,
modelling and simulation

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1 Intruduction

Fluid power technology relies on a small number of basic working principles. First of all, the displacement principle used in hydraulic pumps and motors (including hydraulic cylinders for translator motion). Secondly, the resistance principle used in various valves for control purposes. For basic system design, the story already ends here, and the designers of hydraulic drive systems apply the same principles that were used for conceiving Joseph Bramah's hydraulic press invention in 1795 or Kepler's gear pump invented around 1600. For all these concepts, the hydraulic fluid may be regarded as a mass-less and incompressible ideal medium for transmitting the hydrostatic pressure without adding any dynamic effects to the system. The friction losses of the viscous fluid in pipelines and hoses are however included in classical fluid power system design. There is abundant literature on the so-called minor pressure losses due to long lines and various fittings for branching and connecting these line elements. From the point of view of the hydraulic design engineer, the simple line representing a pipeline connection in a circuit schematic represents an almost ideal connection between two points in the fluid power system: The pressures at these end points are assumed equal except for a pressure loss depending on the flow. This pressure loss is kept small by a proper dimensioning of the line diameter according to line length and expected flow rates.

The described engineering approach neglects pipeline dynamics and in most cases, this is perfectly justified because of the hydrostatic nature of fluid power systems. In some cases, however, either the mass inertia of the oil column in the lines or the wave propagation effect caused by the interplay of inertia and compressibility can cause severe problems.

These effects are known at least since the days of ancient Rome when supply lines for potable water were equipped with devices against the water hammer phenomenon which was mathematically explained by Joukowsky in the late 19th century.

The first approaches for analysis and design of pipeline systems under the influence of fast dynamic processes have been graphical methods in characteristic coordinates, later on computer codes where developed in order to solve systems of partial differential equations. All of these methods are typically regarded as out of scope for the fluid power design engineer and their use has been restricted to two groups of special cases: The first case arises when a system is dimensioned with classical knowledge of hydrostatic behaviour, but some dynamic effects result in malfunction and some sort of trouble-shooting is needed. The second possibility arises, when known hydrostatic solutions are pushed to higher and higher operating frequencies resulting in wave propagation in lines becoming important at some point. Section 3 of this paper will give industrial application examples for both cases.

During the last three or four decades both the broad availability of computer systems capable of numerical simulation and the mathematical modelling skills of mechanical and electrical engineers have improved dramatically. The symbiosis of mechanical and electrical engineering together with computer science, now known as mechatronics and the new paradigm of model-based systems engineering have led to new possibilities in hydraulic circuit design. Computer based simulation systems are increasingly used in system dimensioning. In order to be helpful in the system design phase, these simulation tools need to be easy to understand and use and they must be computationally efficient resulting in very small simulation times.

However, most numerical simulation models for wave propagation available in applied mathematics and engineering literature are either more or less the complete opposite of a fast and computationally efficient tool, or they are over-simplified and fail to capture the real system behaviour with sufficient accuracy.

The remainder of this paper is organised as follows: Section 2 contains a brief overview on the state of the art in hydraulic transmission line modelling and simulation, with a special emphasis on the trade-off between computational efficiency and accuracy of the models. Section 3 presents an example from industrial applications where the dynamics of transmission line plays an important role. Section 4 contains some conclusions.

2 Overview on the state of the art in transmission line modelling

2.1 Modelling assumptions

In the most simple case, a perfectly cylindrical (and thus straight) pipe with a length much bigger than its diameter is passed through by a weakly compressible liquid with Newtonian fluid friction behaviour. The pipe walls are assumed rigid and the temperature is assumed to be constant. Then, the unknowns are the three-dimensional flow velocity and the scalar values of pressure and density inside the pipe. A material law like

$$\rho = \rho_0 \cdot \left(1 + \frac{p - p_0}{K}\right) \tag{1}$$

links the density ρ to the pressure p with a reference density ρ_0 given at a reference pressure p_0. The kinematic viscosity ν is assumed with a constant value due to the constant fluid temperature in this model.

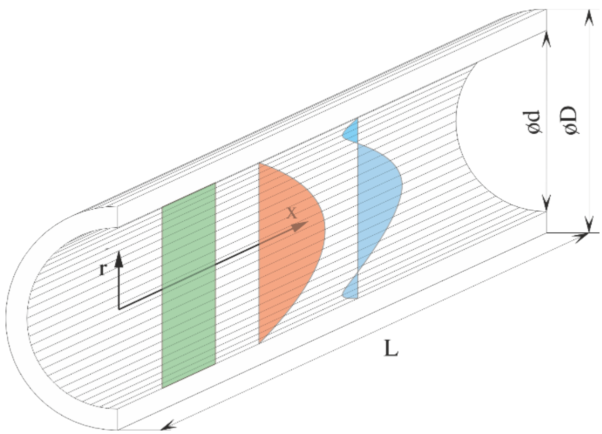

Figure 1: Cylindrical pipe geometry with radial and axial coordinates as well as various assumptions on the axial velocity profile: Plug flow (green), Parabolic Hagen-Poiseuille profile (red), dynamic flow profile under frequency-dependent friction (blue).
Source: own.

When it comes to describing the flow velocity inside the pipe, the Navier-Stokes equations from fluid mechanics must be somehow adapted to the problem at hand. The cylindrical geometry of the pipe calls for the use of a cylindrical coordinate system (axial coordinate x, radial coordinate r, circumferential angle) for formulating the equations of motion for the fluid. In general, this would result in three equations for the momentum balances in axial, radial and circumferential direction. It is common sense in the fluid power community to disregard any swirl flow in the pipe, i. e. to set all circumferential velocity components zo zero a priori. Also the flow velocity in radial direction is omitted in most models and the pressure in the pipeline is assumed to be independent of the radial coordinate at any given axial position and time. All these assumptions result in a problem for the unknown distributions of pressure p and velocity in axial direction

$$p = p(x,t), u = u(x,r,t) \tag{2}$$

of the form

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right) + \frac{4\nu}{3}\frac{\partial^2 u}{\partial x^2}, \tag{3a}$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} = -K\frac{\rho}{\rho_0}\frac{\partial u}{\partial x}. \tag{3b}$$

The first equation (3a) comes from the axial component of the Navier-Stokes momentum balance equation. The second one (3b) is motivated by mass conservation. The parameters in these equations are the fluid density at ambient pressure ρ_0, the kinematic viscosity ν and the bulk modulus K.

At the rigid pipe wall, the flow velocity needs to be zero to fulfil the no-slip condition known from fluid mechanics. A number of over-simplified models disregard this condition by neglecting any viscous friction effects and assuming a so-called plug flow with a constant flow velocity over the whole pipe diameter as shown by the rectangular flow profile in Figure 1 (in green colour). This approach was useful in early water-hammer analysis for hydro-power plants.

In the next level of model complexity the flow profile is assumed to always have a parabolic shape as in the stationary laminar flow case (shown in red colour in Figure 1). Such models are very popular among engineers, because the partial differential equation system (3) can be simplified strongly by a set of easy to understand arguments:

- Following the ancient “divide et impera” principle, the pipeline is divided into a number of identical sections in axial direction.
- Within such a section, the velocity is assumed equal to the parabolic profile in radial direction and with a linear gradient in axial direction:

$$u(x,r,t) = \left(u_0(t)\cdot\left(1-\frac{x}{L}\right) + u_1(t)\cdot\frac{x}{L}\right)\cdot\left(1-\frac{r}{R}\right)\left(1+\frac{r}{R}\right)$$

- For the pressure, a linear axial gradient is used:

$$p(x,t) = p_0(t)\cdot\left(1-\frac{x}{L}\right) + p_1(t)\cdot\frac{x}{L}$$

- The density variations of the hydraulic liquid are small, so the assumption $\frac{\rho}{\rho_0} \approx 1$ simplifies eq. (3b).

Substitution of all these assumptions into eqs. (3), averaging the first equation over the cross sectional area ($\frac{1}{R^2\pi}\int_0^R \dots\, 2\pi r\, dr$) and integrating both equations from 0 to L over x yields a simple system of ordinary differential equations

$$\dot{u}_0 + \dot{u}_1 + \frac{2}{3L}(u_1+u_0)(u_1-u_0) = \frac{4}{\rho_0 L}(p_0-p_1) - \frac{8\nu}{R^2}(u_1+u_0)$$

$$\dot{p}_0 + \dot{p}_1 + \frac{(u_1+u_0)(p_1-p_0)}{2L} = -\frac{K}{L}(u_1+u_0)$$

For the pressures and flow velocities at start (index 0) and end (index 1) of a section. This concept simply needs to be rolled out for N sections, so the second section will have indices 1 and 2, up to the last section with indices N-1 and N. Concepts more or less similar to this are found in a number of commercial simulation software packages including SimScape Fluids.

This simple approach of the segmented pipeline is quite sufficient for simple simulation tasks with low requirements on model fidelity. When it comes to questions where a precise modelling of the real physical behaviour of a viscous, compressible liquid in a pipeline is needed, the assumption of the radial velocity distribution always staying in a parabolic shape is not helpful. The real flow shows a behaviour, where the flow in the core of the pipeline can transiently have an opposite direction as compared to the boundary layer. The profile shown in blue colour in Fig. 1 depicts such an example. The reason for this can be found in a difference between the core flow which is mainly influenced by inertia effects and the boundary layer where the influence of viscous friction is stronger.

Mathematical models taking this dynamic, so-called frequency dependent friction effect into account [4] and software implementations of these models can be found in [2] where a method of characteristics is employed or in [3] where the transmission line matrix method is used. A very compact low order model has been presented in [4].

3 Example

3.1 Pump with long suction line

The example is motivated by the failure of a large hydraulic pump with long suction line. The damaged pump was one of several 1000 cc axial piston units sharing a common suction line. This line was kept on an elevated suction pressure by a feed pump and a pressure limit valve. The damage in the axial piston pump turned out to be induced by cavitation due to strong pressure oscillations in the suction line.

The plant operators reported about the occurrence of strong cavitation noise depending on several factors. First of all, the oil temperature during start-up of the plant: The problem would typically never occur after a cold start of the hydraulic system and it would also not develop when the plant was warming up during continuous operation of the pumps. If, however, the plant was shut down for a short time and then restarted with warm oil conditions, there was a chance of ending up in a situation with strong cavitation. A cavitation-free operation mode could be reached "with good luck" by several start-up trials. And additionally, the problem seemed to occur most likely on the last of three pumps along the common suction line, i. e. the pump with the lowest mean pressure level during operation of all units.

In order to find out, whether the start-up conditions could decide about two different final operation states, one with and one without cavitation, a computer simulation model is built. The focus is not on precisely reproducing the real system behaviour, but on explaining the mechanism that enables at least two different operating conditions for the same pump, at the same speed of rotation and with the same hydraulic load.

For starting up the large 1000 cc constant displacement units, a dedicated start-up hydraulic system is used with a boost pressure pump operated at a 50 bar pressure level and delivering a flow large enough to drive the pump in motor mode and accelerate it together with the inertia of its electrical motor to a speed near the operating speed, in this case 993 rpm. For the model presented here, the dynamics of the large asynchronous induction motor is neglected completely and a simple grid-synchronous speed assumption is used. Figure 2 shows the axial piston pump taking in fluid either in motoring model from a 50 bar start pressure supply or in pumping mode from a long suction line with a 5 bar constant feed pressure at the end. The left half of the Simulink schematics shows the modelling of the angular speed during the start-up phase. The R-S flip-flop block is initially switched on and the torque generated by the pump in motoring mode accelerates the rotary inertia modelled by the single integrator block. The output of this integrator is fed as angular speed back to the pump model. Once the angular speed reaches the synchronous grid speed of 1000 rpm, the flip-flop is reset resulting in the angular speed becoming constant at 1000 rpm, the 50 bar start pressure being cut-off by a switching valve and the load on the high pressure side being increased by reducing the valve opening of the load orifice.

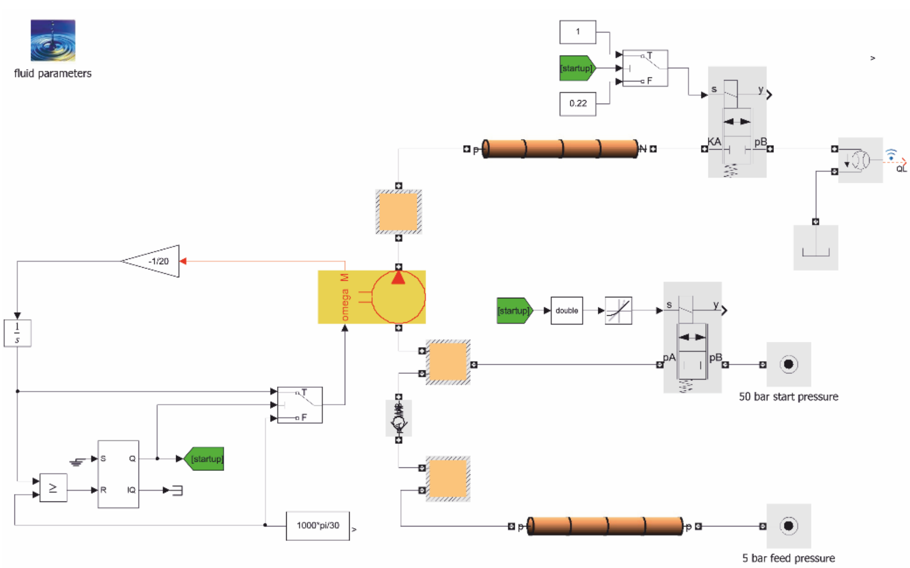

Figure 2: Model of pump with long suction line (MATLAB/Simulink).
Source: own.

The axial piston pump is modelled with nine individual pistons located relative to the crank angle according to

$$\varphi_i = \varphi - i \cdot \frac{2\pi}{9}, \quad i = 0\,..\,8$$

The chamber volumes are

$$V_{c,i} = A_p \cdot (h_0 + r + r \cdot \cos\varphi_i)$$

and the pressure build-up in the chambers is described by orifice equations and the compressibility law as

$$\dot{p}_{c,i}\frac{V_{c,i}}{K} = -\dot{V}_{c,i} + \alpha A_{in}(\varphi_i)\sqrt{\frac{2(p_{in} - p_{C,i})}{\rho}} - \alpha A_{out}(\varphi_i)\sqrt{\frac{2(p_{C,i} - p_{out})}{\rho}}$$

$$\dot{p}_{in}\frac{V_{in}}{K} = Q_{in} - \sum_{i=0}^{8} \alpha A_{in}(\varphi_i)\sqrt{\frac{2(p_{in} - p_{C,i})}{\rho}},$$

$$\dot{p}_{out}\frac{V_{out}}{K} = \sum_{i=0}^{8} \alpha A_{out}(\varphi_i)\sqrt{\frac{2(p_{C,i} - p_{out})}{\rho}} - Q_{out}$$

with opening functions shaped like in Figure 3.

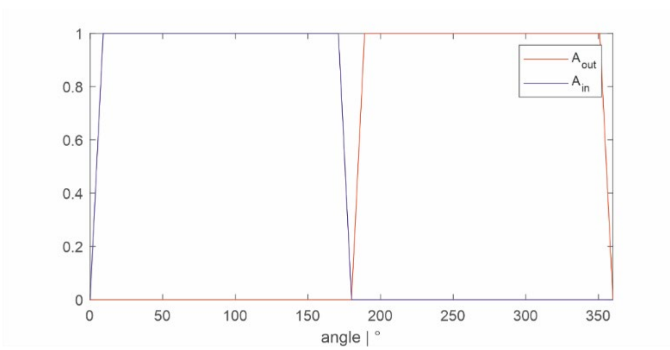

Figure 3: Opening functions (scaled by max. area value).

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The modelling of the switching valve for cutting off the 50 bar supply pressure is a done by an orifice equation controlled by an opening function $u_{cut}(t)$ and the check valve between the suction line and the pump also by an orifice equation with an opening that reacts to the pressure differential without dynamics between a crack pressure p_{cr} where the check valve starts to open and a full opening pressure p_{fo} where the full orifice opening is reached. Together the two valve models result in the flow

$$Q_{in} = u_{cut} \cdot Q_{cut} \sqrt{\frac{50\,\mathrm{bar} - p_{in}}{\Delta p_n}} + Q_{check} \sqrt{\frac{p_{Suc} - p_{in}}{\Delta p_n}} \begin{cases} 0 & p_{Suc} - p_{in} < p_{cr} \\ \frac{p_{Suc} - p_{in} - p_{cr}}{p_{fo} - p_{cr}} & else \\ 1 & p_{Suc} - p_{in} > p_{fo} \end{cases}$$

The pressure $p_{Suc}(t)$ is output by the transmission line model. On the delivery side, a transmission line model connects the volume V_{out} to a load orifice and further back to tank pressure.

3.2 Transmission line modelling by method of characteristics

The initial goal for this paper was to compare different modelling approaches for their ability to explain the phenomenon of operating conditions depending on the start-up procedure observed in the real plant. Unfortunately, only one of the models resulted in a good match between model and reality at acceptable simulation times. This was the Zielke-Suzuki [1, 2] method of characteristics. This method is known to be very accurate but computationally inefficient due to the discretization approach with a detailed friction model being applied over and over again at each node of the axial grid along the pipeline.

The reason, why the other models [3, 5] did not perform well was found in their inefficient coupling with boundary conditions algebraically coupling the pressure with the flow rate at the boundary. Figures 4 and 5 show the results for two different values of the switching time of the valve cutting off the 50 bar start pressure supply.

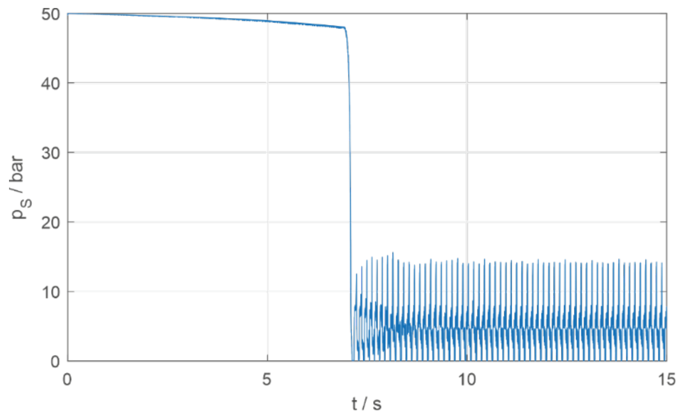

Figure 4: Pressure at pump intake during start-up and shortly thereafter, 200 milliseconds switching time at the cut-off valve.

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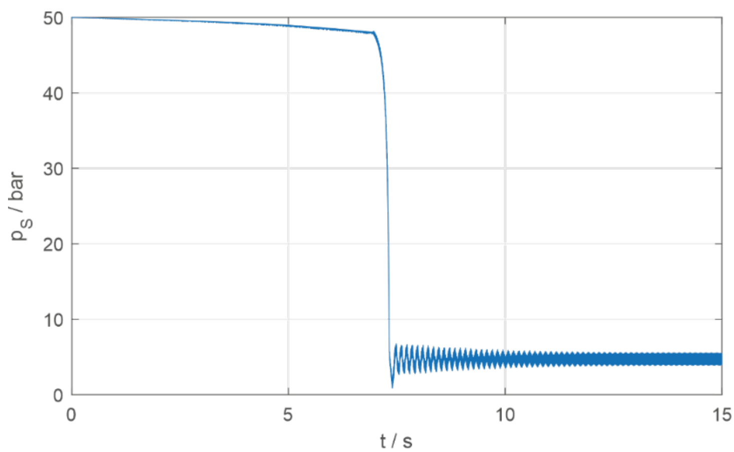

Figure 5: Pressure at pump intake during start-up and shortly thereafter, 500 milliseconds switching time at the cut-off valve.

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4 Conclusions

For an industrial example with high demands on the transmission line model fidelity, a benchmark example has been shown. The classical method of characteristics in an implementation with efficient treatment of valve boundary conditions shows excellent results in explaining the interesting plant behaviour of the final period pressure pulsations at the pump intake having two different amplitudes depending on the switching time between start-up and suction pressure. More modern and theoretically more efficient models failed in the first attempt due to unsolved problems with valve boundary conditions. Further work is under way to resolve this.

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