## A Spatio-Temporal and

## Categorical Correlation

 Computing Method for Induction and Deduction AnalysisYasuhiro Hayashi, ${ }^{1}$ Yasushi Kiyoki, ${ }^{1}$ Yoshinori Harada, ${ }^{2}$ Kazuko Makino, ${ }^{2}$ Seigo Kaneoya ${ }^{2}$<br>${ }^{1}$ Musashino University, Tokyo, Japan<br>yhayashi@musashino-u.ac.jp, y-kiyoki@musashino-u.ac.jp<br>${ }^{2}$ Credit Saison Co., Ltd Tokyo, Japan

We propose a spatio-temporal and categorical correlation computing method for induction and deduction analysis in order to reveal relationships between two sets in past events, thereby finding insights to build new relationships between the two sets in the future. The most significant feature of this method is that it provides a means for inductive and deductive thinking in the cycle of memory recall in which humans unravel the relationship between two entities. Concretely, this method calculates correlations in a 'hypothesis-to-fact' and 'fact-to-hypothesis' approach based on information such as when, how often, who, where, what, and how, and enables to derive the relationship between two sets. For the correlation calculation, this method dynamically creates a multi-dimensional vector space in which the dimensions consist of time, space, and category, and creates a vector consisting of temporal, spatial, and categorical features of independent elements in each attribute from past events containing two attributes with a certain relationship. The strength of relationships between the two sets is calculated as similarity. This method also makes it possible to derive facts from hypotheses by applying context vectors. This paper shows the details of this method and implementation method and assumed applications in commerce activities.

Keywords: spatio-temporal \& categorical correlation computing, induction and deduction analysis, dynamic multidimensional vector space creation, vector composition operator, context-based data mining

## 1 Introduction

We propose a method that realizes deductive and inductive analysis of spatiotemporal and categorical relationships between entities that are related to each other from a certain set. The most significant feature of this method is that it provides a means for inductive and deductive thinking in the cycle of spatio-temporal and categorical memory recall in which humans unravel the relationship between two entities. Concretely, this method calculates correlations in a 'hypothesis-to-fact' and 'fact-to-hypothesis' approach based on information such as when, how often, who, where, what, and how, and enables to derive the relationship between two sets.

For the correlation calculation, this method dynamically creates a multi-dimensional vector space in which the dimensions consist of time, space, and category. This method dynamically creates a vector consisting of temporal, spatial, and categorical features of independent elements in each attribute from past events containing two attributes with a certain relationship. This vector is mapped to the multi-dimensional vector space. Based on the correlation calculation between vectors mapped to that multi-dimensional vector space, the strength of relationships between the two sets is calculated as similarity. Consequently, the method can retrieve past events mapped to a multi-dimensional vector space by time, space, and category conditions, and reveals relationships between the two sets. Namely, this method makes it possible to derive hypotheses from facts.

The method also enables the setting of context vectors describing spatio-temporal and categorical features as hypotheses. This context vector is also mapped to a multidimensional vector space, and a distance calculation between the context vector and other vectors reveals relationships between the two sets. Namely, this method also makes it possible to derive facts from hypotheses. This method was inspired by the following related research.

## 1.1 (1) Semantic Computing by the Mathematical Model of Meaning \& Meta-Level System

The Mathematical Model of Meaning \& Meta-Level System is the core method that inspired this research. The mathematical model of meaning proposed by Kiyoki et al $[1,2]$ is a method for computing semantic associations between data that change
dynamically according to context or situation. A n orthonormal space called the metadata space is created and media data are mapped onto the space. By calculating distances in the metadata space, this method realizes retrieval of media data that are semantically similar to the query. If the context is given along with the query at the time of retrieval, the dimensionality selection control of the space is dynamically executed, and the retrieval of semantically similar media data is executed according to the context.

Furthermore, the meta-level system proposed by Kiyoki et al [2,3] is a method that enables integration and linkage of heterogeneous local database systems by setting up a meta-database system in the upper layer of heterogeneous local database systems. By realizing an integrated semantic space and a mechanism for semantic distance calculation in the meta-database system, the correlation between temporal, spatial, and semantic features obtained from each local database is weighed. With the proposed mathematical model of meaning and the meta-level system, Kiyoki et al. aim to realize a memory processing mechanism that realizes interpretation of dynamically changing meanings and sensitivities depending on context or situation [3].

Our method analyzes data in which two attributes have some relationships. The data is inputted to our method by Table Join process on meta-level of heterogenous relational databases. Plus, our method dynamically controls dimensions to calculate correlations between two sets. Meta-level system and The Mathematical Model of Meaning are fundamental calculation model of this method.

### 1.2 Image-Query Creation Method

The image-query creation method is proposed by Hayashi and Kiyoki [4,5] creates image queries for content-based image retrieval by combining images. In this method, an image-query creation database and image-query creation operators are set up in the query part of the content-based image database system. The combination of the image database and the operators is used to operate the color and shape features. Based on the color and shape features of the images that the searcher wants to focus on, this method dynamically controls dimensions of the image query and the image database to be searched.

In our method, vectors of two sets are created in the integrated space of past events in various fields. Creating the vectors in the image-query creation method and calculating contextual correlation quantities in the orthogonal space of color and shape features is one of the methods that inspired this research.

### 1.3 Emotional MaaS (Mobility as a Service)

The Emotional MaaS, proposed by Kawashima, Hayashi, and Kiyoki [6] is an application of the Mathematical Model of Meaning and Meta-Level System, calculates travel routes and facilities based on the context of tourists. MaaS provides mobility and related services to tourists across the board by highly integrating real space and information space. In this method, the context of the tourist's speed of move, distance in real-space, and purpose are set in advance, and transportations and related facilities that are highly correlated with that context are weighed. In order to create a variety of traveler contexts, the intention and situation of each traveler are described as vectors, and the context is described by composing these vectors.

The correlation calculation and the multi-dimensional vector space creation to describe various contexts are similar to our method in this research. In addition, our method can be applied to the commerce activity field to clarify human behaviors. Research area overlaps with that of data utilization in mobility information services.


Figure 1: The Concept of The Proposed Method

## 2 A Spatio-Temporal \& Categorical Correlation Computing Method for Induction and Deduction Analysis

### 2.1 Data Structure \& Calculation Method

This method executes spatio-temporal and categorical correlation calculation for induction and deduction analysis. The concept of this model is shown in Figure 1. The concrete calculations are defined as follows.

The given a set $\boldsymbol{P}$ is expressed as Formula 1. The elements of the set $\boldsymbol{P}$ are expressed as $p_{i j}$. Where $i:=1 \ldots, \boldsymbol{q}, \boldsymbol{q}$ is the number of attributes of the set $\boldsymbol{P}$. Furthermore, $\boldsymbol{j}$ $:=1 \ldots, r, r$ is the number of elements of the set $\boldsymbol{P}$. Also, the attributes of the set $\boldsymbol{P}$ are expressed as $\boldsymbol{a}_{i}$.

$$
\begin{align*}
& P:=\left(\begin{array}{cccc}
a_{1} & a_{2} & \ldots & a_{q q} \\
p_{11} & p_{21} & \ldots & p_{q 1} \\
p_{12} & p_{22} & \ldots & p_{q 2} \\
\vdots & \vdots & \ddots & \vdots \\
p_{1 r} & p_{2 r} & \ldots & p_{q r}
\end{array}\right)  \tag{1}\\
& p_{i j} \in P \quad \text { where } \quad\left\{p_{i j} i-1 \ldots q, j-1 \ldots r\right\}  \tag{2}\\
& a_{i} \text { where }\left\{a_{i} \mid i-1 \ldots q\right\} \tag{3}
\end{align*}
$$

Based on the given necessary conditions about time, space, and category, selection and projection are executed on the set $\boldsymbol{P}$. The set $\boldsymbol{P}$ reduced in number of elements and attributes by this symbolic filtering is expressed as the set $\boldsymbol{P}^{\prime}$. The set $\boldsymbol{P}^{\prime}$ is assumed to have two attributes $\boldsymbol{a}_{\boldsymbol{x}}$ and $\boldsymbol{a}_{\boldsymbol{y}}$ that have significant relationships as entities. Where $1<=x<q, 1<=y<q$, and $x$ not equals $y$.

$$
\begin{array}{lll}
a_{x} & \text { where } & \left\{a_{x} \mid 1 \leq x<q, x \neq y\right\} \\
a_{y} & \text { where } & \left\{a_{y} \mid 1 \leqq y<q, x \neq y\right\} \tag{4}
\end{array}
$$

Here, the set $\boldsymbol{P}^{\prime}$ aggregated by the independent element $\boldsymbol{P}\left[\boldsymbol{a}_{\boldsymbol{x}}\right]$ in the attribute $\boldsymbol{a}_{\boldsymbol{x}}$ is defined as a set $U$.

$$
U:=P^{\prime} \text { groupby } a_{x}=\left(\begin{array}{ccccc}
a_{x} & a_{1} & a_{2} & \ldots & a_{q}  \tag{5}\\
u_{x 1} & u_{11} & u_{21} & \ldots & u_{q 1} \\
u_{x 2} & u_{12} & u_{22} & \ldots & u_{q 2} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
u_{x m m} & u_{1 m} & u_{2 m} & \ldots & u_{q m}
\end{array}\right)
$$

Elements in the set $\boldsymbol{U}$ is expressed by $\boldsymbol{u}_{i j}$. Where, $\boldsymbol{i}:=\mathbf{1} \ldots, \boldsymbol{q}, \boldsymbol{q}$ is the number of attributes in the set $U$. Furthermore, $j:=1 \ldots, m$, where $m$ is the number of elements in the set $U$.

$$
\begin{equation*}
u_{i j} \in U \quad \text { where }\left\{u_{i j} \mid i=1 \ldots q, j=1 \ldots m\right\} \tag{6}
\end{equation*}
$$

When temporal attributes in the set $\boldsymbol{U}$ are $\boldsymbol{a}_{\boldsymbol{t}}$, the temporal elements are expressed as $u\left[a_{t}\right]$. Where $\left\{a_{t} \mid t=1 \ldots, t t\right\}$ and $t t$ is an arbitrary number. When spatial attributes in the set $U$ are $\boldsymbol{a}_{t}$, the temporal elements are expressed as $u\left[a_{t}\right]$. Where $\left\{\boldsymbol{a}_{t} \mid t=\mathbf{1}\right.$ $\ldots, \boldsymbol{s} \boldsymbol{s}\}$ and $\boldsymbol{s s}$ is an arbitrary number. Furthermore, when categorical attributes in the set $U$ are $\boldsymbol{a}_{t}$, the temporal elements are expressed as $u\left[a_{t}\right]$. Where $\left\{\boldsymbol{a}_{t} \mid t=1 \ldots\right.$, $\boldsymbol{c c}\}$ and $\boldsymbol{c c}$ is an arbitrary number.

With the temporal feature extraction function $t f$, the spatial feature extraction function $s f$, and the categorical feature extraction function $c f$ defined below, the temporal feature $u\left[a_{x j}, t\right]$, the spatial feature $u\left[a_{x j}, s\right]$, and the categorical feature $u\left[a_{x j}, c\right]$ are calculated as follows.

$$
\begin{align*}
& u\left[a_{x j}, t\right]:-t f\left(u\left[a_{x j}\right], u\left[a_{\ell}\right]\right) \\
& \left.u a_{x j}, s\right]:=\operatorname{sf}\left(u\left[a_{x j}\right], u\left[a_{s}\right]\right) \\
& \left.\left.u a_{x j}, c\right]:=c f\left(u a_{x j}\right], u\left[a_{c}\right]\right) \tag{7}
\end{align*}
$$

Note that $u\left[a_{x}\right]=p\left[a_{x}\right], j:=1 \ldots, m$, where $m$ is the number of elements in the set $\boldsymbol{U}$. The spatio-temporal and categorical feature vector $v\left[a_{x j}\right]$ of the set $u\left[a_{x j}\right]$ is created by this process.

$$
\begin{equation*}
v\left[a_{x j}\right]:=\left(u\left[a_{x j}, t\right], u\left[a_{x j}, s\right], u\left[a_{x j}, c\right]\right) \tag{8}
\end{equation*}
$$

Thus, the set $\boldsymbol{P}^{\prime}$ aggregated by the independent element $\boldsymbol{P}\left[\boldsymbol{a}_{\boldsymbol{y}}\right]$ in the attribute $\boldsymbol{a}_{\boldsymbol{y}}$ is defined by a set $W$.

$$
W:=P^{\prime} \text { groupby } a_{y}=\left(\begin{array}{ccccc}
a_{y} & a_{1} & a_{2} & \ldots & a_{q}  \tag{9}\\
w_{y 1} & w_{11} & w_{21} & \ldots & w_{q 1} \\
w_{y 2} & w_{12} & w_{22} & \ldots & w_{q 2} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
w_{y n} & w_{1 n} & w_{2 n} & \ldots & w_{q n}
\end{array}\right)
$$

Elements in the set $W$ are expressed as $\boldsymbol{w}_{i j}$. Where, $\boldsymbol{i}:=\mathbf{1} \ldots, \boldsymbol{q}, \boldsymbol{q}$ is the number of attributes of the set $W$. Plus, $j:=1 \ldots, n, n$ is the number of elements in the set $W$.

$$
\begin{equation*}
w_{i j} \in W \text { where }\left\{w_{i j} i=1 \ldots q, j=1 \ldots n\right\} \tag{10}
\end{equation*}
$$

When temporal attributes in the set $W$ is $\boldsymbol{a}_{t}$, the temporal elements are expressed by $w\left[a_{t}\right]$. Where $\left\{a_{t} \mid t=1 \ldots, t t\right\}$ and $t t$ is an arbitrary number. When spatial attributes in the set $W$ are $\boldsymbol{a}_{t}$, the temporal elements are expressed as $W\left[\boldsymbol{a}_{\boldsymbol{t}}\right]$. Where $\left\{\boldsymbol{a}_{t} \mid \boldsymbol{t}=\mathbf{1}\right.$ $\ldots, s s\}$ and $s s$ is an arbitrary number. Furthermore, when categorical attributes in the set $W$ are $a_{t}$, the temporal elements are expressed as $w\left[a_{t}\right]$. Where $\left\{\boldsymbol{a}_{t} \mid t=1\right.$ $\ldots, c c\}$ and $\boldsymbol{c c}$ is an arbitrary number.

With the temporal feature extraction function $t f$, the spatial feature extraction function $s f$, and the categorical feature extraction function $c f$ defined below, the temporal feature $w\left[a_{y j}, t\right]$, the spatial feature $w\left[a_{y j}, s\right]$, and the categorical feature $w\left[a_{y j}, c\right]$ are calculated as follows.

$$
\begin{align*}
w\left[a_{y j}, t\right] & :=\operatorname{tf}\left(w\left[a_{y j}\right], w\left[a_{t}\right]\right) \\
w\left[a_{y j}, s\right] & :=\operatorname{sf}\left(w\left[a_{y j}\right], w\left[a_{y}\right]\right) \\
w\left[a_{y j}, c\right] & :=\operatorname{cf}\left(w\left[a_{y j}\right], w\left[a_{t}\right]\right) \tag{11}
\end{align*}
$$

Note that $w\left[a_{y}\right]=p\left[a_{y}\right], j:=1 \ldots, n$, where $n$ is the number of elements in the set $W$. The spatio-temporal and categorical feature vector $v\left[a_{y j}\right]$ of the set $w\left[a_{y j}\right]$ is created by this process.

$$
\begin{equation*}
v\left[a_{y j}\right]:=\left(w\left[a_{y j}, t\right], w\left[a_{y j}, s\right], w\left[a_{y j}, c\right]\right) \tag{12}
\end{equation*}
$$

The vectors $v\left[a_{x j}\right]$ and $v\left[a_{y j}\right]$ created by Equations 8 and 12 are mapped to a multidimensional vector space $V$ with time, space, and category as dimensions. The distance between vectors $\boldsymbol{d}_{t}, \boldsymbol{d}_{s}, \boldsymbol{d}_{\boldsymbol{c}}$ is calculated for each temporal, spatial, and categorical feature by the temporal feature distance function $t d$, spatial feature distance function $\boldsymbol{s d}$, and categorical feature distance function $\boldsymbol{c d}$ defined by Formula 13. Plus, to calculate the total correlation score between mapped vectors $v\left[a_{x j}\right]$ and $v\left[a_{y j}\right]$, the similarities $\boldsymbol{d}_{t}, \boldsymbol{d}_{s}, \boldsymbol{d}_{c}$ calculated in different method are normalized and expressed as $\boldsymbol{d}_{t}^{\prime}, \boldsymbol{d}_{s}{ }^{\prime}, \boldsymbol{d}_{\boldsymbol{c}}{ }^{\prime}$ (Formula 14).

$$
\begin{align*}
d_{t} & :=\operatorname{td}\left(v\left[a_{x j}\right], v\left[a_{y j}\right]\right) \\
d_{s} & :=\operatorname{sd}\left(v\left[a_{x j}\right], v\left[a_{y j}\right]\right) \\
d_{c} & :=\operatorname{cd}\left(v\left[a_{x j}\right], v\left[a_{y j}\right]\right)  \tag{13}\\
d_{t}^{\prime} & :-\operatorname{norm}\left(d_{t}\right) \\
d_{s}^{\prime} & :=\operatorname{norm}\left(d_{s}\right) \\
d_{c}^{\prime} & :=\operatorname{norm}\left(d_{c}\right) \tag{14}
\end{align*}
$$

The result of each normalized distance calculation is multiplied by the weights $\boldsymbol{w} \boldsymbol{t}_{\boldsymbol{t}}$, $\boldsymbol{w} \boldsymbol{t}_{\boldsymbol{s}}, \boldsymbol{w} \boldsymbol{t}_{c}$ and calculated as a sum value $\boldsymbol{s c o r e}$.

$$
\begin{equation*}
\text { score }:=\operatorname{sim}\left(v\left[a_{x j}\right], v\left[a_{y j}\right]\right)=w t_{t} \times d_{t}^{\prime}+w t_{s} \times d_{s}^{\prime}+w t_{c} \times d_{c}^{\prime} \tag{15}
\end{equation*}
$$

### 2.2 Context Vector for Induction and Deduction Data Analysis Approach

To realize the inductive and deductive data analysis approach, this method applies a context vector $C \boldsymbol{X}$ which is the originality of this method. The inductive and deductive approaches in this method are defined as follows:

The inductive approach (fact-to-hypothesis): To extract a context vector $\boldsymbol{C X}$ that is similar to the temporal, spatial, and categorical features of the elements of the set $U$ or the set $W$ as model events.

The deductive approach (hypothesis-to-fact): To extract elements of the set $\boldsymbol{U}$ or elements of the set $W$ that are similar to the temporal, spatial, or categorical features of some model events indicated by a context vector $\boldsymbol{C X}$.

The context vector $\boldsymbol{C X}$, consisting of the temporal feature $\boldsymbol{c \boldsymbol { x } _ { t }}$, the spatial feature $\boldsymbol{c \boldsymbol { x } _ { s }}$, and the category feature $\boldsymbol{c \boldsymbol { x } _ { \boldsymbol { c } }}$, is expressed as follows.

$$
\begin{equation*}
C X:=\left(c x_{t}, c x_{s}, c x_{c}\right) \tag{16}
\end{equation*}
$$

Using Formula 15, the similarity calculation between the elements $v\left[a_{x j}\right]$ of the set $U$ and the context vector $C X$, or the elements $\left.v a_{y j}\right]$ of the set $W$ and the context vector $\boldsymbol{C X}$, enables an inductive or deductive data analysis approach.

$$
\begin{align*}
& \text { score }:=\operatorname{sim}\left(v\left[a_{x j}\right], C X\right) \\
& \text { score } \left.:=\operatorname{sim}\left(v \mid a_{y j}\right], C X\right) \tag{17}
\end{align*}
$$

### 2.3 Spatio-Temporal and Categorical Feature Extraction Functions

### 2.3.1 Temporal Feature Extraction Function

This function extracts temporal average and variance. The date in the purchase history data has two facets: the usage date and the usage date interval. The extracted temporal feature vector $V_{T}$ consists of the average $\boldsymbol{t a}$ of the usage dates in the $\boldsymbol{\eta}$ data in the purchase history, its variance $t v$, the average tia of the usage date interval, and its variance tiv. Note that the usage interval date is not constant.

$$
\begin{equation*}
V_{T}-(t a, t v, t i a, t i v) \tag{18}
\end{equation*}
$$

When each usage date data in the purchase history is expressed as $p_{i}$, the average $t \boldsymbol{t a}$ of usage dates in $n$ data is calculated as follows. Where, $i:=1, \ldots, n$. The dtoi function converts a Gregorian date to a standard date integer. The itod function converts an integer value of a date in standard format to a date in Gregorian format. The variance $\boldsymbol{t v}$ of the date of use in $\boldsymbol{n}$ data is calculated by Formula 20. Note that absolute values are used to simplify the calculation.

$$
\begin{align*}
& t a:=i t o d\left(\frac{1}{n} \sum_{i=1}^{n} d t o i\left(p_{i}\right)\right)  \tag{19}\\
& t v:=i t o d\left(\frac{1}{n} \sum_{i=1}^{n}\left|d t o i\left(p_{i}\right)-\frac{1}{n} \sum_{i=1}^{n} d t o i\left(p_{i}\right)\right|\right) \tag{20}
\end{align*}
$$

When each usage date data in the purchase history is expressed as $p_{i}$, the average tia of usage date intervals for $n$ data is calculated as follows. Where $i:=1, \ldots, n$. When $\boldsymbol{n}>\mathbf{0}$, the average tia and its variance $\boldsymbol{t i v}$ are obtained. When $\boldsymbol{n}=\mathbf{0}$, tia and $\boldsymbol{t i v}$ are zero. Additionally, the variance $\boldsymbol{t i v}$ of the usage date interval in $n$ data is calculated as follows.

$$
\begin{align*}
& t i a:=i t o d\left(\frac{1}{n} \sum_{i=1}^{n}\left(d t o i\left(p_{i \mid 1}\right)-d t o i\left(p_{i}\right)\right)\right)  \tag{21}\\
& t i v:=i t o d\left(\frac{1}{n} \sum_{i=1}^{n}\left|d t o i\left(p_{i}\right)-\frac{1}{n} \sum_{i=1}^{n}\left(d t o i\left(p_{i \mid 1}\right)-d t o i\left(p_{i}\right)\right)\right|\right) \tag{22}
\end{align*}
$$

### 2.3.2 Spatial Feature Extraction Function

This function calculates spatial feature. The spatial feature vector $V_{S}$ consists of the center position $\boldsymbol{s a}$ and variance $\boldsymbol{s v o f}$ latitude and longitude of stores included in the $n$ data in the purchase history.

$$
\begin{equation*}
V_{S^{\prime}}=(s a, s v) \tag{23}
\end{equation*}
$$

The latitude and longitude $\boldsymbol{s} \boldsymbol{a}$ corresponding to the center of gravity of the latitude and longitude in the $n$ data is calculated by Formula 24. When the latitude and longitude of two points $p, q$ on the earth are given as $p$ (latitude1, longitude1), $\boldsymbol{q}$ (latitude2, longitude2), the great circle distance $\boldsymbol{s} \boldsymbol{d}$ between $\boldsymbol{p}$ and $\boldsymbol{q}$ is calculated by Formula 25. This formula is also used for spatial similarity degree.

$$
\begin{align*}
& x:=\frac{1}{n} \sum_{i-1}^{n} \cos \left(\frac{\text { latitude }[i] \pi}{180}\right) \times \cos \left(\frac{\text { longitude }[i] \pi}{180}\right) \\
& y:=\frac{1}{n} \sum_{i=1}^{n} \cos \left(\frac{\text { latitude }[i] \pi}{180}\right) \times \sin \left(\frac{\text { longitude }[i] \pi}{180}\right) \\
& z:=\frac{1}{n} \sum_{i=1}^{n} \sin \left(\frac{\text { latitude }[i] \pi}{180}\right) \\
& s a:=\left(\frac{180 \cdot \operatorname{atan} 2\left(z, \sqrt{x^{2}+y^{2}}\right)}{\pi}, \frac{180 \cdot \operatorname{atan} 2(y, x)}{\pi}\right) \tag{24}
\end{align*}
$$

$$
\begin{array}{ll}
r: \quad 6378.137 \mathrm{~km}] \\
x 1:=r \cos \left(\frac{\text { latitude } 1 \pi}{180}\right) * \cos \left(\frac{\text { longitude } 1 \pi}{180}\right) & x 2:=r \cos \left(\frac{\text { latitude } 2 \pi}{180}\right) * \cos \left(\frac{\text { longitude } 2 \pi}{180}\right) \\
y 1:-r \cos \left(\frac{\text { latitude } 1 \pi}{180}\right) * \sin \left(\frac{\text { longitude } 1 \pi}{180}\right) & y^{2}:-r \cos \left(\frac{\text { latitude } 2 \pi}{180}\right) * \sin \left(\frac{\text { longitude } 2 \pi}{180}\right) \\
z 1: r \sin \left(\frac{\text { latitude } 1 \pi}{180}\right) & z 2: r \sin \left(\frac{\text { latitude } 2 \pi}{180}\right) \\
s d:=2 r \text { asin }\left(\frac{\sqrt{(x 1-x 2)^{2}+(y 1-y 2)^{2}+(z 1-z 2)^{2}}}{2 r}\right) \tag{25}
\end{array}
$$

When each latitude and longitude data in the purchase history is expressed as $p_{i}$, the variance $\boldsymbol{s} \boldsymbol{v}$ of latitude and longitude in $\boldsymbol{n}$ data is calculated as follows. Where $\boldsymbol{i}:=1$, $\ldots, n . s d$ is a function to calculate the great distance between $p_{i}$ and the center of gravity $\boldsymbol{s a}$ of latitude and longitude in $\boldsymbol{n}$ data.

$$
\begin{equation*}
s v:=\frac{1}{n} \sum_{i=1}^{n}\left|s d\left(p_{i}, s a\right)\right| \tag{26}
\end{equation*}
$$

### 2.3.3 Categorical Feature Extraction Function

The Category Feature Histogram $V_{C}$ is the sum of n stores' category vector data $C_{i}$ in the purchase history. Where $\boldsymbol{i}:=\mathbf{1}, \ldots, \boldsymbol{k}$. The category is expressed as tree data consisting of four levels: large, medium, small, and detailed. By converting the tree data format to vector data format, distance calculation in the vector space can be applied. When $L$ major category, $M$ medium category, $S$ minor category, and $D$ detailed category consist of $1, m, s$, and $d$ elements, respectively, tree data $T$ is converted to vector data $C_{i}$ consisting of $k:=1+m+s+d$ elements.

$$
\begin{gather*}
k:=l+m-s+d \\
C_{i}:=\left(c_{i 1}, c_{i 2}, \ldots, c_{i k}\right) \\
V_{C}:-\sum_{i=1}^{k} C_{i}-\sum_{i 1}^{k}\left(c_{i 1}, c_{i 2}, \ldots, c_{i k},\right. \tag{27}
\end{gather*}
$$

### 2.3.4 Vector Composition Function

Given $\boldsymbol{k}$ vectors $V_{i}:=\left(v_{i}, v_{i 2}, \ldots, v_{i n}\right)$ consisting of $\boldsymbol{n}$-dimensions, this function that is named Vector Creation Operator composes a new vector by computing the sum of the same elements of those vectors. Where $2<=i<=\boldsymbol{k}$.

$$
\begin{equation*}
V_{\text {composition }}:=\text { composition }\left(V_{1}, V_{2}, \cdots V_{n}\right)=\sum_{i=1}^{k}\left(v_{i 1}, v_{i 2}, \cdots v_{i n}\right) \tag{28}
\end{equation*}
$$

### 2.3.5 Distance Calculation Function

The inner product $i p$ is used to calculate the semantic distance as similarity similarity between two context vectors $V_{a}, V_{b}$ created by this method. Where $D:=$ $2+4+\boldsymbol{k}$. Note that if semantical distance is required between elements of two vectors, Euclidean distance calculation and geographical distance calculation are also used.

$$
\begin{gather*}
V_{a}:=\left(v_{a 1}, v_{a 2}, \cdots, v_{a d}\right) \\
V_{b}:=\left(v_{b 1}, v_{b 2}, \cdots, v_{b d}\right) \\
\text { similarity }:-i p\left(V_{a}, V_{b}\right)-\sum_{i}^{d}\left(v_{a i} \times v_{b i}\right) \tag{29}
\end{gather*}
$$

## 3 Implementation Method and Assumed Applications

This method realizes deductive and inductive analysis of spatio-temporal and categorical relationships between entities that are related to each other from a certain set. One of the implementation fields of this method is the purchase between customers and stores in commerce. Customers and stores have a spatio-temporal and categorical relationship. Concretely, the relationship is that on a certain day (time information), a customer purchased a certain product (category information) at a certain store (place information). In order to implement this method, the system requires a computation layer that enables deductive and inductive analysis of the relationship between customers and stores, an input layer that receives parameters from the analyst, and an output layer that visualizes the analysis results.

Furthermore, the following applications of this method in commerce are assumed.

## (1) Store recommendation based on inductive and deductive analysis of customers

the customer's spatio-temporal and categorical features and the context vector. Furthermore, store recommendations are performed to approach the ideal happiness by calculating the similarity between the context vector and the customer's happiness state.

## (2) Store recommendation based on deductive and inductive analysis of customers

Deductively analyze customers who are similar to the ideal happiness by calculating the similarity between the spatio-temporal and categorical features of the context vector and customers. Furthermore, store recommendations are performed to approach better happiness by calculating the similarity between the customer and other customers.

## 4 Conclusion

We described a spatio-temporal and categorical correlation computing method for induction and deduction analysis. The originality of this method is to realize deductive and inductive analysis of spatio-temporal and categorical relationships between entities that are related to each other from a certain set. The introduction of context vectors enables inductive data analysis (fact-to-hypothesis) and deductive data analysis (hypothesis-to-fact) by spatio-temporal and category features between two sets.

This is corresponding to humans' logical thinking involving the cycle of temporal, spatial, and categorical memory recall to reveal the relationship between two entities. For this reason, in the calculation process, this method dynamically creates the metric space and queries based on the context, consisting of spatio-temporal and categorical features, and calculates correlations between customers and stores. Additionally, the implementation method and assumed applications were shown in this paper.

For the next step, we start to develop a proto-type system that is applied the proposed method and experiments to evaluate effectiveness and feasibility, and business deployment.

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