Adaptive Model Predictive Control with Regularized Finite Impulse Response Models

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Abstract. *The use of regularized finite impulse response models allows to incorporate prior knowledge of the process. This can be used to decrease the variance error of an online parameter estimation and ensures a robust system identification. The online adapted model can be used to control time-variant or nonlinear processes. This approach is named adaptive model predictive control. The investigated method is tested on a nonlinear single tank simulation and is compared to an already established method.*

Keywords. adaptive MPC, regularized FIR models, system identification, leaky recursive least squares, gray-box identification

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1 Introduction

The main task in model predictive control (MPC) is to predict the future output of the process and estimate an optimal sequence for the manipulated variable with respect to an objective function. To be able to control nonlinear processes with an MPC, a linear model which is adapted online can be used. The advantage of a linear MPC is given by a closed-form solution, which is computationally efficient and yields a global optimum.

For the internal model, a finite impulse response (FIR) model structure is chosen. FIR models belong to the class of linear time-invariant models. They are well suited for the identification of stable processes as the model class is inherently stable. Further advantages are e.g. the linearity in the parameters, the output error configuration, and the insensitivity w.r.t. a wrong model order or too small dead times. However, a main drawback of this model structure is the large number of parameters which causes a high variance error. To overcome this problem, regularization can be used by introducing an additional penalty term in the parameter estimation [1]. Thus, prior knowledge can be incorporated and the variance error is decreased.

The use of regularization in a recursive weighted least squares (RWLS) method is investigated. The leaky recursive least squares (LRLS) method enables the use of regularization without loosing its effect over time [2]. The estimation of the FIR model is done online in a closed-loop adaptive model predictive control (AMPC). This method is compared to an AMPC which interpolates between offline identified FIR models. Both methods are tested on a simulation of a nonlinear single tank system [3], [4].

2 AMPC Algorithm

In the presented AMPC algorithm, a single FIR model is used. The parameters of this model are updated with the LRLS in each time step, in order to match the current process behavior of the controlled system.

2.1 FIR Models

The output of a strictly proper FIR model with offset can be calculated by a linear combination of the delayed inputs $u(k-1), \ldots, u(k-n-1)$. Consequently, the output of the model $\hat{y}(k)$ of order n is given by

$$\hat{y}(k) = \theta_{\text{off}} + \sum_{j=1}^{n} \theta_j u(k-j) = \underline{x}(k)^T \, \underline{\theta}, \tag{1}$$

where $\underline{x}(k)^T = \begin{bmatrix} 1 & u(k-1) & \cdots & u(k-n) \end{bmatrix}$ with $u(k<1)=0$ and the $(n+1)$-dimensional parameter vector $\underline{\theta} = \begin{bmatrix} \theta_{\text{off}} & \theta_1 & \cdots & \theta_n \end{bmatrix}^T$. To estimate $\underline{\theta}$, usually a least squares algorithm is used.

2.2 Online System Identification

The parameters $\underline{\hat{\theta}}(k)$ at time k can be estimated by

$$\underline{\hat{\theta}}(k) = \Big(\underbrace{\underline{X}(k)^T \underline{Q}(k) \underline{X}(k)}_{\underline{M}(k)} + \lambda \underline{R}\Big)^{-1} \underbrace{\underline{X}(k)^T \underline{Q}(k) \underline{y}(k)}_{\underline{t}(k)}, \tag{2}$$

with the regressor matrix $\underline{X}(k) = \begin{bmatrix} \underline{x}(1) & \underline{x}(2) & \cdots & \underline{x}(k) \end{bmatrix}^T$ and the measured output vector $\underline{y}(k) = \begin{bmatrix} y(1) & y(2) & \cdots & y(k) \end{bmatrix}^T$. The weighting matrix is defined by $\underline{Q}(k) = \text{diag}\left(\rho^{k-1}, \rho^{k-2}, \cdots, \rho^0\right)$ with the forgetting factor ρ. The $(n+1 \times n+1)$-dimensional regularization matrix $\underline{R}$ incorporates prior knowledge of the process and the strength of it is controlled by λ. Here, a first order impulse response preserving (IRP) kernel with exponential weighting is used. The offset parameter θ_{off} is unregularized. For more detail refer to [5].

The calculation of (2) can be simplified by calculating $\underline{M}(k+1)$ and $\underline{t}(k+1)$ recursively by

$$\begin{aligned} \underline{M}(k+1) =& \rho \underline{M}(k) + \underline{x}(k+1)\underline{x}^T(k+1), \\ \underline{t}(k+1) =& \rho \underline{t}(k) + \underline{x}(k+1) y(k+1). \end{aligned} \tag{3}$$

This approach corresponds to the LRLS algorithm [2].

3 Simulation Results

To investigate the proposed AMPC algorithm, a single tank system is considered. The task is to control the fill level $h(k)$ between 0 and 10 m in a tank with a small hole as an outlet by adjusting the inflow $u(k) = \dot{V}_{\text{in}}(k)$ of the fluid. For more information about the setup and the geometric data refer to [3]. Additionally, the fill level $h(k)$ is disturbed with additive white Gaussian noise.

The closed-loop AMPC, with no ($\lambda = 0$), medium ($\lambda = 50$) and strong ($\lambda = 10^6$) regularization, and an open-loop AMPC are compared. The open-loop AMPC interpolates linearly between previously learned models. These models are estimated at fill levels between 0 and 5 m (training data). For the exact configuration, refer to [3]. As reference trajectory (test data) different fill levels between 0.5 and 9.5 m are specified and held for 100 time steps each. Therefore, for the open-loop AMPC both interpolating and extrapolating behavior are investigated.

The fill level trajectories are depicted in Fig. 1. For a high value of λ the controller leads to overshoots. Furthermore, for $\lambda = 0$ the model is not robust, due to high parameter variance which leads to poor control ($400 < k < 500, k > 900$).

Table 1 shows the different root mean squared error (RMSE) values to compare the methods. RMSE_{all} calculates the RMSE in the interval $0 < k < 1000$, whereas $\text{RMSE}_{\text{inter}}$ only the interval $k \leq 500$ and $\text{RMSE}_{\text{extra}}$ only $k \geq 501$ considers. It can be seen that the RMSE of the closed-loop AMPC with $\lambda = 50$ is nearly the same as of the open-loop AMPC.

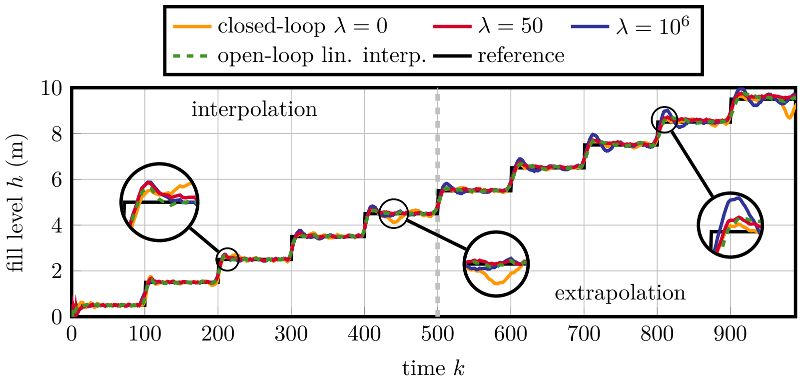

Figure 1. Fill level $h(k)$ for a sequence of steps for the different AMPC methods and various regularization strengths

Table 1. Performance of the different AMPC methods

method	closed-loop AMPC			open-loop
	$\lambda = 0$	$\lambda = 50$	$\lambda = 10^6$	AMPC
RMSE_{all}	0.148	**0.122**	0.153	**0.121**
$\text{RMSE}_{\text{inter}}$	0.128	0.108	0.119	**0.094**
$\text{RMSE}_{\text{extra}}$	0.166	**0.135**	0.182	0.143

During interpolation the open-loop AMPC and during extrapolation the closed-loop AMPC is preferable.

Figure 2 shows the changing model parameters $\hat{\underline{\theta}}(k)$ over time k. For $\lambda = 0$, the model parameters have a high variance error and with $\lambda = 50$ there is a good tradeoff between variance and bias error. In the approach with $\lambda = 10^6$, the wrong prior is weighted to highly which can be seen from the peaks of the model parameters $\hat{\underline{\theta}}(k)$. Additionally, it shows that the the open-loop AMPC do not change its parameters during extrapolation.

4 Conclusion and Outlook

We describe a closed-loop AMPC using regularized linear FIR models to control a nonlinear process. It is shown that incorporating prior knowledge of the process can improve the model quality. The closed-loop AMPC performs better in the extrapolation. Whereas, if a reference trajectory in the interpolation range is given, an open-loop AMPC should be chosen.

Further research will be done on extending this concept to also perform a hyperparameter optimization online.

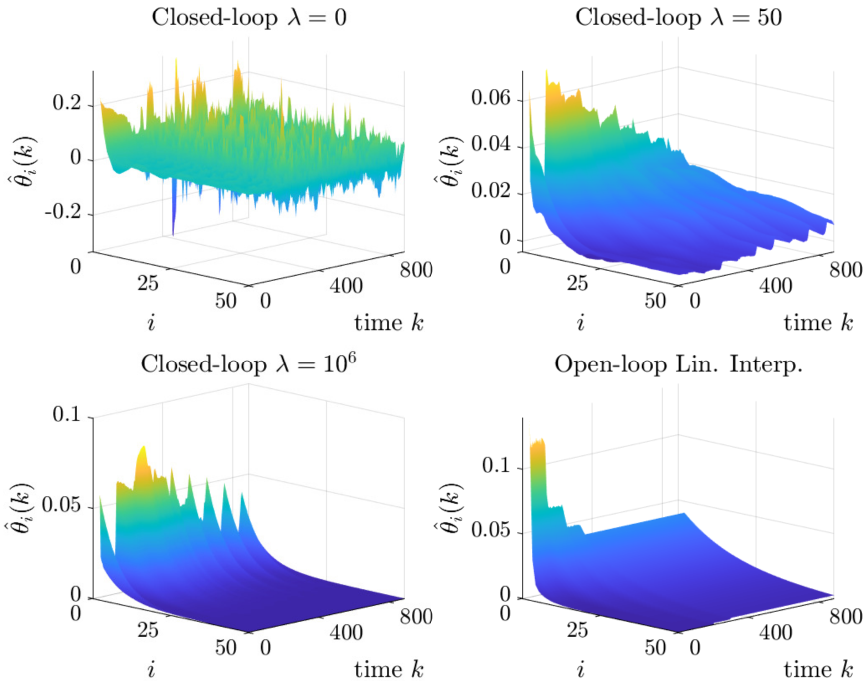

Figure 2. Adapted FIR coefficients during simulation of the sequence of steps for the different AMPC methods and various regularization strengths

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