

# MINLP Optimization of the Underground Lined Rock Cavern

STOJAN KRAVANJA & TOMAŽ ŽULA

**Abstract** The paper presents the cost optimization of a lined rock cavern (LRC), designed for an underground gas storage (UGS). The optimization was performed by the mixed-integer non-linear programming (MINLP) approach. GAMS/DICOPT was used. For this purpose, the MINLP optimization model was developed. The model comprised the cost objective function, which was subjected to geomechanical and design constraints. The rock mass strength stability and safety of the system were assured by these constraints. In the near past, the non-linear programming (NLP) optimization of a single gas cavern, of a whole underground gas storage and of a UGS in different rock environments was performed. Contrary to the mentioned NLP optimizations, where only the theoretical optimal results with continuous variables were obtained, in this paper the MINLP optimization of the LRC is proposed in order to handle the discrete alternatives explicitly. In this way, the solution obtained is a real optimal engineering solution with the calculated discrete values of different design parameters like cavern depth, diameter, height, wall thickness and inner gas pressure. A numerical example at the end of the paper shows the MINLP optimization of the investment costs of the lined rock cavern for the UGS in Senovo, Slovenia.

**Keywords:** • Lined rock cavern • Underground gas storage • Cost optimization • Mixed-integer non-linear programming • MINLP •

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## 1 Introduction

The paper deals with the cost optimization of a lined rock cavern (LRC), designed to be used for an underground gas storage (UGS). While the LRC is a high pressure gas reservoir, the UGS is normally planned with one to four LRCs. The LRC is typically designed in a cylindrical shape with a concrete wall and a steel lining. The concrete wall transports the gas pressure onto the neighbor rock, while the steel lining enables the impermeability, see also Stille and Sturk (1994), Sofregaz US Inc. (1999), Brandshaug et al. (2001), Chung et al. (2003) and Glamheden and Curtis (2006).

In the near past, the non-linear programming (NLP) optimization of a single gas cavern was performed by Kravanja and Žlender (2010), the optimization of a whole UGS was carried out by Žlender and Kravanja (2011), whilst the optimization of the UGS in different rock environments was calculated by Kravanja and Žlender (2012). In addition, analyses of UGS caverns with the adaptive network based fuzzy inference system (ANFIS) were done and reported by Žlender et al. (2012, 2013). In the mentioned NLP calculations, a simple economical objective function was defined to be subjected to geomechanical and design constraints. While the rock mass strength stability and safety of the system were assured by the geomechanical constraints, the relations between cavern dimensions, rock mass and gas pressure were defined by the design constraints. The geomechanical constraints assure that the strength of the rock mass is sufficient, the uplift of the rock above the cavern is prevented, the collapse of the rock between the caverns is prevented and that deformations of the concrete wall and steel lining is limited (large deformation or destruction of the steel lining is prevented). These constraints were derived from a series of the finite element method (FEM) analyses for the combinations between different alternatives of parameters: different gas pressures, cavern depths, cavern diameters, wall thicknesses and different load cases. While the FEM analyses were performed with the computer code Plaxis Version 3D, see Brinkgreve and Broere editors (2008), and Hoek et al. (2002); the NLP optimizations were carried out with the computer program GAMS/CONOPT2 (the general reduced gradient method), Drudd (1994).

Contrary to the above mentioned NLP optimizations, where only the theoretical optimal results with continuous variables (dimensions) were obtained, the mixed-

integer non-linear programming (MINLP) optimization of the LRC is proposed in this paper in order to handle the discrete alternatives explicitly. In this way, the solution obtained is a real optimal engineering solution with the calculated discrete values of different design parameters like the depth, diameter, height and the wall thickness of the cavern, and the inner gas pressure.

## 2 MINLP model formulation

The problem of the lined rock cavern is the non-linear, continuous and discrete optimization problem. For this reason, the MINLP is applied. The general MINLP optimization problem can be formulated as follows:

$$\begin{aligned} & \min z=f(\mathbf{x},\mathbf{y}) \\ & \text{subjected to: } g_k(\mathbf{x},\mathbf{y}) \leq 0 \quad k \in K \\ & \mathbf{x} \in X = \{\mathbf{x} \in R^n: \mathbf{x}^{LO} \leq \mathbf{x} \leq \mathbf{x}^{UP}\} \\ & \mathbf{y} \in Y = \{0,1\}^m \end{aligned} \quad (\text{MINLP})$$

where  $\mathbf{x}$  are the continuous variables and  $\mathbf{y}$  are the discrete (0, 1) variables. Non-linear function  $f(\mathbf{x}, \mathbf{y})$  is the objective function, which is subjected to non-linear (and linear) equality and inequality constraints  $g_k(\mathbf{x}, \mathbf{y})$ .

## 3 Handling discrete alternatives

The MINLP optimization model of the LRC is developed according to the above MINLP formulation. The optimization problem defined is simple, because the optimization of the LRC is proposed to be performed only for the draft phase. In the model, the following design variables ( $\mathbf{x}$ ) are defined: inner diameter of the cavern  $DCAV$  [m], depth of the cavern  $DEPTH$  [m], height of the cavern tube  $HCAV$  [m], thickness of the concrete cavern wall  $TWALL$  [m] and gas pressure  $PGAS$  [MPa], see Fig. 1.

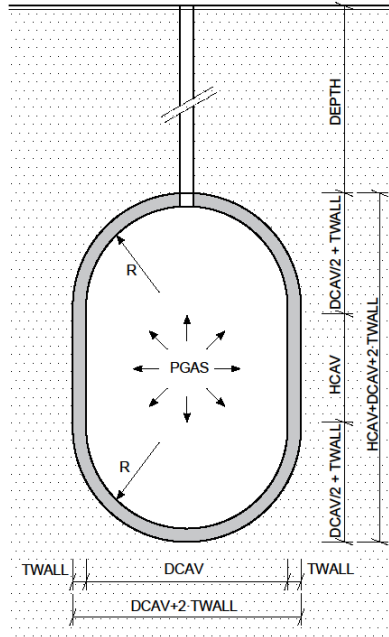


Figure 1: The vertical cross-section of the lined rock cavern

In order that the optimal solution will be a real one, all the above variables have to be rounded on whole discrete values. While the dimensions are proposed to be rounded on whole decimeters (dm), the gas pressure is rounded on one tenth of megapascal (0.1 MPa). For this purpose, a set of discrete value alternatives is defined to each variable, which is then rounded (calculated to be equal) to one of the discrete alternative during the optimization process. This calculation procedure leads to the simultaneous cost and rounded dimension type of the optimization.

The variables are bounded by their lower and upper bounds, see Eqs. (1), (4), (7), (10) and (13). Each variable is calculated as a scalar product between a vector of discrete value alternatives ( $q_{DCAV_i}$ ,  $q_{DEPTH_j}$ ,  $q_{HCAV_k}$ ,  $q_{TWALL_l}$ ,  $q_{PGAS_m}$ ) and a vector of binary variables ( $y_{DCAV_i}$ ,  $y_{DEPTH_j}$ ,  $y_{HCAV_k}$ ,  $y_{TWALL_l}$ ,  $y_{PGAS_m}$ ), see Eqs. (2), (5), (8), (11) and (14). One discrete value is then selected to each variable, because the sum of its binary variables is equal one, see Eqs. (3), (6), (9), (12) and (15).

Inner diameter of the cavern  $DCAV$  [m]:

$$DCAV^{LO} \leq DCAV \leq DCAV^{UP} \quad (1)$$

$$DCAV = \sum_{i \in I} q_{DCAV_i} \cdot y_{DCAV_i} \quad (2)$$

$$\sum_{i \in I} y_{DCAV_i} = 1 \quad (3)$$

Depth of the cavern  $DEPTH$  [m]:

$$DEPTH^{LO} \leq DEPTH \leq DEPTH^{UP} \quad (4)$$

$$DEPTH = \sum_{j \in J} q_{DEPTH_j} \cdot y_{DEPTH_j} \quad (5)$$

$$\sum_{j \in J} y_{DEPTH_j} = 1 \quad (6)$$

Height of the cavern tube  $HCAV$  [m]:

$$HCAV^{LO} \leq HCAV \leq HCAV^{UP} \quad (7)$$

$$HCAV = \sum_{k \in K} q_{HCAV_k} \cdot y_{HCAV_k} \quad (8)$$

$$\sum_{k \in K} y_{HCAV_k} = 1 \quad (9)$$

Thickness of the concrete cavern wall  $TWALL$  [m]:

$$TWALL^{LO} \leq TWALL \leq TWALL^{UP} \quad (10)$$

$$TWALL = \sum_{l \in L} q_{TWALL_l} \cdot y_{TWALL_l} \quad (11)$$

$$\sum_{l \in L} y_{TWALL_l} = 1 \quad (12)$$

Inner gas pressure  $PGAS$  [MPa]:

$$PGAS^{LO} \leq PGAS \leq PGAS^{UP} \quad (13)$$

$$PGAS = \sum_{m \in M} q_{PGAS_m} \cdot y_{PGAS_m} \quad (14)$$

$$\sum_{m \in M} y_{PGAS_m} = 1 \quad (15)$$

#### 4 Numerical example

The example shows the MINLP optimization of the investment costs of the lined rock cavern, designed for the UGS in Senovo, Slovenia. The project of the UGS in Senovo comprises four equal LRCs in order to store 5.56 million m<sup>3</sup> of natural gas each, see Žlender and Kravanja (2011).

**Table 1: Cost items and prices**

Cost item	Price
Upper ground works	2 982 500 EUR
Underground works	2 798 025 EUR
Price of the tunnel excavation	2 440 EUR/m <sup>1</sup>
Price of the tunnel protection	1 340 EUR/m <sup>1</sup>
Price of the cavern excavation	100 EUR/m <sup>3</sup>
Price of the cavern protection	90 EUR/m <sup>2</sup>
Price of the cavern drainage	60 EUR/m <sup>2</sup>
Price of the cavern wall concrete	190 EUR/m <sup>3</sup>
Price of the wall reinforcement	2 000 EUR/t
Price of the steel lining	920 EUR/m <sup>2</sup>

The optimization model is developed. Cost items and prices, defined in the objective function, are the same as they were used in the project and our mentioned previous research works (NLP optimizations), see Table 1. The optimization model includes the same constraints as in Kravanja and Žlender (2010). In order to handle discrete alternatives for variables, the model is extended with Eqs. (2), (3), (5), (6), (8), (9), (11), (12), (14) and (15). The model is modelled in GAMS (General Algebraic Modelling System) by Brooke et al. (1988).

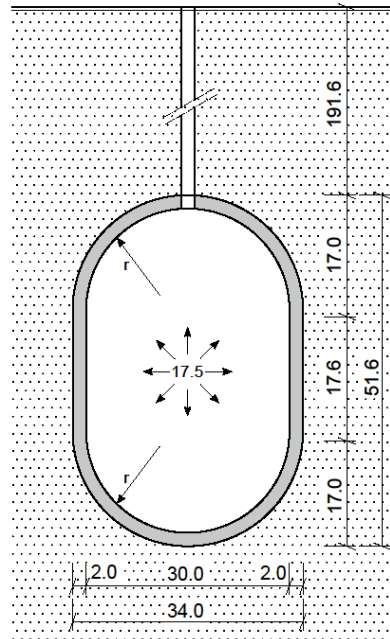


Figure 2: The optimized lined rock cavern

The LRC superstructure comprises 201 different rounded dimension alternatives for the inner diameter of the cavern, 2001 alternatives for the depth of the cavern, 301 alternatives for the height of the cavern tube, 31 alternatives for the thickness of the concrete cavern wall and 201 discrete alternatives for the inner gas pressure. 2735 binary variables are defined. In this way, the combination between the given dimension and gas pressure discrete alternatives gives  $7.54 \cdot 10^{11}$  different LRC structure alternatives. One of them is the optimal one.

For the solution of comprehensive MINLP optimizations of structures, we usually use computer program MIPSYN by Kravanja (2010). Because the non-linear and discrete MINLP problem in the paper is simple, i.e. it discusses the cost and rounded dimension optimization only, GAMS/DICOPT (Grossmann and Viswanathan, 2002) was selected for application.

The optimal result represents the obtained minimal investment costs of 18.22 million EUR per the lined rock cavern. All four LRCs of the UGS in Senovo thus reach 72.88 million EUR. Fig. 2 shows the vertical cross-section of the optimized lined rock cavern. In the figure, the calculated “optimal” variables (dimensions and the inner gas pressure) are shown. The optimal result exhibits

47.7 % of savings when compared to the design, obtained by the classical method (FEM).

## 5 Conclusions

The paper presents the cost optimization of a lined rock cavern (LRC), designed for an underground gas storage (UGS). The optimization was performed by the mixed-integer non-linear programming (MINLP) approach in order to handle discrete alternatives of dimensions and inner gas pressure explicitly. The MINLP optimization model of the structure was modelled and the computer program GAMS/DICOPT was used for the optimization. Advantages of the MINLP optimization approach are noticed. The calculated optimal result exhibits 47.7 % of net savings in investment costs when compared to the design, obtained by the classical method.

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## References

- Brandshaug, T., Christianson, M., Damjanac, B. (2001). Technical review of the lined rock cavern (LRC) - concept and design methodology: mechanical response of rock mass. US Department of Energy.
- Brinkgreve, R.B.J., Broere, W. editors (2008). PLAXIS 3D foundation, version 2. A.A. Balkema, Rotterdam, The Netherlands.
- Brooke, A., Kendrick, D., Meeraus, A. (1988). GAMS - A User's Guide. Scientific Press, Redwood City, CA.
- Chung, I.M., Cho, W., Heo, J.H. (2003). Stochastic hydraulic safety factor for gas containment in underground storage caverns. *J Hydrol*, 284, 77-91.
- Drudd, A.S. (1994). CONOPT - a large-scale GRG code. *ORSA J Comput*, 6(2), 207-16.
- Glamheden, R., Curtis, P. (2006). Excavation of a cavern for high-pressure storage of natural gas. *Tunnelling and Underground Space Technology*, 21(1), 56-67.
- Grossmann, I.E., Viswanathan, J. (2002). DICOPT - Discrete and Continuous Optimizer. Engineering Design Research Center (EDRC) at Carnegie Mellon University, Pittsburgh, PA.
- Hoek, E., Carranza-Torres, C.T., Corkum, B. (2002). Hoek-Brown failure criterion. In: Proc. North American rock mechanics society meeting in Toronto.
- Kravanja, S., Žlender, B. (2010). Optimal design of underground gas storage. In: WIT Transactions on the Built Environment, 112, 389-399. doi.org/10.2495/HPSM100361



- Kravanja, S., Žlender, B. (2012). Optimization of the underground gas storage in different rock environments. In: WIT Transactions on the Built Environment, 125, 15-26. doi.org/10.2495/OP120021
- Kravanja, Z. (2010). Challenges in sustainable integrated process synthesis and the capabilities of an MINLP process synthesizer MipSyn. *Comput Chem Eng*, 34(11), 1831-1848. doi:10.1016/j.compchemeng.2010.04.017
- Sofregaz US Inc. (1999). Commercial potential of natural gas storage in lined rock caverns (LRC). US Department of Energy, Federal Energy Technology Center.
- Stille, H., Sturk, R. (1994). Storage of gas in lined shallow rock caverns - conclusions based on results from the Grängesberg test plant. In: Rock mechanics in petroleum engineering, Delft, Netherlands.
- Žlender, B., Kravanja, S. (2011). Cost optimization of the underground gas storage. *Eng Struct*, 33, 2554-2562. doi.org/10.1016/j.engstruct.2011.05.001
- Žlender, B., Jelušič, P., Boumezerane, D. M. (2012). Planning geotechnical investigation using ANFIS. *Geotechnical and Geological Engineering*, 30(4), 975-989. doi:10.1007/s10706-012-9520-7.
- Žlender, B., Jelušič, P., Boumezerane, D. M. (2013). The feasibility analysis of underground gas storage caverns. *Eng Struct*, 55, 16-25. doi:10.1016/j.engstruct.2013.01.003.

