What Do Slovenian and Croatian Teachers Know About Mathematical Modelling?

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Abstract In today's world, mathematical modelling is essential, and it is necessary to learn how to model. Therefore, considerable efforts must be made to make the mathematical modelling process understandable to all students. Almost all education systems are striving in this direction. Nevertheless, there is still relatively little modelling in early mathematics teaching practice. The paper describes what mathematical modelling is in the context of early mathematics and the benefits it brings to students. The survey was conducted in Slovenia and Croatia on a sample of 887 teachers in the first four/five years of elementary education. The participants answered questions on the general meaning of the term mathematical modelling as they understand it. After the answer, we provided the teachers with the definition of mathematical modelling. We then set up claims about mathematical modelling with which teachers could either agree or disagree on the five-point Likert scale. Finally, the teachers answered some questions about the obstacles in teaching mathematical modelling. The results of the study suggest that teachers in both countries feel a lack of professional skills to teach mathematical modelling efficiently. Besides, the results show the advantages of explicit attention to modelling in the curriculum documents.

Keywords: mathematics, cross-country comparison, curricular documents, teachers' beliefs, socially desirability bias.

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Introduction

Mathematics has been developed to describe the world almost from the beginning of human existence. Processes such as constructing, reasoning, predicting, guessing, organising data, quantifying, and so forth, are becoming increasingly more essential processes in the life of every person. English and Watters (2004) have stated that mathematical modelling provides a rich source of opportunity for developing these crucial processes. The fact that applications and modelling have been, and continue to be, central themes in mathematics education is not at all surprising. Nearly all questions and problems in mathematics education, that is questions and issues concerning human learning and the teaching of mathematics, influence and are influenced by relations between mathematics and some aspects of the real world (Niss et al. 2007, p. 22).

This essential interrelationship between the real world and mathematics has been recognised as of critical importance by many in education and educational research and has given rise to a sub-field of educational research related to the teaching and learning of mathematical applications and mathematical modelling. Niss et al. (2007) suggested that the *ICMI Study on modelling and applications in mathematics education* from 2004 might "formally mark the maturation of applications and modelling as a research discipline in the field of mathematics education" (p. 29). Niss et al. (2007) define applications (and modelling) as being when mathematics is applied to some aspect of the extra-mathematical world for some purpose including "to understand it better, to investigate issues, to explain phenomena, to solve problems, to pave the way for decisions, and so on" (Niss et al. 2007, p. 24).

Mathematical modelling conceived as real-world problem-solving is the process of applying mathematics to a real-world problem with a view of understanding it (Niss et al., 2007). It is more than using mathematics where we also start with a real-world problem, apply the necessary mathematics, but after having found the solution we no longer think about the initial problem except to check if our answer makes sense. With mathematical modelling, the use of mathematics is more for understanding the real-world problem/situation. The modeller also poses the problem(s) and questions (Stillman, 2019). In line with the described emphasis, we have decided for the following definition: "Mathematical modelling is an iterative process that involves the open-ended, real-world, practical problems that students make sense of with mathematics using assumptions, approximations, and multiple representations. Other sources of knowledge besides mathematics can be used as well" (Stohlmann & Albarracin, 2016, p. 2). To better illustrate the process of mathematical modelling, we can use the cycle of mathematical modelling from Blum and Leiss (2007) in Figure 1.

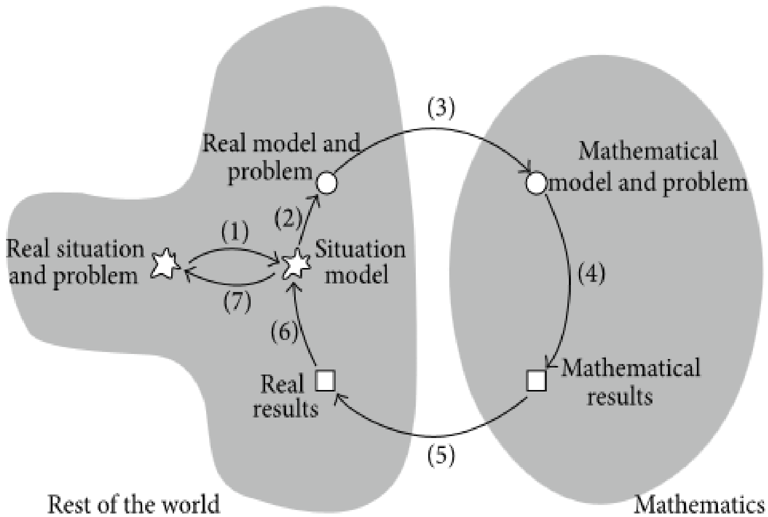

Figure 1: The mathematical modelling cycle according to Blum and Leiß (2007, p. 225)

In the modelling circle, we begin with the problem that is to be solved using math tools. In the first stage, the problem is described by relevant non-mathematical terms. During this phase, it is necessary to choose (simplify) assumptions. The outcome of the first phase is the conceptual model. This conceptual model is then translated into a mathematical model that can be mathematically analysed.

Furthermore, the mathematical solution is translated back into the language of the initial problem, which is called interpretation. Finally, we confirm the answer. If necessary, we will again begin the modelling circle by adjusting one or more steps (Spandaw & Zwaneveld, 2009).

Example: ***Uwe Seller's foot.*** The relevance of metacognition has increased in the context of mathematical modelling. It is essential to involve metacognitive activities during modelling processes to support modelling competencies. Concerning this, Blum (2011) underlines: "There are many indications that meta-cognitive activities are not only helpful but even necessary for the development of modelling competency" (Blum 2011, p. 22). In the example, Uwe Seller's foot, the importance of metacognitive skills is visible (Figure 2). The example shows several characteristics of students' as well as teachers' metacognitive skills (Vorhölter and Schwartz, 2020; Wendt et al., 2020)

> *Since August 2005, there has been a sculpture of the right foot of Uwe Seeler, a famous German soccer player, in front of the football arena in Hamburg, Germany.*

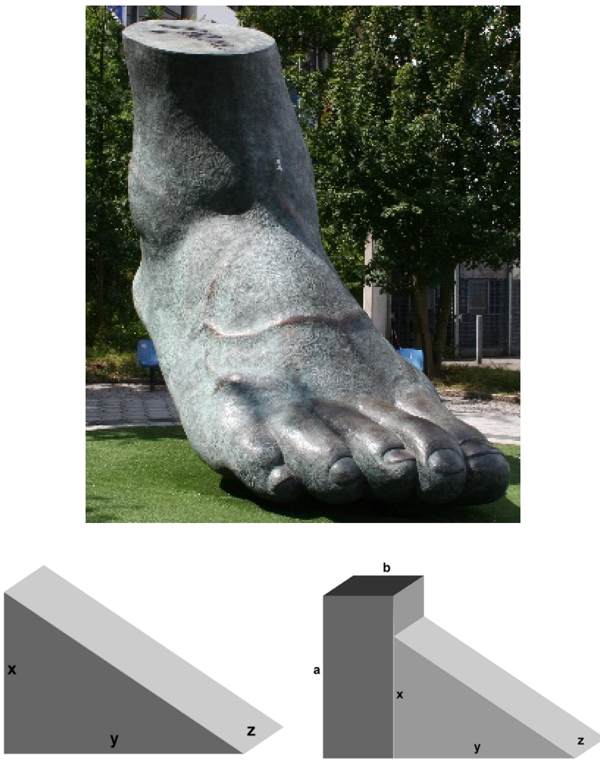

Figure 2: Fuß Uwe Seeler
(https://en.wikipedia.org/wiki/File:Fu %C3 %9F_Uwe_Seeler.JPG) and possible solutions for splitting the foot into geometric bodies (Vorhölter et al., 2019, p.8).

A newspaper, the Hamburger Abendblatt, reported that Uwe Seller's real foot fits exactly 3,980 times into the sculpture. Is it possible? Uwe Seller's shoe size is 10½.

Mathematical modelling helps students understand the world around, and it contributes to the development of different competencies and appropriate attitudes including flow (Csíkszentmihályi's, 1990, as cited in Liu, & Liljedahl, 2019). It fosters metacognitive skills, conceptual understanding, creative and innovative abilities, the socio-cultural role of mathematics, and is linked to several other competencies such as reading, communication, design, and application of problem-solving strategies, which emphasise high cognitive skills (Blum, 1994; Blum & Borromeo Ferri, 2009; English, 2010; English & Watters, 2004). From an educational perspective, the purpose of modelling can be considered as an end in itself or as a method to achieve the goal of constructing mathematical knowledge (Cai et al., 2014). The first purpose is based on the assumption that the ability to model and find solutions, for life situations, is a competence that can serve an individual in daily life and the workplace. Another purpose is achieved when an individual constructs new knowledge or reconstructs the knowledge they have already acquired when engaging in a modelling process (van den Heuvel-Panhuizen, 2003). As modelling requires the use of previously acquired mathematical knowledge in different ways, it promotes a flexible and adaptable mindset for mathematical competences. Challenging modelling problems, however, require the acquisition of new mathematical facts, skills and processes, which involves the construction of new knowledge (Cai et al., 2014). Mathematical modelling tasks enable young students to understand the importance and usefulness of mathematics for individuals as well as for society and create opportunities for students to perceive mathematics as useful and applied rather than abstract and isolated (Asempapa, 2015).

Mathematical Modelling in Elementary School

Teaching mathematical modelling is a demanding job. Teachers need to draw on several dimensions of knowledge, including but not limited to pedagogical content knowledge (Blum, 2012), knowledge of the modelling process, understanding of student background and experience, and knowledge of teaching practices that facilitate individual and group learning (Zawojewski, Lesh, & English, 2003). Besides, the open nature of many modelling activities means that teachers need to take students' ideas into account and respond to them as they emerge. At the same time, understanding of mathematical modelling as an open, exploratory and dynamic process mean that learning to involve children in mathematical modelling had to include opportunities for teachers to learn from their practice. All classroom instruction is relational work (Lampert 2010), and modelling, which involves ongoing negotiations about the meaning and importance of contexts, assumptions, representations and mathematical strategies, intensifies relational work between and among teachers and students. It was, therefore, essential to involve teachers in the process of reflecting on modelling from two perspectives: from the perspective of a student learning to model and from the perspective of a teacher teaching others to model.

Unfortunately, mathematical modelling is rarely taught in elementary school, and even less so in primary school. Until recently, mathematical modelling was often not included in the elementary school curriculum (Brown & Ikeda, 2019). However, the basics of mathematical modelling can and should begin in elementary school where children already possess the necessary competencies on which modelling can be developed (English & Watters, 2004). For instance, Albarracína and Gorgorió (2019) argue that the use of significant number estimation problems is a suitable activity for introducing mathematical modelling processes in primary school classrooms. Results show that students can become active participants in mathematical modelling activities, even if they are fifth-grade underachievers (Zubi et al., 2019). Mellone et al. (2017) investigated whether there is a relationship between Grade 5 students' situation models and the realistic nature of their answers to problems. In clearly defining modelling as the process of creating a mathematical model from a situation model, they found working in pairs and rewording then solving led to an increase in realistic responses, but for only one problem.

Traditionally mathematical modelling in elementary schools was misunderstood as solving arithmetic problems with words (tasks with words) in which concrete materials were presented that would then model the more abstract operating rules (English, 2003; Stohlmann & Albarracin, 2016). Many of the mathematical tasks used in elementary schools are word problems - applications where either the real world does not affect the problem, either there is a clear solution strategy (Tran and Dougherty 2014). In mathematical modelling, problems can be open at the beginning of the investigation. The modellers can ask different mathematical questions about a scenario, open in the middle when the modellers investigate different solution strategies and open at the end when the modellers consider how the models can or cannot be applied to other situations. The struggle with openness in modelling is a feature that conveys the idea that real-world situations do not always have a single, clear beginning, approach, or solution.

Dogan Temur (2012) stated that effective teaching of mathematics during the first grade of primary school is crucial for the formation of students' mathematical thinking. Often solving such problems is not a modelling task for students, but relies on keywords or phrases in the problem, such as times, more, less, and so on (English, 2003); for example, *Suzy saved $12. Maria saved three times more than Suzy. How much did Maria save?* Furthermore, there is often only one way of interpreting the problem, so students are forced into limited mathematical thinking. Although we do not dispute the importance of this type of tasks, they do not address enough mathematical knowledge, processes, fluency and social skills that our children in the twenty-first century need. Below, we present a task that involves mathematical modelling and is appropriate for lower grades of primary school.

Example: *Beans, beans, glorious beans*. *Beans, beans, glorious beans* is a more straightforward mathematical modelling task in which students test their knowledge of the conditions required for plant growth and use minimal mathematical calculations. This assignment, as noted by English and Watters (2004), proved to be exciting and encouraged students to participate in the joint discussion. The task was initially published by English and Watters (2004, p. 5).

> *Farmer Sprout is trying to decide which light conditions are best for growing Butter beans. To help Farmer Sprout make his decision, he went to visit the Farmers' Association who are growing climbing Butter bean plants using two different light conditions. The two light conditions being tested are:*
>
> *a) Growing Butter beans out in the full sun with no shade at all*
> *b) Growing Butter beans underneath shade-cloth.*

Table 1: Butter beans crops.

Sunlight				Shade			
Butter Bean Plants	Week 6.	Week 8.	Week 10.	Butter Bean Plants	Week 6.	Week 8.	Week 10.
Row 1	9 kg	12 kg	13 kg	**Row 1**	5 kg	9 kg	15 kg
Row 2	8 kg	11 kg	14 kg	**Row 2**	5 kg	8 kg	14 kg
Row 3	9 kg	14 kg	18 kg	**Row 3**	6 kg	9 kg	12 kg
Row 4	10 kg	11 kg	17 kg	**Row 4**	6 kg	10 kg	13 kg

Using the data above, determine which of the light conditions is suited to grow Butter beans to produce the greatest crop. In a letter to Farmer Ben Sprout, outline your recommendation of the light condition and explain how you arrived at this decision. Predict the weight of butter beans produced on week 12 for each type of light. Explain how you made your prediction so that Farmer Ben Sprout can use it for other similar situations.

(English and Watters, 2004, p. 5).

With the help of this type of assignment, students can exchange ideas about possible solutions, since a unique solution does not limit the task. They also learn to cooperate and are forced to widen perspectives to solve the task themselves; for example, some students will ask questions asking about rain, whether this will affect the yield of the beans, in which climate the farmer lives, and so on.

Example: *The theme park.* We list another task appropriate for students in early primary education that involves mathematical modelling, as written in Bleier-Baxter et al. (2017, p.22).

The park manager needs your help. His intern was able to fill in only a selection of the information needed. Your group will work to complete the table with missing wait times for Universal Studios Park. You will need to mathematically justify how your group has decided to fill in the table and explain your strategy clearly on chart paper. You can use pictures, words, and symbols to help you make your strategy for filling in the missing data as clear as possible for another group to follow.

Table 2: Wait times for Universal Studio Park.

Ride	Low	Moderate	Busy	Very busy
Production Central				
Despicable Me: Minion Mayhem 3D	30	50	110	155
Transformers: The Ride 3D		30		
Shrek 4D	10			
Hollywood Rip Ride Rockit	20	30	75	110
New York				
Twister: Ride It Out	5	10	15	20
Revenge of the Mummy				90
The Blues Brothers Show	10			
San Francisco				
Beetlejuice's Graveyard Revue		15		
Disaster!			35	
World Expo				
Men in Black™: Alien Attack			55	
The Simpsons Ride	15	35	60	90
Fear Factor Live		10		
Woody Woodpecker's KidZone				
Animal Actors on Location	10	15	15	20
A Day in the Park with Barney			20	
Curious George Goes to Town		20		
Woody Woodpecker's Nuthouse Coaster				45
Fievel's Playland			15	
E. T. Adventure			45	
Hollywood				
Universal Orlando's Horror Make-Up Show	10	20	30	45
Terminator 2: 3D			35	
Lucy -A Tribute	5			

> *Your group will present your strategy for filling in the missing data to the whole class. During the presentation, each member of the group should be prepared to justify the group's strategy mathematically. Our class goal is to find the best possible model for filling in an amusement park wait times. Important note: There is no one correct way to complete this task. Your task is to try to find the best possible strategy.* (Bleier-Baxter et al., 2017, p.22).

Real-world contexts are full of underlying patterns that can be represented mathematically. To successfully "see" such underlying patterns, students must be able to identify relevant quantities in the situation. Then, they must make decisions about the best way to map relationships using mathematical tools, such as diagrams, tables, graphs, flowcharts, and formulas (Bleiler-Baxter et al., 2017). In their research with the theme park task, Bleiler-Baxter et al. (2017) stated that students identified relevant quantities within the table in different ways and that their ways of seeing the table influenced how they mapped relationships. Some students focused on pairs of data in the table and identified a possible multiplicative relationship among those pairs, some concentrate on complete rows of data and identified additive relationships. Bleiler-Baxter et al. recognised that to engage students in modelling with mathematics; it is critical for teachers to value student autonomy and to select a task that represents a complicated real-world situation. Giving students the freedom to make decisions means the decisions they make may vary. It also means the teacher must be patient to let students struggle through the process of weighing pros and cons of their decisions and not to bypass the decision-making process by stepping in and making decisions for students (Bleiler-Baxter et al., 2017).

Teachers and Mathematical Modelling

Teacher training is considered necessary in achieving high-quality teaching of mathematics. A critical step in developing and promoting high-quality modelling experiences for children and youth is to understand the learning opportunities teachers need to facilitate such experiences. Modelling will not become an integral part of students' mathematical learning if their teachers are not prepared to provide classroom leadership in this area (Fulton et al., 2019). In-service training is most effective when it is sustainable, intensive and integrated into the daily work of teachers (Garet Porter, Desimone, Birman, & Yoon, 2001). It should also be student-centred (Hawley and Valli 1999, p. 137) and take into account the existing knowledge, experience and beliefs of teachers.

Regardless of the benefits that mathematical modelling brings to learning mathematics, several situations present challenges for teachers. Teachers must give up most of their traditional ways of engaging their students to achieve goals of mathematical modelling we described previously (Asempapa, 2015).

It is known that teachers (falsely) assume that a large proportion of students finds modelling difficult or challenging, and therefore teachers rarely apply modelling activities in the classroom (Asempapa, 2015; Brown, 2019; Spandaw & Zwaneveld, 2009). Even experienced and skilled teachers may not automatically transfer their knowledge of teaching mathematics to teaching mathematical modelling. Thus, teachers need a variety of experiences and support to effectively engage students in mathematical modelling (Fulton et al., 2019). Additionally, in most textbooks developed for elementary and secondary schools, less emphasis is placed on mathematical modelling activities. Teachers rarely use modelling tasks in their classrooms due to time constraints and their perception that mathematical modelling tasks are complex and demanding (Borromeo Ferri, 2010).

Example: ***Harvesting the eucalypt forest.*** To facilitate teacher's mathematical Stillman (2019, p.2) listed a task that was used in a university mathematics unit for primary pre-service teacher education students in Australia. The students had four weeks to work on the task independently outside the class. The task follows:

> *Those of you who drive the Western Freeway between Ballarat and Ballan will have noticed that a large plantation of Eucalypts has been felled and the logs transported away. Using mathematical modelling pose a problem related to the removal of the forest that can be mathematised and solved [The task was accompanied by several photographs taken before, during and after the felling of the trees.].* (Stillman, 2019, p.2.)

The purpose of mathematical modelling was to analyse an existing real-world situation (the felling of a forest) as a means of answering a practical question. Both mathematical and extra-mathematical knowledge were needed to answer this question. It is also an example of using modelling as empowerment for students to become independent users of their mathematics (Stillman, 2019). Any solution depends on particular assumptions. Different assumptions may lead to a different solution method which may not be what the teacher intended. A critical aspect of mathematical modelling is that the modeller makes decisions, for example, considering some, but not all real-world aspects in one's initial solution, describing how to interpret terms such as the best one. Such mathematical thinking naturally leads to diverse solutions, but the task must be presented in such a way as to allow this (Brown & Ikeda, 2019).

Fulton et al. (2019) identified four features of modelling practice that could be developed and used by the novice, as well as experienced modellers: a) wrestling with openness in modelling, b) posing mathematical problems to address real-world situations, c) making choices creatively while modelling and d) revisiting ideas and revising solutions during the modelling process. Fulton et al. (2019) stated that these features were unlikely to be a part of most teachers as learners' experiences in teaching and learning mathematics. Wrestling with openness in modelling is a feature that conveys the idea that real-world situations do not always have a single, clean-cut beginning, approach, or solution. Problem posing is a central feature not only of mathematical modelling, but of mathematical activity in general that can occur before, during, or after the solution of a problem. Furthermore, making choices creatively while modelling focuses on the ability to determine what mathematics the modeller will use or develop to make progress on a modelling task. Finally, revisiting ideas and solutions during the modelling process involves considering whether the solution makes sense in light of the initial problem (Fulton et al., 2019).

Slovenian school curriculum (Žakelj et al., 2011) explicitly address mathematical modelling. The curriculum also contains a topic entitled *Mathematical problems and real-life problems from first to ninth grade*. The Croatian curriculum devoted less attention to modelling; the term mathematical modelling was not mentioned in the elementary school curriculum at all. After our study, in the school year of 2019/2020, the revamped Croatian curriculum was applied, addressing modelling more explicitly (Ministarstvo znanosti obrazovanja i sporta, Hrvaška [Ministry of Science and Education Croatia], 2018).

The research aimed to examine Slovenian and Croatian teachers' self-reported knowledge on beliefs about mathematical modelling. Participants were teachers in the lower grades of public elementary schools.

Methodology

An exploratory study using a self-constructed questionnaire delivered to teachers in both countries was used. We have used the methods of quantitative empirical pedagogical research. The questions were set in Croatian for teachers from Croatia and in Slovene for teachers from Slovenia. We asked the teachers several questions addressing their knowledge and beliefs following two main research questions:

1. What are teachers' beliefs regarding mathematical modelling?
2. Are there any differences among countries?

Sample

The survey was carried out based on completed questionnaires on a convenience sample of 438 teachers from Slovenia and 449 teachers from Croatia, together 887. In Slovenia, we were interested in the knowledge of first to fifth-grade elementary teachers, because of the nine-year elementary school system of education, In Croatia, we were interested in the knowledge of first to fourth-grade elementary teachers, because of the eight-year primary school system.

Table 3: Sample structure

Country	*f*	*f* %
Slovenia	438	49.4
Croatia	449	50.6
Total	887	100

Instrument

A questionnaire was designed with several types of questions: (a) questions about teachers' basic data (gender, years of work experience in the classroom), (b) general questions about mathematical modelling, (c) claims regarding mathematical modelling given with a five-point Likert scale of agreement/disagreement, (d) questions about teachers' opinions regarding introducing mathematical modelling into the classroom.

Before conducting the survey, we did a pilot survey on 20 primary teachers in Slovenia and Croatia and adapted the questionnaire according to the results. The questionnaires we used to collect the data were online surveys. We surveyed from February 2019 to June 2019. As we collected data using an online survey, teachers from all parts of Slovenia and Croatia were represented. The obtained data were processed and analysed using the IBM SPSS statistics 22 program.

Results

In Table 4, we present the results regarding whether teachers were ever acquainted with the concept of mathematical modelling.

Table 4: Teachers' acquaintance with the concept of mathematical modelling

Are you acquainted with the concept of mathematical modelling?			
		f	*f* %
Slovenia	yes	41	9.4
	no	343	78.3
	barely	54	12.3
	total	438	100
Croatia	yes	32	7.1
	no	373	83.1
	barely	44	9.8
	total	449	100
Slovenia and Croatia	yes	73	8.2
	no	716	80.8
	barely	98	11.0
	total	887	100

The participating teachers from Slovenia and Croatia are not familiar with the concept of mathematical modelling. Only 95 (21.7 %) of Slovenian participants are acquainted or barely acquainted with the concept of mathematical modelling. Similarly, only 76 (16.9 %) of Croatian participants are acquainted or barely acquainted with this concept. A vast majority of the teachers in both countries have not heard about the term mathematical modelling. There was no difference between Slovenian and Croatian teachers regarding this question ($\chi^2 = 3.251, p = .197$). The next question regarding modelling targets teachers' experience in class. Table 5 presents the results.

Table 5: Teachers' incorporation of mathematical modelling into teaching

Do you incorporate tasks/activities that include mathematical modelling in your math class?			
		f	*f* %
Slovenia	yes	34	9.4
	no	276	75.8
	sometimes	54	14.8
	total	364	100
Croatia	yes	39	8.7
	no	344	46.6
	sometimes	66	14.7
	total	449	100
Slovenia and Croatia	yes	73	9.0
	no	620	76.2
	sometimes	120	14.8
	total	813	100

The results in Table 5 show that among Slovenian participants, 87 teachers (20.0 % of 438) include or sometimes include what they believe to be mathematical modelling in the classroom practices. Similarly, 105 (23.4 % of 449) Croatian participants' activities in their teaching include or sometimes include what they believe to be "mathematical modelling" in the classroom practices. We observed that an additional 20 teachers include mathematical modelling into classroom practice even if their answer to the first question was that they are not acquainted with mathematical modelling. There was no difference between Slovenian and Croatian participants regarding this question ($\chi^2 = 0.115, p = .944$).

When teachers completed those two general questions, we provided them with the definition of mathematical modelling by Stohlmann and Albarracin (2016). We set several statements regarding mathematical modelling, which teachers could answer. For that paper, we present the results (Table 3) for three statements.

Statement A: Mathematical modelling is an exact, formal process, or a collection of formulas and rules that have to be applied.

Statement B: Although mathematical modelling activities improve students' ability to solve problems, I find that there are too many obstacles to incorporate such activities into my math classes.

Statement C: I think mathematical modelling is a necessary skill in the 21st century for every student.

The results are presented in Table 6.

Table 6: Agreement with claims regarding mathematical modelling

		Statement A		Statement B		Statement C	
		f	*f* %	*f*	*f* %	*f*	*f* %
Slovenia	Completely disagree	40	20.8	53	27.6	2	1.1
	Partially disagree	34	17.7	39	20.3	6	3.2
	Neither agree nor disagree	45	23.4	34	17.7	21	11.1
	Partially agree	61	31.8	62	32.3	82	43.4
	Completely agree	12	6.3	4	2.1	78	41.3
Croatia	Completely disagree	71	22.3	59	18.2	6	1.9
	Partially disagree	45	14.2	57	17.6	11	3.4
	Neither agree nor disagree	100	31.4	80	24.7	35	10.9
	Partially agree	87	27.4	112	34.6	137	42.5
	Completely agree	15	4.7	16	4.9	133	41.3
Total	Completely disagree	111	21.8	112	21.7	8	1.6
	Partially disagree	79	15.5	96	18.6	17	3.3
	Neither agree nor disagree	145	28.4	114	22.1	56	11.0
	Partially agree	148	29.0	174	33.7	219	42.9
	Completely agree	27	5.3	20	3.9	211	41.3
	Total	510	100.0	516	100.0	511	100.0

Results reported in Table 6 are somewhat disappointing. Although we defined the mathematical modelling before, 73 (16.6 %) of Slovenian participants and 102 (22.7 %) of Croatian participants completely or partially agree with the statement A. That result was surprising, given the definition was written in the survey just before the statement A. It is obvious that statement A contradicts the definition. We also notice that 66 (15.0 %) of Slovenian participants and 128 (28.5 %) of Croatian participants partially agree or completely agree with statement B. However, they know that mathematical modelling improves many student capabilities, such as solving problems. 160 (36.5 %) of Slovenian participants and 260 (57.9 %) of Croatian participants partially agree or completely agree that mathematical modelling is a necessary skill for every student (statement C). The results for statement A and statement C do not differ considering the country. We applied the c^2-test (statement A: $c^2 = 5.136$, $p = .274$; statement C: $c^2 = 0.540$, $p = .969$). There are, however, significant differences regarding statement B ($c^2 = 10.763$, $p = .029$). Slovenian participants are more prone to incorporating modelling activities in teaching. Namely, almost half of the participating Slovenian teachers (47.9 %) completely or partially disagree with statement B. Slightly more than one-third of the participating Croatian teachers (35.8 %) completely or partially disagree with statement B.

The last set of questions was related to the teachers' thoughts on whether they thought they were educated enough to teach mathematical modelling in elementary math classes. Some results are shown in Table 7.

Table 7: Teachers' opinions about their professional knowledge regarding mathematical modelling

I believe that I am sufficiently educated to teach mathematical modelling in elementary math classes.			
		f	*f* %
Slovenia	Yes	43	22.6
	No	147	77.4
Croatia	Yes	85	26.4
	No	237	73.6
Total	Yes	128	25.0
	No	384	75.0
	Sum	512	100.0

As expected, most participants feel that they are not educated enough to teach mathematical modelling (75 %). Some of the reasons they mentioned in the open part of the question were: (1) insufficient workshops/education/professional training on this subject, (2) lack of examples and materials, (3) have never met this term, (4) too little experience in working with mathematical modelling, and so on. There are no significant differences among countries ($c^2 = 0.904$, $p = .342$).

Discussion and Conclusion

As the world -as well as the STEM area -progresses, we see the need for a growing number of young people who are good at connecting, constructing and modelling everyday problems. Modelling is a mathematics teaching strategy that encourages a real-world connection to the abstract world of mathematics. Mathematical models help many sciences solve their problems, which is why mathematical modelling competency is highly desirable in the modern world (Merrit et al., 2017). These are just a few of the reasons why we need mathematical modelling, and why it needs to be practised it in elementary school teaching. Our results show that more than three-quarters of lower elementary school teachers from Croatia and Slovenia have never been acquainted with the concept of mathematical modelling, one-tenth of them have barely been acquainted with mathematical modelling. The result is in line with several other studies (e.g. Stillman et al., 2013). Therefore, teacher education is crucial.

Our study detected a high rate of social desirability bias, which refers to the tendency of research subjects to give socially desirable responses instead of choosing answers that are reflective of their real feeling (Muijs, 2006). The teachers first claimed to be unfamiliar with mathematical modelling and then claimed to be introducing it into teaching (see Table 2). As we expected such a situation, we defined the term mathematical modelling. Nevertheless, teachers still provided unreasonable answers (see statement A). It seems that the questionnaire used in this study is not a sensitive instrument when examining teachers' attitudes beliefs regarding mathematical modelling. This phenomenon also occurred in several different areas of education dealing with novel approaches.

Results, reported by Lüke and Grosche (2018), for instance, show that the attitude of the organisation conducting the survey -as perceived by the participant - outperforms well-documented variables (such as sex, age, and contact to a person with a disability) in predicting the attitudes of the participant towards inclusion. Positive bias in the attitudes of participants was evident when a university surveyed them. Lüke and Grosche (2018) argued that social desirability is a neglected issue in research on attitudes towards inclusive education. Our results confirm that hypothesis in the area of mathematical modelling in education.

After describing the definition of mathematical modelling, we gave the teachers statements regarding mathematical modelling in which they could agree or disagree. In the first claim that mathematical modelling is an exact, formal process or collection of formulas and rules to be applied, more than one third (34.3 %) of the teachers partially or completely agree with the statement. However, earlier in the poll, the definition of mathematical modelling was given. There is support in the literature for the claim that teachers' beliefs appear to act as filters through which teachers interpret and ascribe meanings to their experience as they interact with children and the subject matter (Bergman Ärlebäck, 2010). Depending on the mathematical beliefs held by the teacher, it is more or less likely that they build up obstacles for introducing applications and modelling in their mathematics teaching (Bergman Ärlebäck, 2010).

Our results show that teachers agree with the importance of modelling (84.2 %). However, more than one-third of teachers (37.6 %) (partially or completely) agreed that although mathematical modelling activities improve students' ability to solve problems, there are too many obstacles to incorporate such activities into teaching maths. Students of different ages -even very young students -can learn to model, but it requires effort and investment in the sense of careful and focused teaching design, learning environments, activities and time to develop such activities and tasks (Niss, 2012).

Slovenian and Croatian teachers are aware of the fact that they are not educated enough to teach mathematical modelling (75 %) and that they need additional workshops/training, which is in line with other studies (e.g. Fulton et al., 2019). One way of providing future teachers with the necessary professional knowledge is to offer specific modelling seminars already at the university, with their own compulsory teaching experiences (Blum, 2012). We found that there were almost no significant differences between Slovenia and Croatia. These are traditionally similar school systems. In mathematics, education systems respond similarly, which has been shown in other studies from the area of mathematics education. Lipovec and Ferme (2018) report the results regarding mathematics homework practises in three countries. Slovenia, Croatia and the Slovak Republic. Even though some homework policies differ, Slovenia and Croatia are much closer to each other in homework practices than they are to the Slovak Republic. On the other hand, Sabo and Lipovec (2017) provided evidence for differences in opinions of Croatian and Slovenian teachers on the differences between curricular mathematical content.

Nevertheless, differences occurred regarding the statement B. We believe that the curricular framework in Slovenia helped Slovenian teachers shape their beliefs more accurately. In Slovenia, curricular renewal occurred in 2008, and in 2011, the documents were updated. Notably, in 2011, much attention was paid to integrating mathematics with real-world experience and applicability. In the last three years of elementary education, mathematics modelling was added as obligatory content and was also explicitly mentioned in the national mathematics curriculum (Žakelj et al., 2011).

National Educational Institute in Slovenia provided teachers with many resources. In the book *Upgrading teaching in elementary school practice* (Suban & Kmetič, 2014), the whole Chapter 4 is dedicated to mathematical modelling. Unfortunately, almost all sources focus too much on the last three years of elementary education, and mathematical modelling in Slovenia, also, has not come to life in primary school classrooms. Slovenia offers mathematical modelling materials for free-to-all users in e-textbooks (I-textbooks, 2014). The task example *Beans, beans, glorious beans,* as well as several other examples could be found in the i-textbook for fifth grade (Bajramović et al., 2014, pp. 484, 488).

The teacher plays a vital role in supporting student engagement in mathematical modelling, the reflection on it, decisions related to technology use and mathematical modelling and the interactions between these (Brown & Ikeda, 2019). We believe that it is necessary to give greater importance to the education of teachers for the implementation of mathematical modelling in teaching, and to encourage them in the introduction, work and performance of mathematical modelling tasks in the classroom. Teachers' knowledge about modelling activities can be developed through their active engagement in modelling activities. Several researchers have proposed interventions in which practising and prospective teachers engage in modelling activities as learners.

The teachers develop an understanding of the nature of mathematical modelling, of the relationship between mathematical modelling and meaningful understanding, and the nature of mathematical modelling tasks (Shahbari & Tabach, 2019). We propose increasing and developing available assignments and tasks that include mathematical modelling for elementary school. We also emphasise the need to research mathematical modelling in elementary schools, since such research is not currently available in either Slovenia or Croatia.

A more in-depth teachers' knowledge of mathematical modelling could perhaps be obtained through a similar experiment to the one conducted by English and Watters (2004) in Australia. Elementary school teachers presented four mathematical modelling tasks to their students (one of which was introduced in the introductory section of this paper) for six months, and tracked both the teachers' and their students' progress. The teachers agreed that the children enjoyed the activities, although they were perceived to be challenging. They also stated that there were substantial social gains concerning group work, social interaction, reporting and questioning skills. They considered the students at the end of the year ready to engage in more mathematically oriented tasks. They were more prepared to question assumptions and each other's interpretation of the data (English et al., 2004).

Our results show, among other things, the importance of curricular documents. In Slovenia, the curriculum contains mathematical modelling, which has shaped a more positive attitude towards the introduction of modelling into school practice. Nevertheless, the results are still disappointing, since the vast majority of teachers in Slovenia and Croatia (approximately 75 %) do not feel competent enough to teach mathematical modelling.

A crucial next step in developing and promoting quality modelling experiences for children and young people is to understand the learning opportunities that teachers need to enable such experiences. Modelling will indeed not become an integral part of students' mathematical learning if their teachers are not prepared to take the lead in this area in the classroom.

When students have opportunities to model with mathematics, they can improve their problem-solving abilities, reason mathematically, and make connections to real-world problems (Bleiler-Baxter et al., 2017). We want all students to be involved in solving real-life problems. Teachers and students should develop shared expectations for mathematical modelling in elementary school (Bahmaei, 2011). In modelling, students will face the problems that matter to them and the society in which they live. They will have to decide which information is relevant, make approximations and use appropriate mathematical tools wisely. As teams, students will persevere through challenges, and surprise us with the ways they can use mathematics to improve the world in which all of us live. Undoubtedly, we know that mathematical modelling is particularly tricky and complicated for teachers, but many studies show and point to its benefits and well-being for students. However, to achieve those goals, we first have to convince (and educate) their teachers about the importance of mathematical modelling in lower grades of elementary school.

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