Sustainability Profit Generated by the Optimization of Continuous Beams

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Abstract This paper deals with the optimization of sustainability profit generated by the production of continuous beams. A number of beams designed from three alternative materials, laminated timber, reinforced concrete and structural steel, are considered. Three different optimization models are developed for each of the materials. Furthermore, two different objectives are defined for each material alternative: for economic profit and for sustainability profit (which includes the eco costs of global warming). The design, resistance and deflection constraints of these beams are based on specifications prescribed by Eurocode standards. A mixed-integer non-linear programming (MINLP) approach using GAMS/Dicopt is applied to find most advantageous material alternative for the beams. The results of the numerical analysis clearly show that reinforced concrete beams provide the highest economic profit and the highest sustainability profit.

Keywords: Sustainability profit, GHG emissions, optimization, Mixed-integer non-linear programming, MINLP.

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1 Introduction

We present a model for the optimization of sustainability profit generated by the production of continuous beams for civil engineering. Sustainability profit is the sum of economic profit and the eco costs of global warming. A number of beams designed from three different materials, laminated timber, reinforced concrete and structural steel, are included. Furthermore, two different objectives are defined for each material alternative: economic profit and sustainability profit (which includes the eco costs of global warming).

Different techniques and objectives for optimization and sustainability have been introduced. Zaforteza *et al.* (2009) took into account a simulated annealing algorithm (SA) applied with two objective functions, embedded CO_2 emissions and the economic cost of reinforced concrete structures. Camp and Huq (2013) used a hybrid big bang-big crunch algorithm (BB-BC) for the optimal design of reinforced concrete frames to minimize either the total cost or the CO_2 emissions. Alonso and Berdasco (2015) proposed a carbon footprint for sawn timber products. Li *et al.* (2017) presented a topology optimization in order to obtain the best-possible welded box-beam structures that emit less greenhouse gases by using an improved ground structure method (IGSM).

2 MINLP model formulation

It is assumed that a general non-linear and non-convex continuous/discrete optimization problem for the production of continuous beams can be formulated as an MINLP problem:

$$\min \; z = f(\mathbf{x},\mathbf{y})$$
$$\text{subjected to: } g_k(\mathbf{x},\mathbf{y}) \le 0 \quad k \in K$$
$$\mathbf{x} \in X = \{\mathbf{x} \in R^n \text{: } \mathbf{x}^{LO} \le \mathbf{x} \le \mathbf{x}^{UP}\}$$
$$\mathbf{y} \in Y = \{0,1\}^m$$

where **x** is the vector of the continuous variables and **y** is the vector of the discrete (0, 1) variables. Function $f(\mathbf{x}, \mathbf{y})$ is the objective function, which is comprised of the economic and sustainability profits. g_k (**x**,**y**) represents the design, resistance and deflection constraints of the beams.

3 Numerical example

The example illustrates the optimization of 300 equal continuous beams; see Fig. 1. Each beam is a two-span (2L) structure having altogether a length of 2·5.0=10.0 m, subjected to the combined effect of dead-weight, permanent continuous load of 10.0 kN/m (g) and imposed variable continuous load of 12.0 kN/m (q).

Each continuous beam is made from one of three different materials: laminated timber, reinforced concrete and structural steel, see Fig. 2. At this point, a comparison of competitiveness between these three materials was calculated for various material and dimension alternatives, and for two different objectives: optimization of economic profit and of sustainability profit.

For comprehensive topology optimization problems, we usually use the MipSyn program (Kravanja, 2010). Because the optimization problem of beams in this study is a simple discrete and non-linear problem, the Dicopt application (Grossmann, 2002) was selected. Six optimization models (CONBOPT) were developed for a combination of three different materials (timber, reinforced concrete and steel) and two different objective functions. GAMS (General Algebraic Modelling System), (Brooke *et al.*, 1988), was used for mathematical modelling. The objective functions of the models were subjected to design, loading and resistance constraints known from structural analysis. The dimensioning and deflection constraints were performed according to Eurocode specifications: Eurocode 5 (2004) for timber, Eurocode 2 (2004) for reinforced concrete and Eurocode 3 (2005) for steel. The beams were checked for shear, bending moment and lateral torsional buckling resistances as well as for vertical deflections.

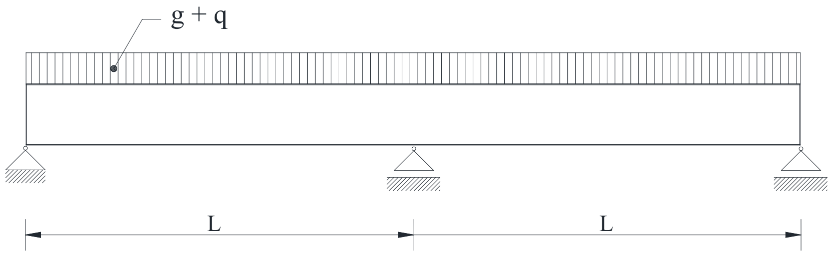

Figure 1: Continuous beam.

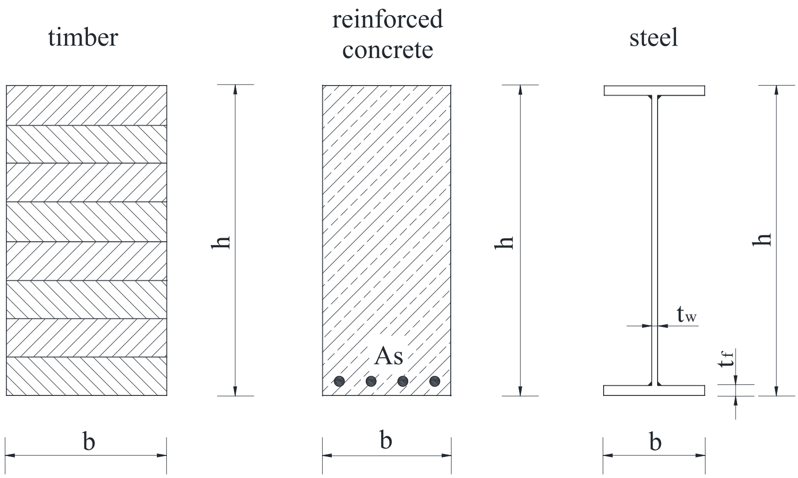

Figure 2: Cross sections of the continuous beam.

The continuous beam superstructure is comprised of three different materials. The laminated timber beam superstructure consists of 101 different rounded dimension alternatives for the cross-section width and 131 rounded dimension alternatives for the cross-section height. The reinforced concrete beam superstructure contains 7 different concrete grades, 13 standard reinforcing steel bars, 131 rounded dimension alternatives for the cross-section height and 101 rounded dimension alternatives for the cross-section width (rounding up by whole centimeters). In addition, 3 different steel grades, 8 different dimension alternatives of the standard steel plate thicknesses for flanges and webs separately, 1051 rounded dimension alternatives for the width of the flange and 1301 rounded dimension alternatives for the height of the web are included in the steel beam superstructure.

The material and dimension alternatives (binary variables) give 13,231 structure alternatives for the timber beam, 1,204,021 different structure alternatives for the reinforced concrete beam and 262,531,392 different structure alternatives for the steel beam.

Two different objective functions were used for two different defined criteria. The first criterion of optimization includes maximization of economic profit (P_E [€]) of 300 equal beams. The economic profit is defined as the sum of the selling price, minus the self-manufacturing material and labor costs and overheads. The objective function was determined for three different materials separately; see Eq. (1). N is the number of continuous beams (N = 300), C_S [€] is the selling price of a single continuous beam, and C_{Mi} [€/kg] represents the material unit prices of ($i \in I$: laminated timber, impregnation and protection paint for the timber beam; concrete, reinforcing steel bars and formwork slab-panels for the concrete beam and structural steel, electrodes, gas consumption and anticorrosion-resistant paint for the steel beam). ϱ_i [kg/m^3] is the corresponding unit mass and V_i [m^3] is volume. C_{Lj} represents the hourly labor costs [€/h], t_j [h] are times required for ($j \in J$: impregnating and painting the timber beam; placing, curing and vibrating the concrete, cutting and placing the reinforcement, paneling the concrete beam, and cutting, welding and painting the steel beam), and f_O is an indirect cost factor for overheads (f_O = 2). For more detail about cost items used in the economic objective function, see (Jelušič, 2017) and (Kravanja, 2017).

$$\max P_E = N \cdot \left(C_S - C_{Mi} \cdot \varrho_i \cdot V_i - C_{Lj} \cdot t_j \cdot f_O \right) \quad (1)$$

The second criterion is the maximization of sustainability profit (P_{SUS} [€]), calculated for 300 beams as the sum of the economic profit and eco costs of global warming (EVR, 2018) caused by beam production. The objective function was determined for three materials separately; see Eq. (2). C_{GW} (€/kg CO_2 eq.) is the price of global warming of 0.116 €/kg CO_2 eq. (EVR, 2018), ϱ_k [kg/m^3] and V_k [m^3] are the corresponding unit masses and volumes, respectively, and f_{CFEFk} is a carbon footprint emission factor ($k \in K$; for the timber beam, reinforced concrete beam and for the steel beam). The carbon footprint emission factors used in the study are 0.69 kg CO_2 eq./kg for timber, 0.11 kg CO_2 eq./kg for concrete, 2.43 kg CO_2 eq./kg for reinforcing steel bars and 1.72 kg CO_2 eq./kg for steel.

$$\max P_{SUS} = P_E + N \cdot \left(-C_{GW} \cdot f_{CFEF\,k} \cdot \varrho_k \cdot V_k \right) \qquad (2)$$

Table 1: Results of the continuous beam optimizations.

Criterion		Laminated timber GL24h	Steel S 355	Reinforced Concrete C 50/60
	Economic profit (€)	**100,638**	**147,281**	**179,791**
1.*	b (cm)	36.0	14.9	30.0
	h (cm)	38.0	25.7	40.0
	Sustainability profit (€)	**86,303**	**135,623**	**141,164**
2.*	b (cm)	35.0	14.9	30.0
	h (cm)	39.0	25.7	40.0

* 1. Economic profit; 2. Sustainability profit

Table 1 displays the results of the optimization for three different materials and two different objective functions. The results show that concrete beams exhibit the highest economic profit and the highest sustainability profit, while laminated timber beams exhibit the worst results in all criteria.

4 Conclusion

Our analysis of the optimization of sustainability profit generated by the production of continuous beams, calculated using two different objective functions, economic profit and sustainability profit with a mixed-integer non-linear programming (MINLP) approach, clearly shows that reinforced concrete beams exhibit the highest economic profit and the highest sustainability profit.

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